SAM and the Savings-Investment Balance

- A Social Accounting Matrix (SAM) provides the *data* for macromodels in a convenient, one-stop, format.
- It summarizes the *National Income and Product Accounts* (NIPA) in matrix form.
- SAMs are available for many countries and for some countries for several years, but are often elusive.[1]

0.1 Agents of the SAM

- SAMs are income and expenditure balances for economic agents; the agents are

  1. *Firms* firms combine *factors of production,* labor, land and capital, to produce *goods.*
  2. *Households* own the factors of production and receive income from firms in exchange for their use. Households spend this income on the very same goods produced by the firms, establishing a *circular flow of income.*
  3. *Government* taxes households, firms and foreign and buy goods and makes transfers to firms (subsidies) and households (transfers).
  4. *Foreigners* sell goods to households, firms and govt and buy goods from firms.

0.2 Balancing a SAM

- *Rows* of SAM contain sources *income*; *columns* are itemized *purchases.*
- Basic balance for each agent: *Income less expenditure is equal to savings.*
- *SAM is balanced* when savings is equal to income minus expenditure for all agents. *Walras law: when SAM is balanced, total savings equals total investment.*

0.3 Example: Simple Crusoe Economy

- *Firms* Friday and Crusoe form three firms to produce cocos, corn and fish. The factors are labor and a fish hook (capital owned by Friday)

  1. Intermediate goods: fish is used to fertilize corn. Corn is used as fish bait.

[1] The worldwide web can be a place to start looking, but there are no guarantees that a SAM for the appropriate country and year exists or can be found.

2. Some goods, like corn can be consumed or saved in the form of seed for future production (investment).
3. Firms pay all taxes.

- *Households* goods, fish, cocos and corn, have utility for Crusoe and Friday. Crusoe is the owns the firms; he hires Friday, pays him a wage and gives Friday a share of what they both produce.
- *Government:* musket retrieved from ship is basis of govt (defense). Patrol island one day per week. Food is provided by govt on that one day.
- *Foreign:* firm trades goods with neighboring island. Fish exchanged for salt.
- Labor theory of value: if it takes two hours to catch a fish and one to fell a coco, the opportunity cost of fish in terms of cocos is 2.
- SAM (all opp costs are 1)

	Firms			Cons					Total
	Coco	Corn	Fish	Fri	Cru	Inv	Govt	Exp	
Coco	0	0	0	2	3	0	1	1	7
Corn	0	0	1	5	8	10	2	0	26
Fish	0	2	0	3	3	0	2	2	12
Wage	2	5	5						12
Profit	4	17	4						25
Savings					10		0	0	10
Taxes	1	2	2						5
Imports				2	1				3
Total	7	26	12	12	25	10	5	3	

0.4 Points of Interest in the Crusoe SAM

- A SAM is in balance since *savings equals investment;* if savings is not equal to investment, then there is an error in one of the income-expenditure balances for the four agents.
- There is no govt savings (no surplus or deficit).
- There is no foreign savings (no trade surplus or deficit.)
- There are no direct taxes on indirect (all taxes paid by firms).
- Firms do not save.
- Crusoe is richer than Friday since all firms are profitable.
- Friday does not save; does not accumulate assets.
- Gross Domestic Product by Expenditure: Table 2.1 p. 23.

1. GDP = 42
2. Private consumption (C) = 27
3. Investment (I) = 10
4. Govt consumption (G) = 5
5. Net Exports (NX) = 0
   Exports = 3
   Imports = 3

- Gross Domestic Product by Industry (value added) Table 2.2 in Sachs and Larrain.

1. Coco = 7
2. Corn = 24
3. Fish = 11
4. Govt = 0 (no govt production or payments to households-could be included in firms)
5. $Y = 42$

- GDP by Incomes Table 2.3 in Sachs and Larrain.

1. wages 12
2. profits 25 (including firm saving)
3. net factor payments (workers hired from neighboring island)

- GDP v. GNP–now say that Friday is a "resident" of the island but not a citizen. Crusoe is the only citizen and doesn't work abroad.

1. GDP is still 42
2. GNP is GDP + net factor payments. GNP = 42-12 =30.

- Now let Crusoe earn some interest on an investment from abroad = 18. Here is how the SAM would be adjusted.

	Firms			Cons					Total
	Coco	Corn	Fish	Fri	Cru	Inv	Govt	Exp	
Coco	0	0	0	2	3	0	1	1	7
Corn	0	0	1	5	8	10	2	0	26
Fish	0	2	0	3	3	0	2	2	12
Wage	2	5	5						12
Profit	4	17	4					18	43
Savings					28		0	-18	10
Taxes	1	2	2						5
Imports				2	1				3
Total	7	26	12	12	43	10	5	3	

0.5 Why Savings is Equal to Investment for the Economy as a Whole

- Commutative Property for Addition:

$$a + b = b + a$$

When you add two numbers, order doesn't matter.

Example: $7 + 3 = 3 + 7$.

- This works for a matrix as well. Let the matrix have n rows and m columns. Define the *ith row sum,* R_i, and the jth column sum C_j. We have[2]:

$$\sum_{i=1}^{n} R_i = \sum_{j=1}^{m} C_j$$

that is *the sum of the row sums* is equal to the *sum of the column sums.*

- Example: Consider the $2x2$ (read: two-by-two) matrix.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The first row sum is $R_1 = a + b$; the second row sum is $R_2 = c + d$. The sum of the row sums $\sum_{i=1}^{n} R_i = a + b + c + d$. The first column sum is $C_1 = a + c$ the second is $C_2 = b + d$. The sum of the column sums is $\sum_{j=1}^{m} C_j = a+c+b+d$. Thus from the commutative property for addition above we have: $\sum_{i=1}^{2} R_i = \sum_{j=1}^{2} C_j$.

- Now SAMs have *four agents:* firms, households, government and foreign. To each agent there corresponds a *row* for income and *column* for expenditure. The *fifth* row and column of the SAM is for savings and investment respectively.

- Since for each agent, *income is balanced with expenditure*, the row and column sums are the same; that is: $R_i = C_i$ for i = 1,2,3,4.

- Now consider the SAM as a $5x5$ matrix with $R_i = C_i$ for i = 1,2,3,4. Since from the commutative property, we have $\sum_{i=1}^{5} R_i = \sum_{j=1}^{5} C_j$. The first four rows sums plus the sum of the savings row S is equal to the first four columns I, plus the sum of the investment column, I. We then have:

$$\sum_{i=1}^{4} R_i + S = \sum_{j=1}^{4} C_j + I$$

[2] Summation notation is defined by the $\sum_{i=1}^{n}$ which just says "sum what follows" according to the *index* i as it runs from 1 to n. The choice of letters for the index is usually i, j or k and has no significance other than if one letter has already been used in the expression, and we want to use a second $\sum$ then we choose a different index.

Since the sum of the first four rows must be the sum of the first four columns, we have:

$$S = I$$

The *sum of savings* equals the *sum of investment.*

- Note that inventories are the adjusting factor, or residual, in the rows of the SAM. Savings are is the residual in the columns.
- Example: Show that savings is equal to investment in a small economy in which there is no government and no foreign sector. The SAM is

	Firms	HH	Inv	Total
Firms		C	I	Y
HH	VA			Y_h
Sav	S_f	S_h		S
Total	Y	Y_h	I	

where C = consumption, I = investment, Y = GDP, Y_h = income of households; VA = value added. S_f = savings of firms; S_h = savings of households, S = total savings. Write the row and column sums (income equals expenditure) for firms and households :

$$\begin{aligned} C + I &= VA + S_f \\ VA &= C + S_h \end{aligned}$$

Now take the sum of the row sums on the left and the sum of the column sums on the right

$$C + I + VA = VA + S_f + C + S_h$$

and cancel like terms on both sides.

$$I = S_f + S_h$$

Investment is equal to the sum of savings if each individual agent is in income-expenditure balance. This is called *Walras' Law.*