Investment in Human Capital in a Macrodynamic Framework: Redistributive Taxation, Public Debt and Welfare

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Abstract

Efficient investment in human capital is a subject of great concern among economists. By means of an overlapping-generations macrodynamic model with credit constraints, imperfect insurance and exogenous labor supply, we appraise inefficiencies related to misinvestment in human capital and propose a simple scheme of redistributive taxation to mitigate them. A numerical simulation is calibrated in order to match stylized facts of the quite unequal Brazilian economy and shows that, in steady-state, with a flat-tax mechanism and lump-sum transfers, government intervention is beneficial to the extent it maximizes our utilitarian measure of welfare and reduces both inefficiency associated with misdirected investment in human capital and standard inequality indexes. After considering the possibility of decomposing our utilitarian measure of welfare and of allowing for public debt, we show that reduced inequality is the driving force which accounts for welfare improvement and that public debt plays no role. Robustness analysis shows that endogenizing labor supply does not lead to substantial changes in previous results.

Key Words: investment in human capital; idiosyncratic risk; efficiency; overaccumulation of capital; redistributive taxation; public debt; welfare measures; social insurance.

JEL Classification Numbers: D1, D31, D52, H21, H63, J24.

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1 Introduction

Charles Dickens, the famous and skillful 19th century English writer, living in an impoverished family, was sent to work at the age of twelve in 1824 and prevented from attending a school for about one year. Judging by the outstanding quality of his published works as a writer, and despite the well documented fact that much of a person's cognitive ability is developed early in life (Heckman, 1999), Dickens' productivity in his later craft was not affected by such short fate of his. But low-income people frequently do not have the means for providing an adequate intellectual support for their children, and, given a poor assistance by the public sector in many developing countries nowadays (World Bank, 2004), a large fraction of the latter, including the ablest ones, surely become unskilled workers when they grow up. Especially in the age of a skill-biased technology, it is crucial to efficiently provide the means for a proper intellectual development for each one in its due time.

The purpose of this paper is to assess, in a macrodynamic framework, the possible sources of inefficiency in the investment in human capital of people early in life and to investigate to what extent there is a role for redistributive public policies. In particular, we argue, following Aiyagari, Greenwood and Seshadri (2002), that the equilibrium of a decentralized economy is not efficient: much of the investment in human capital is misdirected in spite of an overinvestment phenomenon. But, extending the model so as to allow for a public sector, we show that a simple scheme of uniform taxation mitigates the problem by the choice of an appropriate level of the tax rate. Moreover, decomposing the utilitarian welfare gains in three components (gains from reduced inequality, lower uncertainty and increasing levels of consumption), we find that reduced inequality explains the most of welfare improvement.

The majority of related theoretical studies are built on the pathbreaking works by Gary S. Becker and Theodore W. Schultz in the 1960s. Their analysis has drawn our attention to an important type of investment that was being overlooked then: investment in the human capital of people.[1] Such idea is construed as investment of resources (physical or monetary) in one's productive capacity, and is often identified with the concept of education itself. Surely, it has intrinsic characteristics diverse from the ones associated with investment in physical capital. Particularly, it is frequently argued that human capital is not a good collateral for loans, implying that, for example, a father is not seldom borrowing constrained when investing in the human capital of his children.

[1] See Lewis (1988) for a slightly different historical claim about the origins of the human capital concept.

Recent research has investigated to what degree the presence of credit constraints, endogenous or exogenous, affects the equilibrium allocation in a decentralized system and, depending upon the degree of inefficiency achieved, whether there is any role for government intervention. A similar line of reasoning is followed, for example, by Lochner and Monge-Naranjo (2003) and Hanushek, Leung and Yilmaz (2002).[2]

The former work has constructed a dynamic general equilibrium overlapping-generations model with one-sided altruism. In this environment, agents have the choice of investing in the human capital of their children, and the more they invest, the higher are the latter's labor earnings. When credit constraints are not present, the equilibrium allocation in complete markets will be efficient provided every individual is fully committed to repay his debts. On the other hand, the presence of endogenous or exogenous borrowing constraints makes investment in human capital to depend on the father's total wealth, and not only on the child's ability. Therefore, this fact gives rise to misinvestment, in the sense that many children who receive investment in their human capital do not deserve it most. Numerical simulations in a framework with endogenous credit constraints are parametrized in order to replicate key features of US data and evaluate the steady-state impacts mainly in terms of welfare of changes in government policies as educational subsidies and bankruptcy policy.

Hanushek, Leung and Yilmaz's (2002) basic framework differs from Lochner and Monge-Naranjo's in the number of periods a typical agent lives and in the kind of borrowing constraints considered: the latter are mainly concerned with endogenous credit constraints, whereas the former assume an environment with exogenous credit constraints. Hanushek, Leung and Yilmaz's analysis evaluates the steady-state impact on several variables - e.g., welfare, number of agents constrained and college successfulness - of competing government policies as education subsidies and income-contingent loans. One important point is emphasized at the same time: the need for a complete general equilibrium set-up to fully appraise the economic impacts of public policies aimed at fostering human capital accumulation. Although several studies, chiefly on the micro level, still rely on a partial equilibrium framework, some macrodynamic models like the preceding ones provide an approach which seems to be much more general and its premises are more realistic.

The approach developed here contains elements from both previously quoted papers: exogenous credit constraints and general equilibrium analysis. As it has been already stated,

[2] In fact, many studies have endeavored in a macrodynamic analysis of investment in human capital with market imperfections. Loury (1981) and Laitner (1992) are well-known early examples.

we modify the basic set-up of the Aiyagari, Greenwood and Seshadri (2002) - hereafter AGS - framework to answer this simple question: is there any role for public policies in a simple macrodynamic overlapping-generations model with investment in human capital, heterogeneous agents and incomplete markets?

This is indeed an important question not only in the previously quoted models but also in the AGS framework, which has a distinguishing feature: overaccumulation of human and physical capital. When individuals are subject to idiosyncratic shocks - for example, to changing ability levels - in an incomplete markets world (that is, with liquidity constraints and absence of perfect insurance) they overaccumulate capital to create a buffer of assets against negative income shocks in future. Such fact leads to an aggregate level of capital higher than what would be otherwise efficient and, hence, the equilibrium interest rate is lower than the one implied by time preference. Aiyagari (1995) has shown that, in such environment, it is generally true that the optimal long-run level of capital income taxation is strictly positive. By means of numerical simulations, calibrated to match some features of the Brazilian economy, we show this is true as well for our model when we consider only labor income taxation.

In a framework without human capital, a few studies (e.g., Aiyagari and McGrattan (1998), Flodén (2001) and Flodén and Lindé (2001)) have applied this idea to evaluate the optimal size of government debt to GDP level and transfers. Initially, we simply suppose there is no public debt at all and allow for a labor income taxation, but we extend the analysis by taking into account investment in human capital and showing that standard inequality statistics for income and wealth are improved when government imposes an optimal welfare improving tax rate level. Indeed, the link among market imperfections, investment in human capital and inequalities is a topic of much concern in Brazil, at least since the 1970s with an ensuing debate spurred by Carlos Langoni, for whom Brazilian measures of income inequality worsened due to increasing asymmetries in educational level of the population (Langoni, 1973). In fact, we show that this problem can be mitigated somewhat with government intervention, especially with respect to the Gini index and the correlation of labor income across generations.

The taxation scheme we suggest is closely related to the literature on redistributive taxation (Eaton and Rosen (1980), Varian (1980), among others), which argues that, when individuals are heterogenous with respect to their labor income as in the set-up we develop in this paper, a proportional earnings tax acts as social insurance by improving risk sharing in the economy. Indeed, if the government keeps a balanced budget each period with lump-sum transfers, a redistributive effect takes place, because the disposable income of poor agents is increased, whereas an opposite effect occurs with the rich. Moreover, if the objective function of the typical agent is increasing and concave in income, he is clearly better off with such redistributive mechanism.

At the same time, it has been noticed that public debt is to some degree beneficial in liquidity constrained economies, since it increases the liquidity level (Woodford, 1990). Even though our main concern is with taxation, this last claim is also assessed in this paper by allowing public sector not only to levy taxes, but to accumulate debt. The results on debt are that the coefficient of variation of total consumption is increased and overall welfare is lower, contrary to what was expected.

In order to completely gauge the predictive power of the model, we ask the following question: Can the theoretical structure generate Brazilian-like statistics if we preserve the parameterization for the US economy provided by AGS but introduce governmental distortions? To this end, we suppose government undertakes very distortional public policies in the sense of transferring resources from society to top 10% richer individuals. Even though the results suggest inequality statistics do not exactly fit Brazilian data, the model explains a little the unequal environment.

We proceed as follows. In the next section the basic framework is presented carefully and extended to include a public sector through uniform taxation and lump-sum transfers. Next (section 3), we calibrate the model and simulate it both with and without government. The main results are surveyed in section 4, jointly with a discussion of what has been learnt and the limitations of our approach. Section 5 provides further thoughts on welfare decomposition and evaluates previous results on welfare under this perspective. In section 6 we extend the set-up so as to allow for public debt. In section 7 we consider a modified framework with endogenous labor supply and show the new results. Section 8 carries out an appraisal of the predictive power of the model by looking at other way round as we mentioned in the previous paragraph. The conclusion sets forth some ideas for further research and summarizes the findings up to this point.

2 Framework

The AGS model builds on a general set-up originally conceived by S. Rao Aiyagari (Aiyagari and Greenwood, undated). Aiyagari's idea intended to explain the mechanics of human capital investment in a world with borrowing constraints and imperfect insurance. His first step was to characterize the efficient allocation without mentioning preferences, only describing the economy's basic structure. This is also the approach in AGS, which we follow in this paper.

Simply put, the economy's structure is a standard overlapping-generations model with one-sided altruism where agents live for three periods. Each person lives as a child, a young adult (y) and an old adult (o). At each point in time, the three generations are living together and the mass of young and old agents is each one normalized to 1.

The choice set-up is as follows. When a child, an agent does not make any choice: he is merely apt to receive investment in his human capital. There are two channels to increase the human capital of a child: direct investment of physical resources, labeled m, and spending labor resources in child-care, n. Investment via m implies in a fixed cost ϕ. We assume the existence of a competitive market for providing child-care resources.[3] A young individual's choices are standard: how much to consume, to save and to invest (if any investment is made at all) in his offspring. Lastly, an old agent chooses his level of consumption and bequests to be inherited by his son.

There is only one good in this economy ($\mathbf{o}$), whose production obeys a neoclassical production function: $\mathbf{o} = O(\mathbf{k}, \mathbf{l})$, where $\mathbf{k}$ is the aggregate level of capital and $\mathbf{l}$ represents total labor supply in efficient units, O is strictly increasing in its arguments and strictly concave.

A young agent's labor supply is between goods production and child-care, that is, every person chooses whether to be a "hard" worker or a "baby-sitter". The total amount of efficient units of labor offered by the young (π) is a predetermined variable in the following sense: $\pi = H(a_{-1}, m_{-1}, n_{-1})$, where a stands for the young's ability and the subscript in a_{-1} reflects the fact that the son's ability is known to his father.[4] If π' stands for productivity

[3] AGS, in the last section of the their paper, investigate whether the absence of such market implies any kind of inefficiency. They conclude it does so, but we will not replicate this experiment here: we assume there is not a problem of lack of child-care facilities in our world.

[4] See Levhari and Weiss (1974) for an early use of a similar assumption about investment in human capital, ability and future labor income.

next period, the function H is said to satisfy: (i) $H(a,0,0) = a \leq \pi'$; (ii) $H(a,m,0) = H(a,0,n) = a$; (iii) $H_i > 0$, $i = 1,2,3$; (iv) H_{12}, H_{13} and $H_{23} > 0$; (v) H is strictly concave in its arguments, jointly and separately. It is also assumed that an agent with productivity π and ability a_{-1} working in the child-care sector supplies in efficient units only a_{-1} whatever is his actual productivity level. This, jointly with (i), implies the existence of comparative advantage for skilled agents ($m, n > 0$) over unskilled (n and/or $m = 0$) on working in the production of goods.

In this economy, it must be noticed that agents are heterogenous at least in one sense: they do not have the same level of ability. To this kind of heterogeneity it is given a well-behaved dynamic structure: we assume that $(a_t)_t$ follows a stationary Markov process with transition function $A_1(a|a_{-1})$.

Finally, capital accumulation and resources constraints are given, respectively, by $\mathbf{k}' = (1-\delta)\mathbf{k}+\mathbf{i}$ and $\mathbf{c}+\mathbf{m}+\mathbf{i} = \mathbf{o}$. Here, δ is the geometric depreciation rate, assumed constant; $\mathbf{c}$ and $\mathbf{i}$ stand as usual for aggregate consumption and physical investment; $\mathbf{m}$ is the aggregate level of investment in human capital through m, and takes into account the fixed cost ϕ.

2.1 Efficient Allocation

How does an efficient allocation look like in this environment? Possible sources of inefficiency are certainly the idiosyncratic shocks, against which every agent would like to insure himself, and liquidity constraints, in the sense of non-negative bequests and total wealth. In complete markets, these problems are ruled out by definition, and we assume they are for a moment.

Each young parent derives utility from present (c^y) and future consumption ($c^{o\prime}$), and also from his son's lifetime expected utility (V'). Thus, his objective function is: $U(c^y) + \beta E\left[U(c^{o\prime}) + \theta V'\right]$, where U is strictly increasing and strictly concave, $U_1(0) = \infty$, $\beta \in (0,1)$ is a intertemporal discount factor and $\theta \in (0,1]$ represents the degree of altruism towards his son. The dynamic programming problem facing a young agent living in a world with complete markets is:

$$
\begin{aligned}
V(\pi, a, b) = \max_{c^y, m, n, s(a'|a)} &\{U(c^y) + \beta \int J(\pi', a', b + s(a'|a)) A_1(a'|a) da'\} \\
s.t.\ c^y + m + \phi I(\pi', a) + wn + &\int q(a'|a) s(a'|a) da' = wn \\
\pi' &= H(a, m, n) \\
J(\pi', a', b + s(a'|a)) &= \max_{c^{o\prime}, b'} \{U(c^{o\prime}) + \theta V(\pi', a', b')\} \\
s.t.\ c^{o\prime} + \frac{b'}{1+r} &= s(a'|a) + b,
\end{aligned}
\tag{CP1}
$$

where $I(\pi', a) = \begin{cases} 1, \text{ if } \pi' > a \\ 0, \text{ otherwise} \end{cases}$, $s(a'|a)$ is contingent claim to the amount s in case of occurring state a' next period given the current ability state a, $q(a'|a)$ represents its price, b is the bequest received by the old from his parent, and b' denotes the amount of resources (or debts) the old leaves to his son.

If we assume an actuarially fair price for the contingent claims, $q(a'|a) = \frac{A_1}{1+r}$, and a perfect-pooled steady-state (Lucas, 1982)[5], then first-order necessary conditions, jointly with Benveniste-Scheinkman and envelope theorems, imply (AGS, p.299-301):

$$
\begin{aligned}
(1+r)\beta &= \frac{1}{\theta} && (1) \\
U_1(c^y) &= (1+r)\beta U_1(c^{o\prime}) && (2) \\
1 + r &= w H_2(a, m, n), \text{ when } m > 0 && (3) \\
1 + r &= H_3(a, m, n), \text{ when } n > 0 && (4) \\
wn + m + \phi &< \frac{w[H(a, m, n) - a]}{1+r}, \text{ for } m, n > 0 && (5) \\
wn + m + \phi &\geq \frac{w[H(a, m, n) - a]}{1+r}, \text{ for } m, n = 0. && (6)
\end{aligned}
$$

When $\theta = 1$ (pure altruism), equation (1) yields $1 + r = \frac{1}{\beta}$, the standard result for the neoclassical growth model; (2) is a usual Euler equation for a simple two-period economy. (3) and (4) stand for the non-arbitrage conditions which an equilibrium allocation must obey. The derivation of equations (5) and (6) are not straightforward (AGS, p. 301), but their economic interpretation is very simple. Initially notice that a typical agent in our economy

[5]Every young and every old individual consume the same amount: c^y and $c^{o'}$ respectively.

will invest in the human capital of his children only if m and n are both strictly greater than zero: it will never be the case of $m = 0$ and $n > 0$, or $n = 0$ and $m > 0$. This occurs because, for example, $H(a, 0, n) = a = H(a, 0, 0)$: since the same outcome in terms of future productivity of the offspring would be achieved if no investment is made at all, to make no investment will afford the father to consume more presently. But when are $m, n > 0$? Investment in children occurs only if current costs of becoming skilled $(wn + m + \phi)$ falls below the present value of the skill-premium $(\frac{w[H(a,m,n)-a]}{1+r})$. As long as there are two forms to invest in the human capital of the offspring to enhance their skills, an efficiency condition always holds in this case:

$$wH_2(a, m, n) = H_3(a, m, n). \tag{7}$$

There is a distinguishing feature displayed by the efficient allocation: (5) and (6) imply decision rules $M(a)$ and $N(a)$ as well as a threshold, a^*, for the level of ability such that $M(a), N(a) > 0$ if $a > a^*$ and $M(a), N(a) = 0$ otherwise. As we shall stress below (sections 2.2 and 4), this condition is not necessarily valid in an incomplete markets environment, giving rise to misinvestment.

An efficient allocation in a stationary equilibrium is therefore described by equations (1) to (6), standard profit maximization conditions for a competitive firm,

$$r + \delta = O_1(\mathbf{k}, \mathbf{l}) \tag{8}$$

$$w = O_2(\mathbf{k}, \mathbf{l}), \tag{9}$$

and the constraints

$$\mathbf{l} = \int \pi d\mathbf{A}(a) - \int_{\mathcal{S}} N(a) d\mathbf{A}(a) \tag{10}$$

$$\int_{\mathcal{S}} N(a) d\mathbf{A}(a) \leq \int_{\mathcal{U}_{-1}} a d\mathbf{A}(a) \tag{11}$$

$$\mathbf{c} + \mathbf{i} + \int_{\mathcal{S}} [M(a) + \phi] d\mathbf{A}(a) \leq O(\mathbf{k}, \mathbf{l}), \tag{12}$$

where $\mathcal{S} = \{a : H(a, M(a), N(a)) > a\}$ is the set of skilled agents in steady-state, $\mathcal{U} = \mathcal{S}^{\,c}$ and $\mathbf{A}(a)$ is the stationary distribution associated with the Markov process for ability. Condition (10) is the labor supply for goods production and (11) represents market-clearing in chid-care market. Also notice that, given the definition of $\mathcal{S}$, $\int N(a) d\mathbf{A}(a) = \int_{\mathcal{S}} N(a) d\mathbf{A}(a) +$

$\int_{\mathcal{U}} N(a)d\mathbf{A}(a) = \int_{\mathcal{S}} N(a)d\mathbf{A}(a)$, since $N(a) = 0$ for all $a \in \mathcal{U}$ (an analogous property holds for $M(a)$).

We end the discussion of the efficient allocation with two observations:

1. Only unskilled agents will work in the child-care sector: no skilled individual will rationally reject the skill premium his ~~father's~~ choice ~~imparts~~ him. But it is not true to say that only skilled agents work in the goods production sector: (11) does not necessarily ~~binds~~.

2. With pure altruism, which will be the case considered for numerical simulations later on, $1 + r = \frac{1}{\beta}$ holds. Since AGS framework has the flavor of Aiyagari's 1994 influential paper, it seems that in incomplete markets we shall have an interest rate below the one implied by time preference as far as overaccumulation of capital is also present.

2.2 Incomplete Markets Structure with Government

Suppose now that bequests are required to be non-negative and there is no private insurance market, so that we have an incomplete markets structure. Moreover, there is a government which taxes young people's income uniformly by a tax rate $\tau \geq 0$ and returns them the proceeds via identical lump-sum transfers (χ). In this setting with a time-invariant government policy rule and assuming a stationary environment, the dynamic programming problem facing a young agent is:

$$\begin{aligned} V(\pi, a, b) &= \max_{c^y, m, n, s \geq -b} \{U(c^y) + \beta \int J(\pi', a', b+s) A_1(a'|a) da'\} \\ s.t.\ c^y + m + \phi I(\pi', a) + wn + \frac{s}{1+r} &= (1-\tau) w\pi + \chi \\ \pi' &= H(a, m, n) \\ J(\pi', a', b+s) &= \max_{c^{o\prime}, b' \geq 0} \{U(c^{o\prime}) + \theta V(\pi', a', b')\} \\ s.t.\ c^{o\prime} + \frac{b'}{1+r} &= s + b. \end{aligned} \tag{CP2}$$

A recursive stationary competitive equilibrium is defined by:

(i) Decision rules for s, m, n and b, respectively $s = S(\pi, a, b)$, $m = M(\pi, a, b)$, $n = N(\pi, a, b)$ and $b' = B(\pi', a', s + b)$, such that, given r, w, χ and τ, they solve (CP2).

(ii) Invariant distributions, D^y and D^o, which satisfy:

$$\begin{aligned} D^y(\pi', a', b') &= \int T^y(\pi', a', b'|\pi, a, b) dD^y(\pi, a, b) \qquad (13) \\ D^o(\pi', a', s+b) &= \int T^o(\pi', a', s+b|\pi, a, b) dD^y(\pi, a, b), \qquad (14) \end{aligned}$$

where T^y and T^o stand for transition functions induced by $\mathbf{A}(a'|a)$ and decision rules in (i).

(iii) Real numbers w and r such that: $r = O_1(\mathbf{k}, \mathbf{l}) - \delta$, $w = O_2(\mathbf{k}, \mathbf{l})$, $\mathbf{k} = E\left(\frac{s}{1+r} + \frac{b}{1+r}\right)$ and $\mathbf{l} = E(\pi) - E(n)$.

(iv) Government keeps a balanced budget:

$$\chi = \tau \int w\pi dD^y(\pi, a, b). \qquad (15)$$

AGS show that, when government is not present ($\tau = 0$), the equilibrium in the incomplete markets world is such that there is overinvestment in human capital of children, which is misdirected, in the sense that a fraction of low-ability agents become skilled whereas there is also a non-null mass of high-ability agents unskilled. Overaccumulation of physical capital also emerges because of the idiosyncratic shocks and the absence of perfect insurance: in this context, the agent accumulates more assets than it would be optimal in complete markets to make a buffer so as to (imperfectly) insure himself against negative shocks. This fact drives interest rate down to a level below $\frac{1}{\beta} - 1$. The investment in human capital is misdirected due to credit constraints: no father is allowed to desave more than $|b|$ or leave negative bequests. Also notice that, now, m and n are functions not only of a but also of π and b: poor agents with high-ability children will not invest enough in their offspring since they lack resources to do so.

2.3 Taxation and Welfare

Now we pose three questions: What features does a recursive stationary competitive equilibrium display when $\tau > 0$? Is there any "optimal" (in the sense of maximizing a given measure of social welfare) level $\tau^* > 0$ which helps to alleviate the inefficiency associated with the incomplete markets structure? Can government intervention, when comparing steady-states, be beneficial to individuals?

Our measure of welfare, as it is usual in literature, will be the compensating variation in consumption.[6] Suppose that government intervention is indeed beneficial in terms of welfare and let $\lambda \geq 0$ be the constant fraction of consumption arising in incomplete markets when $\tau = 0$ that equalizes agents mean utility in this world and its expected utility when $\tau > 0$ (EV_τ). That is, we want to find λ, our measure of welfare, satisfying:

$$\int E_0 \sum_{t=0}^{\infty} (\beta\theta)^t \left[U\left((1+\lambda)c(i)_t^y\right) + \beta U\left((1+\lambda)c(i)_{t+1}^o\right)\right] dF(i) = EV_\tau, \tag{16}$$

where $(c(i)_t^y, c(i)_{t+1}^o)_t$ is a (possibly uncertain) consumption sequence which solves consumer's i problem in incomplete markets with $\tau = 0$, and F is the distribution function of agents at $t = 0$. Without loss of generality, we may suppose that F simply corresponds to the invariant distribution D^y and that each agent is indexed by $i \in \Omega \subset \mathbb{R}$ (Ω is not necessarily a countable or denumerable set) in the place of its state-vector when young, (π, a, b). With such notation, the average of agents' value functions is:

$$EV = \int E_0 \sum_{t=0}^{\infty} (\beta\theta)^t \left[U\left(c(i)_t^y\right) + \beta U\left(c(i)_{t+1}^o\right)\right] dF(i). \tag{17}$$

Notice that, the higher is λ, for a given $\tau > 0$, the stronger will be an agent's willingness to live in an incomplete markets world with government instead of an incomplete markets world without a public sector. If we set $U(c) = \frac{c^{1-\mu}-1}{1-\mu}$, it is easy to show that (16) and (17) imply:

$$\lambda = \lambda(\tau) = \left[\frac{EV_\tau + \frac{1+\beta}{(1-\mu)(1-\beta\theta)}}{EV + \frac{1+\beta}{(1-\mu)(1-\beta\theta)}} \right]^{\frac{1}{1-\mu}} - 1.^{7} \tag{18}$$

Our experiment to answer the above questions consists in making a search in $\tau \in [0, 1]$ to check whether we can pick some τ^* such that $\lambda(\tau^*) \geq \lambda(\tau)$, $\forall \tau \in [0, 1]$, which would maximize our welfare criterion. Since the problem described in (CP2) does not have a closed-form solution, we simulate it for a given set of parameters properly chosen in order to match some features of Brazilian data. The calibration and simulation procedures undertaken are described in the next section.

[6] See Imrohoroglu (1989) for an early use of such measure in a macrodynamic model with heterogenous agents.

[7] Appendix B.

3 Calibration and Simulation Procedures

Time unity was set to 20 years, and $U(c) = \frac{c^{1-\mu}-1}{1-\mu}$. We pick $\mu = 2$, which is contained in the interval analyzed in Cunha and Ferreira (2003) for Brazil, and $\beta_{year} = 0.88 = \beta^{\frac{1}{20}}$ was selected so that incomplete markets economy without government has an equilibrium interest rate near 9% observed in Brazil. Only the case of pure altruism is considered: $\theta = 1$. The production function has the specification as AGS for the US economy, except for a lower TFP: $\mathbf{o} = O(\mathbf{k}, \mathbf{l}) = z\mathbf{k}^{\nu}\mathbf{l}^{1-\nu}$, where $z = 0.7$ and $\nu = 0.36$. Capital depreciates at a rate of 6.5% each year (Val and Ferreira, 2001), implying $\delta = 1 - (1 - 0.065)^{20} \cong 0.74$.[8]

The stochastic process driving ability satisfies $\log a' = \iota(1-\omega) + \omega \log a + \sigma\sqrt{1-\omega^2}\zeta$, $\zeta \sim N(0,1)$. After Tauchen (1986) we know this process is amenable to an approximation by a discrete Markov process, which will be useful in simulations. Productivity function is $H(a,m,n) = \eta a^{\chi}[\xi n^{\varepsilon} + (1-\xi)m^{\varepsilon}]^{\frac{\rho}{\varepsilon}} + a$, $\varepsilon \leq 1$, as in AGS.[9] The fixed cost associated with m is arbitrarily selected: $\phi = 0.01$. ι, ω, σ, χ, η, ξ, ε and ρ are chosen so that the incomplete markets economy with $\tau = 0$ replicates income Gini index in Brazil (0.6). Table 1 summarizes the model's calibration.

Remark 1 *Certainly the above parameterization does not support any exhaustive matching of the Brazilian economy. In this being the case, we claim that our economy may be construed as a rough approximation to a quite unequal (with respect to income distribution) society, as the targeted income Gini suggests. Furthermore, it may also be interpreted as a developing economy, since it displays a lower TFP vis-à-vis United States and a higher interest rate.*

Given its characterization in section 2.1, it is not a difficult task to find the equilibrium allocation in complete markets numerically relatively to the problem (CP2). To handle the incomplete markets dynamic programming problem, several methods are available nowadays.[10,11] Despite the large number of state variables in our problem - they are indeed four:

[8] We are aware of Cooley's (1997) advises regarding calibrating a model by referring to prior studies. On the other hand, given that macroeconomic models with heterogenous agents represent a recent area of research in Brazil, we consider it is still fair to proceed as we did.

[9] This specification does not satisfy $H(a,m,0) = H(a,0,n) = a$. But it is easily checked that, in equilibrium, given the efficient allocation condition (7) that holds both in complete and incomplete markets, it will never be the case that $m > 0$ and $n = 0$, or $n > 0$ and $m = 0$.

[10] See Marimon and Scott (1999) for an introductory presentation of most of them, and Rust (1996) for a survey.

[11] The basic computation of consumer's problem is based on a C++ code written for the AGS original

π, a, b and $s+b$ -, the state-space discretization approach worked well. We construct grids for the state variables and solve the agent's problem for points inside them. The discrete dynamic programming is:

$$V(\pi_i, a_j, b_k) = \max_{v \in \bar{\mathcal{S}}, \pi' \in \mathcal{P}} \left\{ U\left((1-\tau) w \pi_i + \chi + \frac{b_k}{1+r} - C(a_j, \pi'; w) - \frac{v}{1+r} \right) + \right.$$
$$\left. + \beta \sum_{l=1}^{15} A_{jl} J(\pi', a_i, v) \right\} \tag{19}$$

$$J(\pi_i, a_j, v_k) = \max_{b' \in \mathcal{B}} \left\{ U\left(v_k - \frac{b'}{1+r} \right) + \theta V(\pi_i, a_j, b') \right\}, \tag{20}$$

where: $C(a, \pi'; w) = \begin{cases} \min_{m,n} \{m + \phi + wn : \pi' = H(a, m, n)\} & \text{, if } \pi' > a \\ 0 & \text{, if } \pi' = a \end{cases}.$

The discrete sets above have some properties, which we list:

1. Ability $a \in \mathcal{A} = \{a_1, \ldots, a_{15}\}$.
2. Productivity $\pi \in \mathcal{P} = \{\pi_1, \ldots, \pi_{100}\}$, and $\mathcal{P}$ satisfies $\mathcal{W} \cup \mathcal{A} \subset \mathcal{P}$, where $\mathcal{W}$ stands for the set of productivities arising in efficient markets.
3. $v = s + b \in \bar{\mathcal{S}} = \{v_1, \ldots, v_{125}\}$.
4. Bequest $b \in \mathcal{B} = \{b_1, \ldots, b_{125}\}$.

To solve the consumer's problem, we want to find simultaneously two fixed points, one for each value function. As long as one value function depends on the order, we proceed as AGS suggest, given wage, interest rate, tax rate and lump-sum transfer: 1st) Enter an initial guess for V, denoted V^j, and solve (20) for J, represented by J^j; 2nd) Given J^j compute a revised guess, V^{j+1}, for V solving (19); 3rd) Repeat the process until convergence is obtained in V and J. Although the consumer's problem loop is easily solved this way, it remains to find the equilibrium r, w and χ. This is accomplished by initial guesses for them and, given (8) to (10) and (13) to (15), subsequent updates of their values after each consumer's problem loop. Lastly, we find the invariant distributions defined in equations (13) and (14). To do

framework by its authors, and we are indebted to them. However, any misuse or misinterpretation of their code is ours.

this, we use Monte Carlo simulation, which is a powerful instrument to compute invariant distributions in stochastic dynamic programming problems.[12]

4 Results

4.1 Incomplete Markets without Government

Interest rate and income Gini index found by the simulations are 7% and 0.61 respectively in incomplete markets, numbers very close to Brazilian data. Although income Gini index generated by the model is indeed near empirical estimates for family per capita income Gini index, 0.6 in 1999 (see Menezes-Filho, 2001), the interest rate is rather on the low side. A group of extra simulations was unsuccessfully run to overcome this result: even after giving up to calibrate β to match both r and $\frac{\text{Investment}}{\text{GDP}}$ as AGS do, the model does not generate a more plausible interest rate to Brazilian data if annual time discount factor were not set to very low - and irrealistic - levels.

As it is well documented in AGS, the number of skilled agents increases with incomplete markets due to an overinvestment phenomenon: here they represent 60% in complete and 76% in incomplete markets. The efficient allocation is characterized by a cut-off rule for ability, $a^* = 2.25$, such that everyone with a lower level of ability ($a \leq a^*$) does not become skilled. In incomplete markets, 50% of agents with $a \leq 2.25$ receive positive amounts of investment in human capital, suggesting that a large fraction of such investment is misdirected. Total physical capital stock is 470% higher in incomplete markets, characterizing overaccumulation. Investment in human capital stock measured by $\mathbf{m}$ rises 35% in incomplete markets, and investment in child-care (wn) increases at the same time by 50%.

Labor earnings ($w\pi$) have a Gini index of 0.64, which is the same for productivity as long as they differ only up to a multiplicative factor. 65% of total wealth ($s + b$) is made up of intergenerational transfers (b).[13] The intergenerational correlation of long-run labor income is 0.79 in incomplete markets (17% higher than in efficient steady-state), which is barely in line with empirical evidence in Brazil according to Andrade et al. (2003), whose estimate

[12] Geweke (1996) discusses the main aspects of Monte Carlo simulation and its applications.

[13] Figures 1 and 2 plot invariant distributions for π and $s + b$.

is 0.6.[14,15] We also notice that 76% of agents are constrained (in the sense $b = 0$) in our incomplete markets world: Issler and Rocha (2000) report that 74% of adult population in Brazil are also constrained. Table 2 summarizes inequality data for income and also presents the ratio of income appropriated by the top and bottom 20%: for the last twenty years in Brazil (PNAD data), we find 27.4 as mean, and the incomplete markets economy suggests 27.1.

4.2 Incomplete Markets with Government

Having portrayed incomplete markets inefficient structure, it remains to answer the question we posed before: is there any role for public policies? If we were to compare the mean utility derived from the efficient complete markets structure with the mean utility level from incomplete markets, we would be tempted to say that an agent prefers to live in the last world: steady-state utility is higher with incomplete markets due mainly to the excessive level of aggregate capital there. If this line of reasoning were correct, evidently there will not be any role for a public sector: agents are better off in a decentralized market economy with imperfect insurance and liquidity constraints.

Fortunately, AGS suggest ~~us to do~~ the following experiment. Suppose that, starting from incomplete markets, the efficient equilibrium allocation is suddenly implemented. Taking into account the transition path from incomplete to complete markets, does an agent still prefer to live in an incomplete markets world? Our task is to compute a T-period transition sequence for consumption, $(c_t^y, c_{t+1}^o)_{t=1}^T$ and find λ_{tr} such that

$$\sum_{t=0}^{\infty} (\beta\theta)^t \left[U\left((1+\lambda_{tr})c_t^y\right) + \beta U\left((1+\lambda_{tr})c_{t+1}^o\right)\right] = EV, \tag{21}$$

where EV represents an agent's mean utility in incomplete markets with $\tau = 0$ and the subsequence $(c_t^y, c_{t+1}^o)_{t=T+1}^{\infty}$ stands for an efficient allocation. From (21) and utility parametrization presented above (section 2.3), it is easily checked that λ_{tr} is given by the following

[14] For the US economy, Solon (1992) found a correlation of 0.4 or even higher for labor earnings, but lower than 0.6.

[15] Every autocorrelation described in this paper was taken, as it is usual in the literature, with respect to the logarithm of the relevant variable.

expression:

$$\lambda_{tr} = \left[\frac{EV + \frac{(1+\beta)}{(1-\mu)(1-\beta\theta)}}{\sum_{t=0}^{\infty} (\beta\theta)^t \left[(c_t^y)^{1-\mu} + \beta \left(c_{t+1}^o \right)^{1-\mu} \right] / (1-\mu)} \right]^{\frac{1}{1-\mu}} - 1. \tag{22}$$

We have found from (22) and simulation $\lambda_{tr} = -0.72$.[16] Then a typical individual will be willing to pay 72% of his consumption in an incomplete markets world in order to live in an efficient one (including the transition path). The intuition for this result is simple: the aggregate level of consumption has a boom as the economy's stock of physical capital runs down, affording each one a higher level of utility earlier, which has a greater impact on the summation in (21). Figure 3 plots the transition path for consumption and physical capital.

But what can be said about government's role? To find an optimal design for public policies would be a fairly complicated exercise, mainly for two reasons. Firstly, if we were to take into account only steady-state welfare, the problem is not well-defined: steady-state utility is higher in incomplete markets than in an efficient complete markets structure. Secondly, to allow for a dynamic problem will require us to depart from the standard microeconomic approaches like De Fraja (2002) and his critics, who consider only stationary distributions. Therefore, we follow Aiyagari's (1995) suggestion and other studies thereafter to promote a steady-state analysis, thereby investigating whether a uniform income taxation scheme alleviates incomplete markets' distortions and increases the agent's welfare, which is measured by compensating variation in consumption (section 2.3).

After simulating an incomplete market economy with government for tax rate levels ranging from $\tau = 0$ to $\tau = 1$, we found, following (16) and (18), $\lambda(\tau)$ to be a concave function that is maximized at $\tau^* = 52\%$, and $\lambda(\tau^*) = 100\%$ (see figure 4). Therefore, an individual would prefer to live in an incomplete markets economy where there is a public sector taxing his labor income in 52% and returning the proceeds via lump-sum transfers rather than in an incomplete markets structure with no government at all ($\tau = 0$).

As we hinted before, not only an agent will be willing to live in such world, but also the problems associated with misinvestment and inequality are mitigated when we set $\tau = \tau^*$. Interest rate rises from 7% to 8%, reflecting a stock of physical capital 35% lower.[17] Income Gini index decreases to 0.53, and wealth Gini is now 0.38, nearly half its previous value

[16] AGS perform a similar exercise and find $\lambda_{tr} = -0.63$ in their parametrized economy, suggesting that our value is not too high.

[17] See figure 7.

(0.64). Correlation of total income decreases from 0.9 to 0.83.[18] Now, the ratio of income appropriated by the top and bottom 20% is 16, 40% lower. Concerning the efficiency of the allocation achieved, now we have only 5% with $a \leq a^*$ becoming skilled, contrasting with 50% when $\tau = 0$.[19] Thus the government's intervention ameliorates to some extent the inefficiency associated with incomplete markets, and, at the same time, is welfare-improving according to our criterion.[20]

The economic intuition for the previous results is quite simple. Firstly, notice that the proposed taxation scheme penalizes richer agents, by reducing their disposable income, whereas it gives a "bliss" to poorer ones. For a rich individual the net effect of taxation and lump-sum transfers is negative, that is, $\chi - \tau w \pi^{rich} < 0$. On the other hand, poor agents have an opposite net effect: $\chi - \tau w \pi^{poor} > 0$. Now, a typical agent spends a lower fraction of his lifetime in an unfavorable situation, affording him a greater utility.[21] The distinct aspect of this redistributive mechanism of making some agents less rich an other less poor has to do with the format of the steady-state utility, our value function V. Since V is steeper in the low-end, poor individuals' "marginal utility" is greater than rich's. Therefore, given the skewed distribution of labor income, the net effect of our taxation mechanism in terms of utility for a "mean agent" will be positive.

Secondly, regarding the positive outcomes in terms of efficiency, notice that, as long as rich people accumulate relatively more assets - policy functions for b and s are increasing in $w\pi$ -, to reduce their disposable income is equivalent to prevent them of accumulating too much. Thus, the interest rate is raised, lowering the costs of the precautionary savings for everyone and increasing their effectiveness in smoothing consumption. At the same time, government provides an additional insurance to individuals, χ, improving consumption smoothing: the coefficient of variation of total consumption is 50% lower when $\tau = 52\%$.

Lastly, with respect to efficient investment in children, note that the taxation scheme is more onerous for the rich, who are keen to invest in the human capital of their offspring whatever their ability levels. Top 50% richer individuals in terms of labor income invest 280%

[18] See figure 6.

[19] Unfortunately, this scheme also has perverse effects. When $\tau = 0$, 90% of the agents with ability level above a^* become skilled. Now, with $\tau = 0.52$, only 85%, illustrating an obvious limitation in terms of efficiency of the proposed mechanism. Furthermore, the fraction of constrained agents increases from 76% to 88%.

[20] See table 3 for a summary of most statistics presented in this section.

[21] See figure 5.

more in children relatively to the bottom 50% in incomplete markets without government. When $\tau = 0.52$, the former invest "only" 100% more. Consequently, lowering disposable income of the rich, we prevent them to misinvest too much, and efficiency in the allocation of resources is increased to some degree.

5 Welfare Decomposition

Up to this point in our analysis, we have accomplished the following tasks: (i) We have shown that a simple scheme of labor income taxation in a steady-state analysis with incomplete markets can mitigate the inefficiency in this economy; (ii) Taking into account a utilitarian welfare measure (λ), there is one level of tax rate that maximizes our criterion. Particularly, it was shown there are both a reduced-inequality effect (via income redistribution) and a reduced-uncertainty effect (in the sense of a lower variability in consumption). The latter welfare criterion encompasses all them together.

But one may say, following Flóden (2001), whose analysis is inspired by Bénabou (2002), that it is valid to try to decompose this measure in order to assess each effect separately. To do so, we show that, under a simple assumption about how an agent weights consumption in youth (c^y) and old age ($c^{o\prime}$) - see below -, the measure λ can be decomposed in three components: the previously mentioned (uncertainty, λ_{unc}, and inequality, λ_{ine}) and a level effect (λ_{lev}):

$$(1 + \lambda) = (1 + \lambda_{unc})(1 + \lambda_{ine})(1 + \lambda_{lev}).$$

To motivate the decomposition, some definitions are in order.

Definition 1 *The expected lifetime utility of an individual born at time -1 and indexed (without loss of generality) by $i \in \Omega \subset \mathbb{R}$ (Ω probably is a non-enumerable set) is given by*

$$W(\{c(i)_t^y, c(i)_{t+1}^o\}_{t=0}^{\infty}) = E_0 \sum_{t=0}^{\infty} (\beta\theta)^t \, u(c(i)_t^y, c(i)_{t+1}^o), \tag{23}$$

where $u(c^y, c^{o\prime})$ is simply $U(c^y) + \beta U(c^{o\prime})$ as in section 2.1 and $\{c(i)_t^y, c(i)_{t+1}^o\}_{t=0}^{\infty}$ is a (possibly uncertain at $t = 0$) stream of consumption that solves agent i's problem as in (CP2).

It is straightforward to define a utilitarian social welfare function:

Definition 2 *The utilitarian social welfare function in our framework is*

$$WS^u = EV = \int W(\{c(i)_t^y, c(i)_{t+1}^o\}_{t=0}^{\infty})dF(i), \tag{24}$$

with F being again the distribution function of agents at $t = 0$.

In section 4, we endeavored a steady-state analysis so as to evaluate welfare gains of changing from a policy A ($\tau = 0$) to B ($\tau > 0$). The utilitarian welfare gain criterion, λ, of changing from a benchmark economy (policy A) to an economy where other public policy is present (B) is now defined more rigorously:

Definition 3 *The utilitarian welfare gain, λ, of policy change (A to B) is computed by*

$$\int W(\{(1+\lambda)c(i)_t^{y,A}, (1+\lambda)c(i)_{t+1}^{o,A}\}_{t=0}^{\infty})dF_A(i) = \int W(\{c(i)_t^{y,B}, c(i)_{t+1}^{o,B}\}_{t=0}^{\infty})dF_B(i).^{22} \tag{25}$$

Note that, as before in section 2.3, an average agent will be more willing to live in a world with policy B for (positive and) higher values of λ. The next two definitions are rather technical and they have a microeconomic flavor.

Definition 4 *A certainty-equivalent consumption bundle for agent i is a period-invariant deterministic sequence, $\{\bar{c}(i)^y, \bar{c}(i)^o\}_{t=0}^{\infty} = \{\bar{c}(i), \bar{c}(i)\}_{t=0}^{\infty}$, such that $W(\{\bar{c}(i), \bar{c}(i)\}_{t=0}^{\infty}) = W(\{c(i)_t^y, c(i)_{t+1}^o\}_{t=0}^{\infty})$. That is, it represents a non-stochastic stream of consumption that gives the agent i the same level of expected utility - "expected for i" - as of (CP2).*

Remark 2 *Here our analysis departs from Flodén (2001). He considers a univariate stream of deterministic consumption and, therefore, for each $W(\{c(i)_t^y, c(i)_{t+1}^o\}_{t=0}^{\infty})$ (which is a number), one also finds a unique $\bar{c}(i)$ after a level of labor supply is fixed in his model. Now, we have a continuum of pairs $(\bar{c}(i)^y, \bar{c}(i)^o)$ satisfying our condition. By choosing $\bar{c}(i)^y = \bar{c}(i)^o = \bar{c}(i)$, we implicitly assume that each agent weights consumption in both periods alike. But this assumption is also implicit in AGS (p.311) and here in Definition 3 above. Furthermore, assuming pure altruism ($\theta = 1$) and a mass of young and old both equal to 1 makes such hypothesis less implausible.*

[22] Analogously as in section 2.3, equation (18), λ is given by: $\lambda = \left[\frac{EV_B + \frac{1+\beta}{(1-\mu)(1-\beta\theta)}}{EV_A + \frac{1+\beta}{(1-\mu)(1-\beta\theta)}}\right]^{\frac{1}{1-\mu}} - 1$.

Definition 5 *$\bar{C}$, the mean level of certainty-equivalent, and C, average of consumption, are respectively expressed by*

$$\bar{C} = \int 2\bar{c}(i)dF(i), \tag{26}$$

$$C = \int (c^y(i) + c^o(i))\, dF(i). \tag{27}$$

The following definitions are means of measuring separately the effects of a change in policy and they are similar to Flodén (2001, p.90-1):

Definition 6 *The cost of uncertainty is p_{unc}, as defined by*

$$W\left(\left\{(1-p_{unc})\frac{C}{2}, (1-p_{unc})\frac{C}{2}\right\}_{t=0}^{\infty}\right) = W\left(\left\{\frac{\bar{C}}{2}, \frac{\bar{C}}{2}\right\}_{t=0}^{\infty}\right). \tag{28}$$

The related welfare gain of reduced uncertainty is

$$\lambda_{unc} = \frac{1-p_{unc}^B}{1-p_{unc}^A} - 1. \tag{29}$$

Definition 7 *The cost of inequality, associated with the redistributive effects, is p_{ine}, as defined by*

$$W\left(\left\{(1-p_{ine})\frac{\bar{C}}{2}, (1-p_{ine})\frac{\bar{C}}{2}\right\}_{t=0}^{\infty}\right) = \int W\left(\{\bar{c}(i), \bar{c}(i)\}_{t=0}^{\infty}\right) dF(i), \tag{30}$$

The related welfare gain of reduced inequality is

$$\lambda_{ine} = \frac{1-p_{ine}^B}{1-p_{ine}^A} - 1. \tag{31}$$

Given the definitions above and the assumption about U in section 2.1, it is easily checked that the costs of uncertainty are $p_{unc} = 1 - \frac{\bar{C}}{C}$. The higher is aggregate certainty-equivalent consumption vis-a-vis C, the lower are the costs associated with uncertainty. Notice that the limiting case of a perfect consumption smoothing, we have simply $\bar{C} = C$ and the costs of uncertainty are null. With respect to the costs of inequality, notice they are computed from the distribution of certainty-equivalent consumption and that merely redistributing consumption from a rich to a poor agent has no effect on the expression of p_{unc}, but changes the distribution of certainty-equivalent consumption. Thus, given the concavity of W, the right-hand side of (30) increases and p_{ine} is reduced.

The third effect has to do with changing levels of mean consumption and is defined below.

Definition 8 *The welfare gain of increased levels of equilibrium consumption is*

$$\lambda_{lev} = \frac{C^B}{C^A} - 1. \tag{32}$$

The following proposition suggests a simple way of decomposing λ:

Proposition 1 *Assume $u(c^y, c^{o\prime}) = U(c^y) + \beta U(c^{o\prime})$, with U given as in section 2.1. Then*

$$(1 + \lambda) = (1 + \lambda_{unc})(1 + \lambda_{ine})(1 + \lambda_{lev}). \tag{33}$$

Proof. *Appendix C.* ■

When we perform this decomposition to analyze the results of section 4, we notice that the main effect of our redistributive taxation policy is on reducing inequality (see figure 8). According to (33), at $\tau = \tau^* = 52\%$ the contributions from reduced uncertainty, inequality and increasing levels of consumption to our measure of utilitarian welfare gains are respectively -1%, 166% and -23%. Despite the fact that standard deviation of aggregate consumption is reduced, the effects on our measure of uncertainty is approximately null, that is, $\lambda_{unc} = -1\%$. Moreover, we observe that the measure of gains in terms of "pure economic efficiency", $(1 + \lambda_{unc})(1 + \lambda_{lev})$, is maximized at a quite low level for τ: 0.7% (figure 9).[23] Therefore, equity effects apart, the taxation scheme proposed above is rather distortionary in terms of efficiency.

5.1 Sensitivity

In decomposing utilitarian welfare above, we used a restrictive definition of certainty-equivalent. Since the definition of the three effects hinges on the fact that we give both young and old the same deterministic consumption bundle, one may ask whether our results would change if we define certainty-equivalent differently from Definition 4 and modify subsequent definitions accordingly. To this end, we assume a young agent's certainty-equivalent stands

[23] The component associated with "pure economic efficiency" is labeled after Bénabou (2002) and puts no value on equity effects (λ_{ine}). In a work analyzing the equity and efficiency effects of redistributive policies, Seshadri and Yuki (2003) emphasize this measure, which consists in comparing the aggregate level of certainty-equivalent ($\bar{C}$).

in a fixed proportion to the certainty-equivalent of the old. That is, we take $\bar{c}(i)^y$ and $\bar{c}(i)^o$ such that, for all $i \in \Omega$,

$$\bar{c}(i)^y = \alpha \bar{c}(i) \text{ and } \bar{c}(i)^o = (1-\alpha)\bar{c}(i), \tag{34}$$

where $\alpha = \frac{E(c^y)}{E(c)}$, and

$$W\left(\{\bar{c}(i)^y, \bar{c}(i)^o\}_{t=0}^{\infty}\right) = W\left(\{c(i)_t^y, c(i)_{t+1}^o\}_{t=0}^{\infty}\right). \tag{35}$$

Some redefinitions are in order.

Definition 9 *$\bar{C}$, the mean level of certainty-equivalent, is expressed by*

$$\bar{C} = \int \bar{c}(i) dF(i). \tag{36}$$

The defintion of the mean level of consumption does not change.

Definition 10 *The cost of uncertainty is p_{unc}, as defined by*

$$W\left(\{(1-p_{unc})\alpha C, (1-p_{unc})(1-\alpha)C\}_{t=0}^{\infty}\right) = W\left(\left\{\alpha \bar{C}, (1-\alpha)\bar{C}\right\}_{t=0}^{\infty}\right). \tag{37}$$

Definition 11 *The cost of inequality, associated with the redistributive effects, is p_{ine}, as defined by*

$$W\left(\left\{(1-p_{ine})\alpha \bar{C}, (1-p_{ine})(1-\alpha)\bar{C}\right\}_{t=0}^{\infty}\right) = \int W\left(\{\alpha \bar{c}(i), (1-\alpha)\bar{c}(i)\}_{t=0}^{\infty}\right) dF(i). \tag{38}$$

Given the definitions above, we have the following proposition for decomposing λ:

Proposition 2 *Suppose $U(c)$ to be CRRA momentary utility function as defined in section 3 and let μ be its coefficient of relative risk aversion. Then, under definitions 1 to 3, (34), (29), (31), (32), and definitions 9 to 11, the utilitarian welfare gain of policy change (A to B) amounts to:*

$$(1+\lambda) = \kappa(1+\lambda_{unc})(1+\lambda_{ine})(1+\lambda_{lev}), \tag{39}$$

where $\kappa = \left(\frac{a_B^{1-\mu}+\beta(1-\alpha_B)^{1-\mu}}{a_A^{1-\mu}+\beta(1-\alpha_A)^{1-\mu}}\right)^{\frac{1}{1-\mu}}$.

Proof. *Appendix D.* ■

When we perform this new decomposition, the utilitarian measure of welfare is again maximized at $\tau = 0.52$ and the driving force which accounts for welfare improvement is reduced inequality. However, the welfare gains associated with reduced uncertainty are no longer negative or negligible. Indeed, gains from less uncertainty are always increasing and attain a maximum of 4%. Lastly, the gains associated with pure economic efficiency are now maximized at a significative tax rate: $\tau = 2\%$.

6 Public Debt

In order to allow for a "public sector cum debt" set-up, notice that the budget constraint of government will no longer need to be balanced every period as in (15). Indeed, if x_t stands for total taxes revenue and d_t is the amount of debt in period t, the government budget equation is given now by

$$d_{t+1} = (1+r)(d_t + \chi - x_t), \tag{40}$$

where χ is the lump-sum transfer as before. (40) simply states that, given an initial level of public debt, d_0, the value of debt next period is the sum of its net expenses today.

Following Flodén (2001) we examine only the choice of the average level of debt. Then, together with (15) the last equation yields

$$\chi = \tau \int w\pi dD^y(\pi, a, b) - \frac{r}{1+r}\bar{d}. \tag{41}$$

A recursive stationary competitive equilibrium is defined analogously to section 2.2, except for the fact that now government holds both τ and $\bar{d}$ fixed, and the consumer takes them as given. Besides, our definition of equilibrium in assets market is slightly different:

$$k = E\left(\frac{s}{1+r} + \frac{b}{1+r}\right) - \frac{\bar{d}}{1+r}. \tag{42}$$

Our illustrative experiment consisted in varying $\frac{\bar{d}}{GDP}$ over the interval [0.005 0.3] for our benchmark economy with "optimal" redistributive taxation ($\tau = 0.52$). We follow the same procedures of simulation as described in section 3, except for the new definitions (41) and (42).

The utilitarian welfare measure in this case decreases, implying that an agent would prefer to live in an incomplete markets economy when $\tau = 0.52$ and $\bar{d} = 0$ rather than in an environment with the same tax rate and strictly positive public debt.[24] With respect to inequality indexes, both income and wealth Gini remain relatively constant, whereas the correlation across generations for income slightly increases (figure 11). Furthermore, coefficient of variation of total consumption also increases (figure 12). By applying our welfare gains decomposition from Proposition 1 to the present experiment, there is evidence

[24]See figure 10 for the values of λ that solve equation (18) when our benchmark economy is the one with incomplete markets with $\tau = 0.52$ instead of $\tau = 0$. Negative values suggest that an agent prefer to live with no public debt at all when tax rate is set to its "optimal" redistributive level.

that all components are less than 1 and decreasing in $\frac{\bar{d}}{GDP}$, implying there is no role for public debt when $\tau = 0.52$ - figure 13 shows the decomposition.[25]

7 Robustness Analysis

Our robustness check consists in endogenizing labor supply in the model. In particular, we consider the case where young and old agents choose the amount (in the intensive margin) of effort they do and derive disutility from working. Momentary utility function has the form introduced by Greenwood, Hercowitz and Huffman (1988),

$$u(c^i, l^i) = U(c^i - g(l^i)) = \frac{\left(c^i - \frac{l^{i1+\tilde{\theta}}}{1+\tilde{\theta}}\right)^{1-\mu}}{1-\mu} \; (i = y, o), \tag{43}$$

where l^i stands for labor supply.

Although (43) implies that labor supply choice is independent of the consumption-savings, it is by no means obvious how our results up to this point would change. Most of previous results relied upon the fact that, in incomplete markets without government, there was an overaccumulation of both human and physical capital, giving rise to an inefficient allocation of resources. When we allow for an endogenous labor supply, to overaccumulate human capital decreases utility, because labor supply of the young will be a positive function of $w\pi$. Indeed, investment in human capital measured by $\mathbf{m}$ rises by the same rate as in section 4 with respect to the efficient equilibrium, and investment in child-care increases *only* 1.5%, contrasting with the rise of 50% when labor supply is exogenous.[26]

The transition path along the lines described in section 4.2, equation (22), is also computed. We find that the welfare gains from completing the missing markets are still substantial - $\lambda_{tr} = -0.62$ - but lower than before. This is indeed the case because now, as labor supply exhibits a boom in the beginning of transition path, the agent enjoys a lower level of consumption net of disutility of working than he would do otherwise with exogenous labor supply.[27]

[25] An analogous result holds if we vary $\bar{d}$ when $\tau = 0$.

[26] See the Addendum (Nascimento, 2004) for a detailed presentation of the new framework, results and calculations.

[27] All the computations of welfare gains are done in terms of levels of consumption net of disutility of working $(c - g(l))$. This is not a distinct feature here: see, for example, Gomes et al. (2001) for a similar procedure in a heterogenous agents framework with both agreggate and idiosyncratic shocks.

The exercise of varying τ is carried out and the findings are that $\lambda(\tau)$, again our utilitarian measure of welfare, is maximized at $\tau = \tau^* = 0.55$, and $\lambda(\tau^*) = 98\%$, still representing substantial gains. With respect to welfare decomposition (Proposition 1), the driving force for increased levels of welfare is the reduced inequality (see figure 14). However, the effects of reduced uncertainty are not negligible and represent gains of 3.2% of consumption when τ is around 25%. The redistributive taxation scheme also reduces inequality indexes and inefficiency associated with misinvestment in human capital, and at the same time ameliorates consumption smoothing (see table 4).

Lastly, when we insert public debt in the model, there are no gains from doing so: welfare measures decrease holding τ fixed at τ^* or $\tau = 0$. For example, if government runs a debt to GDP ratio of 18%, overall welfare is cut by half when $\tau = 55\%$. Therefore, the analysis of redistributive policy carried out in previous sections seems to be quite robust to the specification of preferences.

8 The Other Way Round

In this section we ask the following question: if we assume that Brazil and US have the same production function of human capital and the same stochastic process for abilities, but differ with respect to the goods production technology and depreciation rate of capital, what is the degree of distortion introduced by the government that generates Brazilian-like inequality statistics? In order to assess this question, we assume the model as in section 2 and that government taxes people's labor income by a flat-tax rate τ as before, but returns the proceeds through lump-sum transfers only to the top 10% richer individuals in terms of labor income arising in incomplete markets without public sector ($\tau = 0$).

The results are displayed in table 5. Although wealth Gini increases substantially when we vary τ, this is not the case with income Gini, which attains a maximum of 0.47 at $\tau = 0.6$, far below Brazilian income Gini (0.6). With respect to the correlation of labor income across generations, the model replicates data for τ between 0.4 and 0.6.

One may infer from the above results that the model is flawed if our task is to replicate Brazilian inequality indexes. However, since government policies are actually much more complex, this is not necessarily true: false assumptions frequently account for erroneous inferences. Therefore, a better and accurate understanding of the distortions caused by

Brazilian public policies is crucial to appraise the implications of our model, and it is a topic of further research.

9 Concluding Remarks

This study was concerned with investment in human capital of people early in life and the inefficiencies associated with an incomplete markets economy. Our artificial economy with government was calibrated to Brazilian data and its numerical simulations showed that a uniform taxation scheme alleviate those inefficiencies with an income tax of 52%. How the mechanism works is a straightforward question: it creates a disincentive for the rich to invest in low ability children, increasing efficiency and providing a welfare-improving redistributive effect. Moreover, it also provides a constant insurance for the young (χ, the lump-sum transfer), thereby making them less willing to accumulate capital, increasing interest rate and lowering the costs associated with accumulating assets when the interest rate falls below the one implied by time preference.

By performing a decomposition of the welfare measure, it was found that reducing inequality is the most important factor explaining the preference of the typical agent for an incomplete markets economy with income tax of 52%. Furthermore, extending the model so as to allow for public debt, if we keep $\tau = 0.52$ and let the public debt to GDP level to vary, we conclude there is no role for public debt as long as the welfare gains from $\bar{d} > 0$ are negative.

Evidently, our proposal of a uniform taxation scheme is not derived optimally like a optimal mechanism design, but it displayed desirable outcomes as we have already mentioned. The model also does not permit us to investigate other problems which are particularly relevant in developing countries such as Brazil. One possible extension of the model would be to allow for an employment choice, since the question of high unemployment rates in Brazil is of great concern at the present time. Furthermore, it is not less important to understand the mechanics which links public debt, welfare, inequality and efficiency in developing nations, whose government's budget is frequently unbalanced and a nationwide pledge for more equity in terms of opportunities is often made as it can be inferred, for instance, from Brazil's last presidential election. In this sense, we intend to vary both tax rates and debt to GDP levels together so as to address such question in ulterior research.

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A Appendix

Table 1

Parameter	Value	Source or Target
Time Unity	20 years	AGS
Tastes		
β	$0.88^{\frac{1}{20}}$	interest rate $\cong$ 9%
μ	2	Cunha and Ferreira (2001)
θ	1	pure altruism
Production		
z	0.7	
ν	0.36	AGS
δ	0.74	Val and Ferreira (2001)
Ability and Productivity		
ι	1	income Gini index
ω	0.7	"
σ	0.45	"
χ	3	"
η	0.4	"
ξ	0.65	"
ε	0.5	"
ρ	0.2	"
ϕ	0.01	

Table 2

Inequality Statistics for Income			
	Gini	$\frac{\text{Top 10\%}}{\text{Bottom 40\%}}$	$\frac{\text{Top 20\%}}{\text{Bottom 20\%}}$
Data*	0.6	24	27.4
Model (incomplete markets with $\tau = 0$)	0.61	6.7	27.1

*PNAD (1981-2001)

Table 3

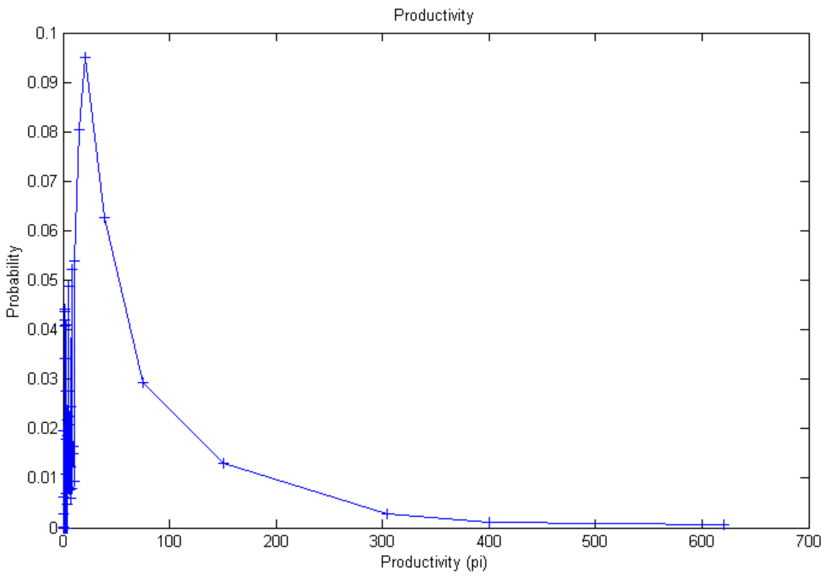
Summary of Statistics			
Statistics	Complete Markets	Incomplete Markets with $\tau = 0$	Incomplete Markets with $\tau = 0.52$
Gini of Inc.	-	0.61	0.53
Gini of Lab. Inc.	0.70	0.64	0.64
Gini of Wealth	-	0.64	0.38
Coeff. of Var. of Cons.	-	2.01	1.02
$corr(\log a, \log a_{-1})$	0.70	0.70	0.70
$corr(\log w\pi, \log w\pi_{-1})$	0.68	0.79	0.75
$corr(\log inc, \log inc_{-1})$	-	0.90	0.83
Annual Interest Rate	13.6%	7%	8%
Physical Capital	0.041	0.235	0.148
Skilled Agents	60%	75%	53%
Constrained	-	76%	88%

Table 4

Statistics - Endogenous Labor Supply			
Statistics	Complete Markets	Incomplete Markets with $\tau = 0$	Incomplete Markets with $\tau = 0.55$
Gini of Inc.	-	0.61	0.48
Gini of Wealth	-	0.67	0.25
Coeff. of Var. of Cons.	-	2.09	0.84
$corr(\log w\pi, \log w\pi_{-1})$	0.63	0.80	0.78
$corr(\log inc, \log inc_{-1})$	-	0.90	0.84
Skilled Agents	43%	56%	30%

Table 5

Statistics as a Function of Tax Rate						
Statistics	$\tau = 0$	$\tau = 0.1$	$\tau = 0.2$	$\tau = 0.4$	$\tau = 0.6$	$\tau = 0.7$
Gini of Inc.	0.38	0.41	0.44	0.48	0.47	0.43
Gini of Wealth	0.41	0.50	0.57	0.68	0.77	0.73
$corr(\log w\pi, \log w\pi_{-1})$	0.65	0.64	0.63	0.62	0.51	0.44



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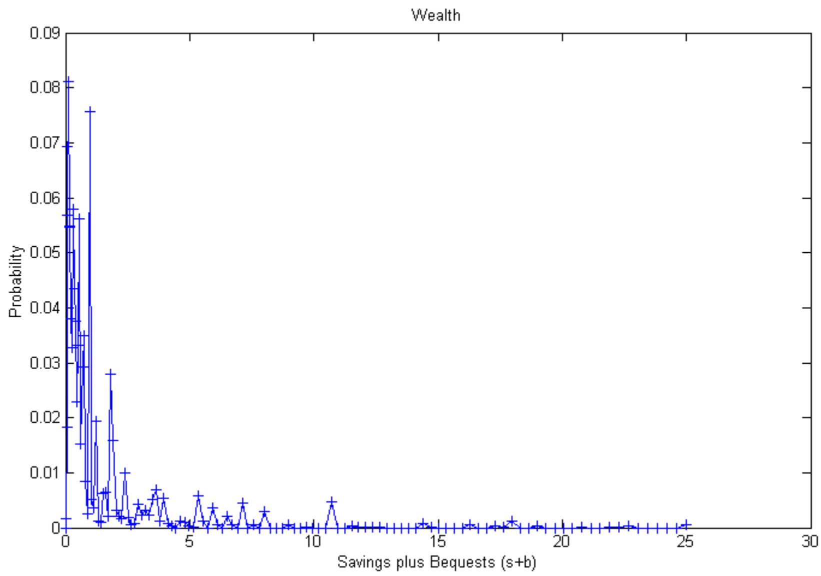

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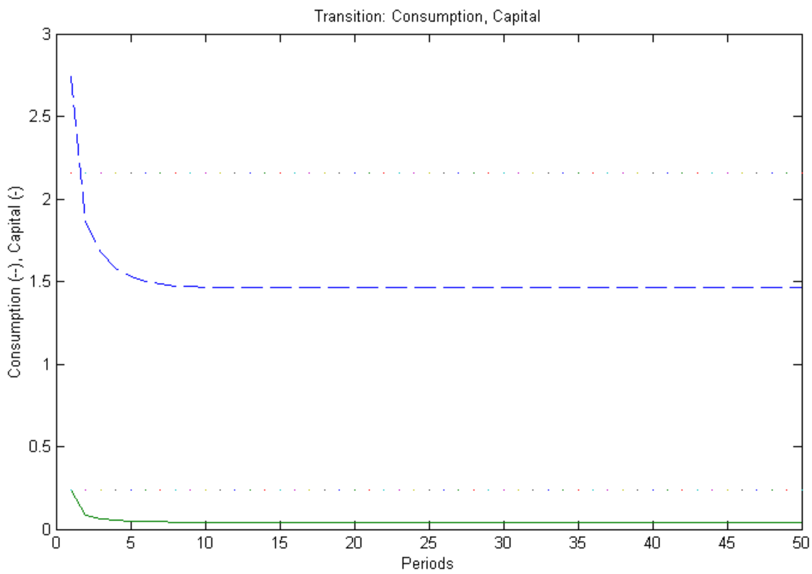

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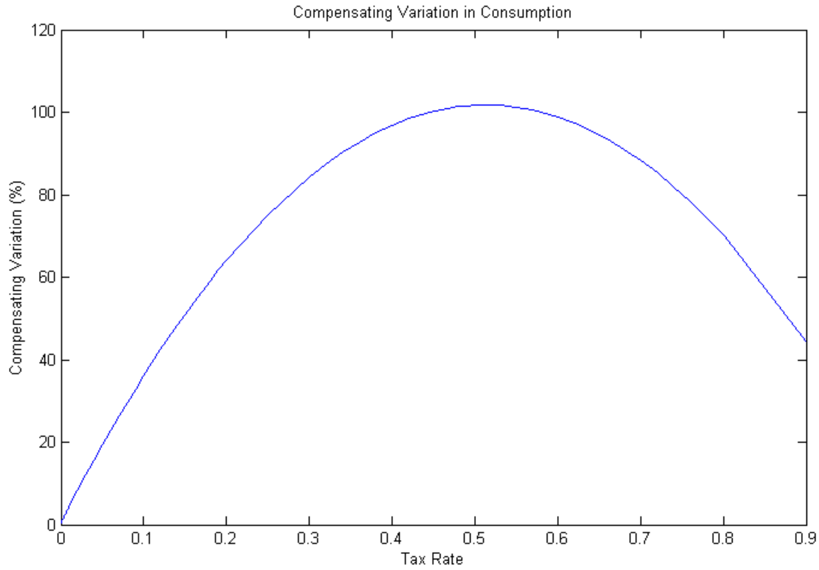

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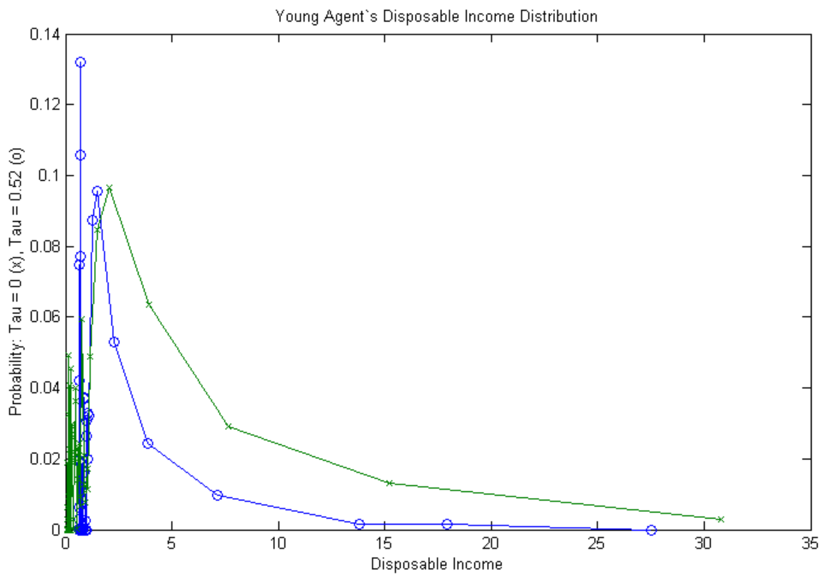

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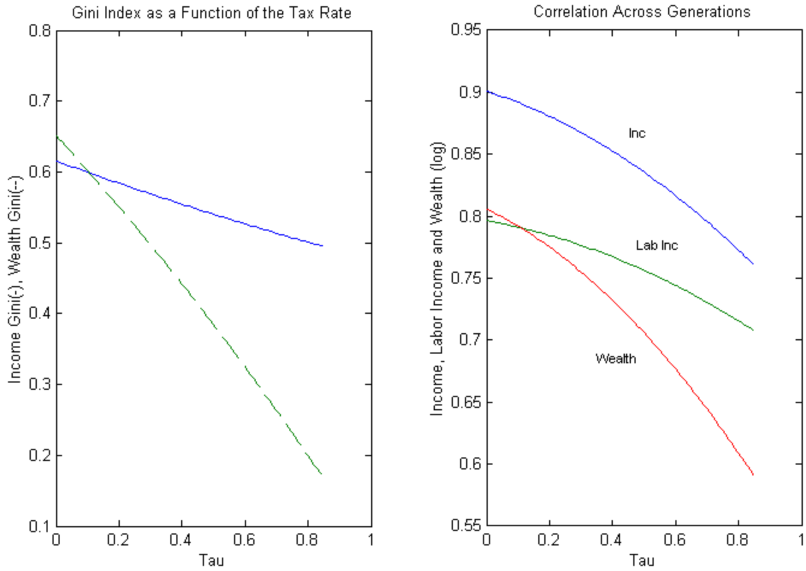

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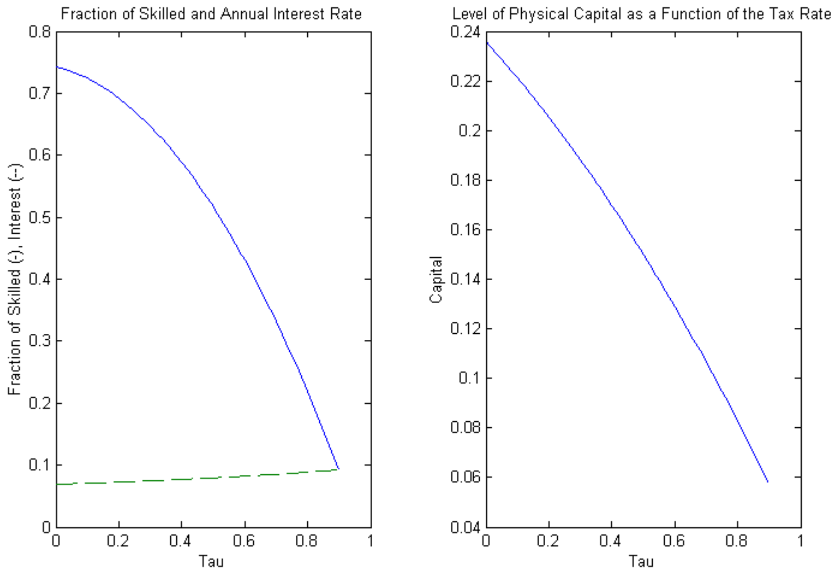

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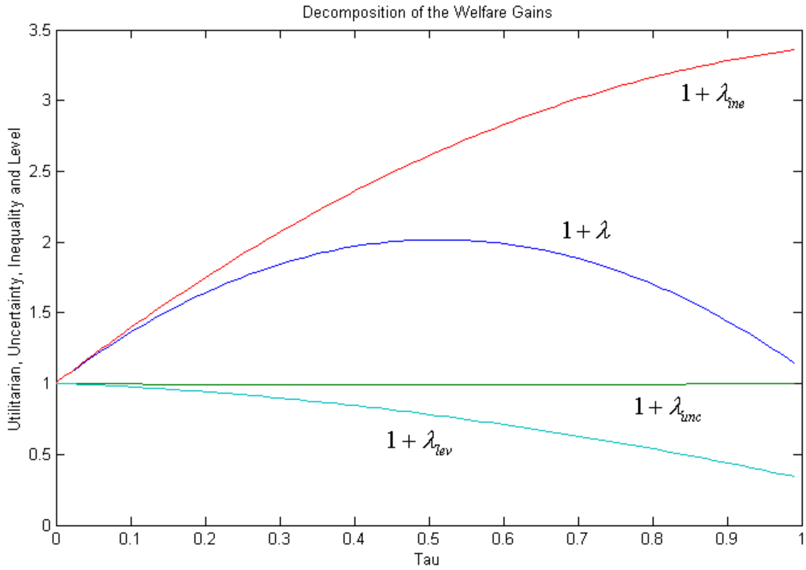

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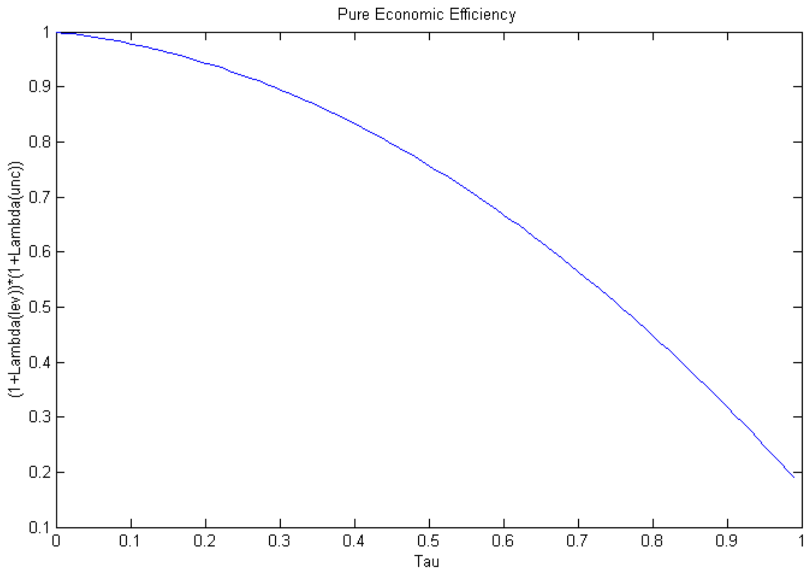

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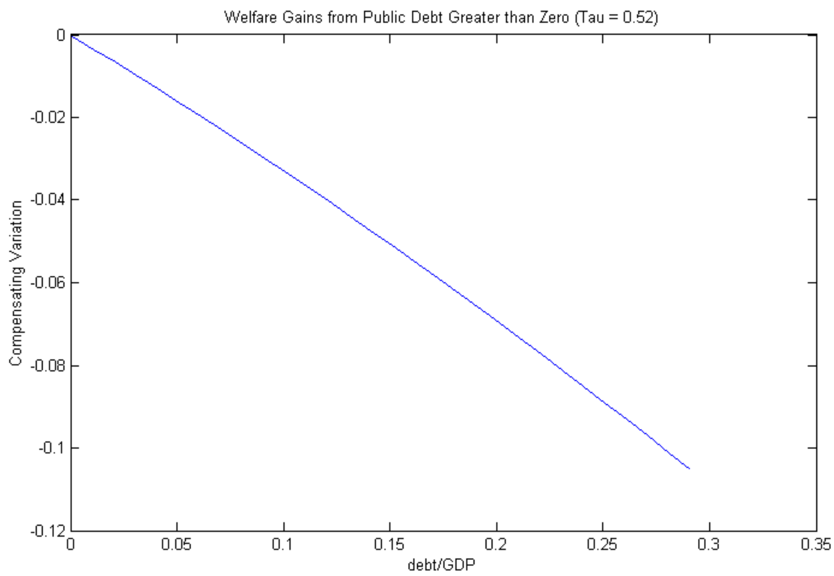
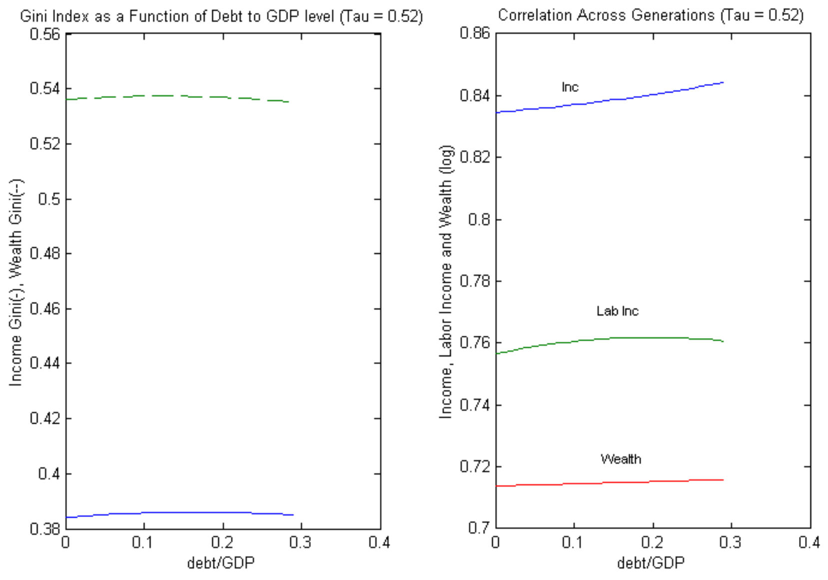


Figure 10



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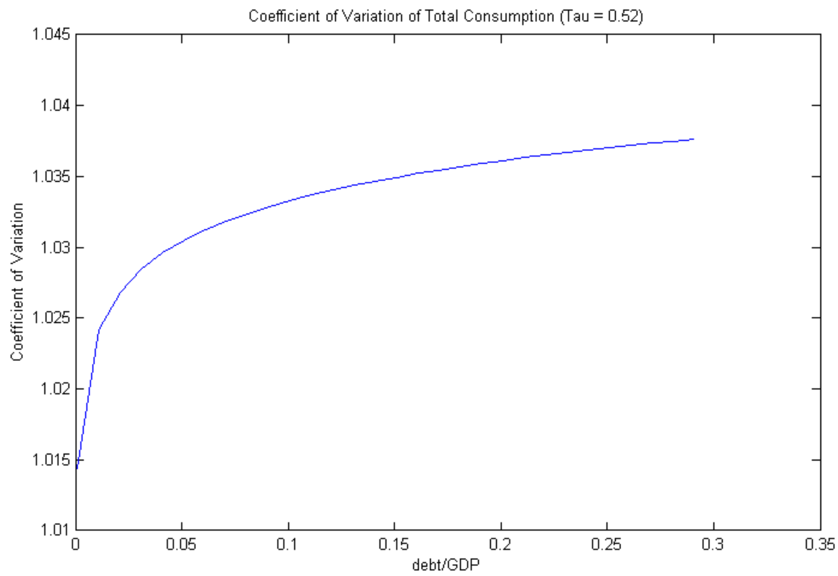

Figure 12

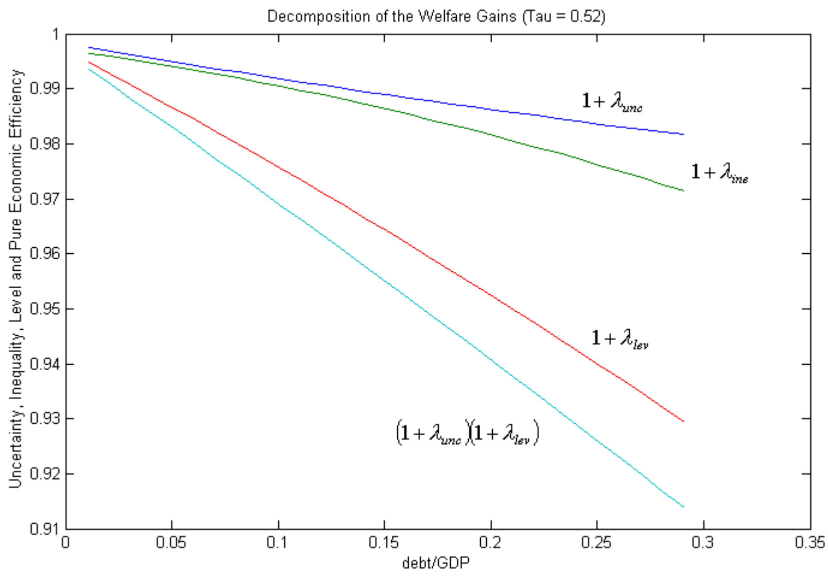

Figure 13

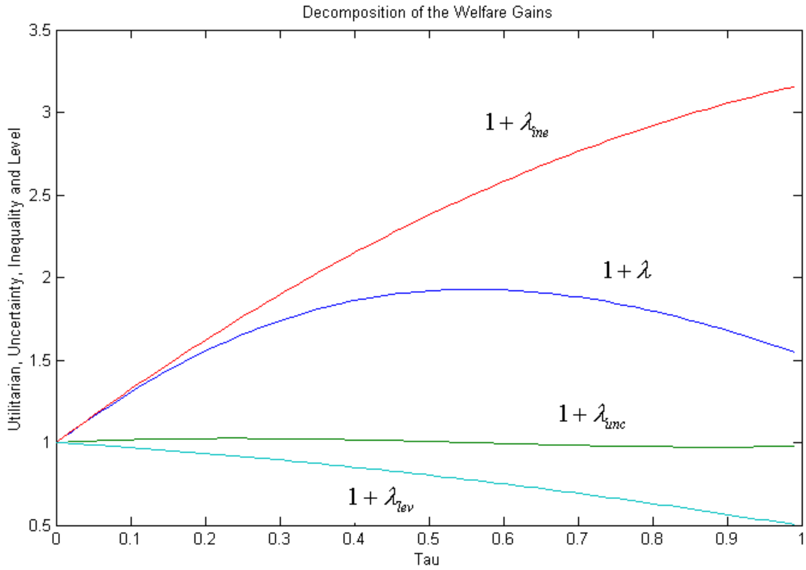

Figure 14

B Appendix

From (16) and $U(c)=\frac{c^{1-\mu}-1}{1-\mu}$:

$$\int E_0 \sum_{t=0}^{\infty}(\beta\theta)^t \left[\frac{[(1+\lambda)c(i)_t^y]^{1-\mu}-1}{1-\mu}+\beta\frac{\left[(1+\lambda)c(i)_{t+1}^o\right]^{1-\mu}-1}{1-\mu}\right] dF(i)=EV_\tau. \tag{B1}$$

But $U((1+\lambda)c)=(1+\lambda)^{1-\mu}U(c)+\left(\frac{1}{1-\mu}\right)[(1+\lambda)^{1-\mu}-1]$, which, together with (B1) and (17), implies $(1+\lambda)^{1-\mu}EV+\left(\frac{1+\beta}{(1-\beta\theta)(1-\mu)}\right)[(1+\lambda)^{1-\mu}-1]=EV_\tau$, whence (18) follows.

C Appendix

Proof of Proposition 1. Firstly note that (23) and (30) give:

$$\sum_{t=0}^{\infty}(\beta\theta)^t\, u\left((1-p_{ine})\frac{\bar{C}}{2},(1-p_{ine})\frac{\bar{C}}{2}\right)=\int E_0\sum_{t=0}^{\infty}(\beta\theta)^t\, u(\bar{c}(i),\bar{c}(i))dF(i)$$

$$\Longrightarrow u\left((1-p_{ine})\frac{\bar{C}}{2},(1-p_{ine})\frac{\bar{C}}{2}\right)=\int u(\bar{c}(i),\bar{c}(i))dF(i). \tag{C1}$$

From (23) and (28):

$$E_0 \sum_{t=0}^{\infty} (\beta\theta)^t \, u\left((1-p_{unc})\frac{C}{2}, (1-p_{unc})\frac{C}{2}\right) = E_0 \sum_{t=0}^{\infty} (\beta\theta)^t \, u\left(\frac{\bar{C}}{2}, \frac{\bar{C}}{2}\right)$$
$$\Longrightarrow u\left((1-p_{unc})\frac{C}{2}, (1-p_{unc})\frac{C}{2}\right) = u\left(\frac{\bar{C}}{2}, \frac{\bar{C}}{2}\right).$$

Since $u(c^y, c^{o\prime}) = U(c^y) + \beta U(c^{o\prime})$ and $U_1 > 0$, the above equation implies:

$$(1-p_{unc})C = \bar{C}. \tag{C2}$$

Using (C2) and (C1):

$$u\left((1-p_{ine})(1-p_{unc})\frac{C}{2}, (1-p_{ine})(1-p_{unc})\frac{C}{2}\right) = \int u(\bar{c}(i), \bar{c}(i))dF(i). \tag{C3}$$

Definition 4, jointly with (C3), implies:

$$E_0 \sum_{t=0}^{\infty} (\beta\theta)^t \, u\left(\bar{c}(i), \bar{c}(i)\right) = E_0 \sum_{t=0}^{\infty} (\beta\theta)^t \, u(c(i)_t^y, c(i)_{t+1}^o)$$
$$\Longrightarrow E_0 \sum_{t=0}^{\infty} (\beta\theta)^t \int u\left(\bar{c}(i), \bar{c}(i)\right) dF(i) = E_0 \sum_{t=0}^{\infty} (\beta\theta)^t \int u(c(i)_t^y, c(i)_{t+1}^o) dF(i)$$

$$\Longrightarrow \sum_{t=0}^{\infty} (\beta\theta)^t \, u\left((1-p_{ine})(1-p_{unc})\frac{C}{2}, (1-p_{ine})(1-p_{unc})\frac{C}{2}\right)$$
$$= E_0 \sum_{t=0}^{\infty} (\beta\theta)^t \int u(c(i)_t^y, c(i)_{t+1}^o) dF(i) \tag{C4}$$

But (C4), (25), $u(c^y, c^{o\prime}) = U(c^y) + \beta U(c^{o\prime})$ and $U_1 > 0$ yield:

$$\int \sum_{t=0}^{\infty} (\beta\theta)^t \, u\left((1+\lambda)(1-p_{ine}^A)(1-p_{unc}^A)\frac{C^A}{2}, (1+\lambda)(1-p_{ine}^A)(1-p_{unc}^A)\frac{C^A}{2}\right) dF_A(i)$$
$$= \int \sum_{t=0}^{\infty} (\beta\theta)^t \, u\left((1-p_{ine}^B)(1-p_{unc}^B)\frac{C^B}{2}, (1-p_{ine}^B)(1-p_{unc}^B)\frac{C^B}{2}\right) dF_B(i)$$

$$\Longrightarrow U\left((1+\lambda)(1-p_{ine}^A)(1-p_{unc}^A)\frac{C^A}{2}\right) = U\left((1-p_{ine}^B)(1-p_{unc}^B)\frac{C^B}{2}\right)$$
$$\Longrightarrow (1+\lambda) = \frac{(1-p_{ine}^B)}{(1-p_{ine}^A)} \frac{(1-p_{unc}^B)}{(1-p_{unc}^A)} \frac{C^B}{C^A}. \tag{C5}$$

Therefore, (33) follows from (C5), (29), (31) and (32). ■

D Appendix

Proof of Proposition 2. The steps are analogous to the previous proof. Given the specification of $U(c)$, (23) and (38) yield

$$\alpha^{1-\mu}U\left((1-p_{ine})\bar{C}\right)+\beta(1-\alpha)^{1-\mu}U\left((1-p_{ine})\bar{C}\right)+U(\alpha)+\beta U(1-\alpha)$$
$$=\int\left(\alpha^{1-\mu}U(\bar{c}(i))+\beta(1-\alpha)^{1-\mu}U(\bar{c}(i))\right)dF(i)+U(\alpha)+\beta U(1-\alpha)$$
$$\Longrightarrow U\left((1-p_{ine})\bar{C}\right)=\int U(\bar{c}(i))dF(i). \tag{D1}$$

From (23) and (37),

$$u\left((1-p_{unc})\alpha C,(1-p_{unc})(1-\alpha)C\right)=u\left(\alpha\bar{C},(1-\alpha)\bar{C}\right)$$
$$\Longrightarrow U((1-p_{unc})C)=U(\bar{C})$$
$$\Longrightarrow (1-p_{unc})C=\bar{C}. \tag{D2}$$

Using (D1) and (D2):

$$U\left((1-p_{ine})(1-p_{unc})\alpha C\right) = \int U(\alpha\bar{c}(i))dF(i),\ \text{or}$$
$$U\left((1-p_{ine})(1-p_{unc})(1-\alpha)C\right) = \int U((1-\alpha)\bar{c}(i))dF(i). \tag{D3}$$

It is easily verified that (34), (35) and (44) imply:

$$E_0\sum_{t=0}^{\infty}(\beta\theta)^t\left(U(\alpha\bar{c}(i))+\beta U((1-\alpha)\bar{c}(i))\right)=E_0\sum_{t=0}^{\infty}(\beta\theta)^t u(c(i)_t^y,c(i)_{t+1}^o)$$

$$\Longrightarrow E_0\sum_{t=0}^{\infty}(\beta\theta)^t\int\left(U(\alpha\bar{c}(i))+\beta U((1-\alpha)\bar{c}(i))\right)dF(i)$$
$$=E_0\sum_{t=0}^{\infty}(\beta\theta)^t\int u(c(i)_t^y,c(i)_{t+1}^o)dF(i)$$

$$\Longrightarrow E_0\sum_{t=0}^{\infty}(\beta\theta)^t\left(U\left((1-p_{ine})(1-p_{unc})\alpha C\right)+\beta U\left((1-p_{ine})(1-p_{unc})(1-\alpha)C\right)\right)$$
$$=E_0\sum_{t=0}^{\infty}(\beta\theta)^t\int u(c(i)_t^y,c(i)_{t+1}^o)dF(i). \tag{D4}$$

Then (25) and (D4) yield:

$$\sum_{t=0}^{\infty} (\beta\theta)^t \, u \left((1+\lambda)(1-p^A_{ine})(1-p^A_{unc})\alpha_A C^A, (1+\lambda)(1-p^A_{ine})(1-p^A_{unc})(1-\alpha_A)C^A \right)$$
$$= \sum_{t=0}^{\infty} (\beta\theta)^t \, u \left((1-p^B_{ine})(1-p^B_{unc})\alpha_B C^B, (1-p^B_{ine})(1-p^B_{unc})(1-\alpha_B)C^B \right)$$

$$\Longrightarrow \left(\alpha_A^{1-\mu} + \beta(1-\alpha_A)^{1-\mu}\right) \left((1+\lambda)(1-p^A_{ine})(1-p^A_{unc})C^A\right)^{1-\mu}$$
$$= \left(\alpha_B^{1-\mu} + \beta(1-\alpha_B)^{1-\mu}\right) \left((1-p^B_{ine})(1-p^B_{unc})C^B\right)^{1-\mu}$$

$$(1+\lambda) = \left(\frac{a_B^{1-\mu} + \beta(1-\alpha_B)^{1-\mu}}{a_A^{1-\mu} + \beta(1-\alpha_A)^{1-\mu}} \right)^{\frac{1}{1-\mu}} \frac{(1-p^B_{ine})}{(1-p^A_{ine})} \frac{(1-p^B_{unc})}{(1-p^A_{unc})} \frac{C^B}{C^A}. \tag{D5}$$

Therefore, (39) follows from (D5), (29), (31) and (32). ■

Addendum to “Investment in Human Capital in a Macrodynamic Framework: Redistributive Taxation, Public Debt and Welfare”: Endogenous Labor Supply

Leandro Gonçalves do Nascimento*

First Version: March 15, 2004
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Abstract

We carry out a robustness analysis of the results derived in a previous paper by adding labor supply decision to the general set-up. It is shown that the basic results do not change significantly. But, in addition, by choosing an optimal redistributive income tax rate, the correlation between labor supply and ability from efficient equilibrium is restored, implying that through a simple taxation scheme government is able to improve labor supply quality.

Key Words: investment in human capital; labor supply; idiosyncratic risk; efficiency; overaccumulation of capital.

JEL Classification Numbers: D1, D31, D52, H21, H63, J24.

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1 Expanded Framework

The model builds on the general framework proposed by Aiyagari et al. (2002). Economy's structure is a standard overlapping-generations model with one-sided altruism where agents live for three periods. Each person lives as a child, a young adult (y) and an old adult (o). At each point in time, the three generations are living together and the mass of young and old agents is each one normalized to 1.

Choice set-up is as follows. When a child, an agent does not make any choice: he is merely apt to receive investment in his human capital. There are two channels to increase the human capital of a child: direct investment of physical resources, labeled m, and spending labor resources in child-care, n. Investment via m implies in a fixed cost ϕ. We assume the existence of a competitive market for providing child-care resources. Young and old agents have both one unit of time and choose the intensity of their labor supply, l^y and $l^{o\prime}$ respectively. A young individual's remaining choices are standard: how much to consume, to save and to invest (if any investment is made at all) in his offspring. Lastly, an old agent chooses his level of consumption and bequests to be inherited by his son.

There is only one good in this economy ($\mathbf{o}$), whose production obeys a neoclassical production function: $\mathbf{o} = O(\mathbf{k}, \mathbf{l})$, where $\mathbf{k}$ is the aggregate level of capital and $\mathbf{l}$ represents total labor supply in efficient units, O is strictly increasing in its arguments and strictly concave.

A young agent's total labor supply, πl^y, where π stands for his productivity, is between goods production and child-care, that is, every person chooses whether to be a "hard" worker or a "baby-sitter". The productivity of labor offered by the young is a predetermined variable in the following sense: $\pi = H(a_{-1}, m_{-1}, n_{-1})$, where a stands for the young's ability and the subscript in a_{-1} reflects the fact that the son's ability is known to his father. If π' stands for productivity next period, the function H is said to satisfy: (i) $H(a, 0, 0) = a \leq \pi'$; (ii) $H(a, m, 0) = H(a, 0, n) = a$; (iii) $H_i > 0$, $i = 1, 2, 3$; (iv) H_{12}, H_{13} and $H_{23} > 0$; (v) H is strictly concave in its arguments, jointly and separately. It is also assumed that an agent with productivity π and ability a_{-1} working in the child-care sector supplies in efficient units only $a_{-1} l^y$ whatever is his actual productivity level. This fact, jointly with (i), implies the existence of comparative advantage for skilled agents ($m, n > 0$) over unskilled (n and/or $m = 0$) on working in the production of goods. An old agent's productivity is assumed to be independent of his human capital and normalized to 1, and his total labor supply, $l^{o\prime}$, is also between goods production and child-care.[1] Notice that, because old agents do not have comparative advantage on working in any sector, they are indifferent to any "specialization" given the assumption of competitive markets for child-care.[2]

In this economy, it must be noticed that agents are heterogenous at least in one sense: they do not have the same level of ability. To this kind of heterogeneity it is given a well-behaved dynamic structure: we assume that $(a_t)_t$ follows a stationary Markov process with transition function $A_1(a|a_{-1})$.

Finally, capital accumulation and resources constraints are given, respectively, by $\mathbf{k}' = (1-\delta)\mathbf{k}+\mathbf{i}$ and $\bar{\mathbf{c}}+\mathbf{m}+\mathbf{i} = \mathbf{o}$. Here, δ is the geometric depreciation rate, assumed constant; $\bar{\mathbf{c}}$ and $\mathbf{i}$ stand as usual for aggregate consumption and physical investment; $\mathbf{m}$ is the aggregate level of investment in human capital through m, and takes into account the fixed cost ϕ.

1.1 Efficient Allocation

How does an efficient allocation look like in this environment? Possible sources of inefficiency are certainly the idiosyncratic shocks, against which every agent would like to insure himself, and liquidity constraints, in the sense of non-negative bequests and total wealth. In complete markets, these problems are ruled out by definition, and we assume they are for a moment.

Our analysis of labor supply relies on the intensive margin only. The disutility of supplying l units will be given by $g(l)$, where $g', g'' > 0$ and $g(0) = g'(0) = 0$. Momentary utility function is $\widetilde{U}(\bar{c}, l) = U(\bar{c} - g(l)) = U(c)$ as introduced by Greenwood, Hercowitz and Huffman (1988), such that:

$$U(c) = \frac{c^{1-\mu} - 1}{1 - \mu} \tag{1}$$

$$g(l) = \frac{l^{1+\tilde{\theta}}}{1+\tilde{\theta}}, \tag{2}$$

where $\mu > 1$ is the coefficient of relative risk aversion and $\tilde{\theta} > 0$ is the inverse labor supply elasticity. Given that each young parent also derives utility from his son's lifetime expected

[1] This independence hypothesis can be ruled out straigthforwardly, but doing so would complicate a little the formulation of incomplete markets consumer's problem. As a topic of further research we intend to permit an old agent's productivity to be given by $(1 + \psi)\pi$, where $\psi > 0$, in line with theoretical studies such as Lochner and Monge-Naranjo (2003).

[2] The results do not change if we assume that only young agents work in the child-care sector.

utility (V'), his objective function is: $U(c^y) + \beta E\left[U(c^{o\prime}) + \theta V'\right]$, where $\beta \in (0,1)$ is a intertemporal discount factor and $\theta \in (0,1]$ represents the degree of altruism towards his son. The dynamic programming problem facing a young agent living in a world with complete markets is:

$$
\begin{aligned}
V(\pi, a, b) = \max_{\bar{c}^y, l^y, m, n, s(a'|a)} & \{U(c^y) + \beta \int J(\pi', a', b + s(a'|a)) A_1(a'|a) da'\} \\
s.t.\ \bar{c}^y + m + \phi I(\pi', a) + wn + & \int q(a'|a) s(a'|a) da' = w\pi l^y \\
\pi' &= H(a, m, n) \\
c^y &= \bar{c}^y - g(l^y) \\
J(\pi', a', b + s(a'|a)) &= \max_{\bar{c}^{o\prime}, l^{o\prime}, b'} \{U(c^{o\prime}) + \theta V(\pi', a', b')\} \\
s.t.\ \bar{c}^{o\prime} + \frac{b'}{1+r} &= wl^{o\prime} + s(a'|a) + b \\
c^{o\prime} &= \bar{c}^{o\prime} - g(l^{o\prime}),
\end{aligned}
\tag{CP1}
$$

where $I(\pi', a) = \begin{cases} 1, \text{ if } \pi' > a \\ 0, \text{ otherwise} \end{cases}$, $s(a'|a)$ is contingent claim to the amount s in case of occurring state a' next period given the current ability state a, $q(a'|a)$ represents its price, b is the bequest received by the old from his parent, and b' denotes the amount of resources (or debts) the old leaves to his son.

Given the utility specification in (1) and (2), it is easily checked that labor supply is independent of the consumption-savings decision and is given by:

$$
l^y = l^y(w\pi) = (w\pi)^{\frac{1}{\theta}}, \ l^{o\prime} = l^{o\prime}(w) = (w)^{\frac{1}{\theta}}. \tag{3}
$$

Hence, (CP1) can be rewritten in terms of individual consumption net of disutility of working:

$$
\begin{aligned}
V(\pi, a, b) = \max_{c^y, m, n, s(a'|a)} & \{U(c^y) + \beta \int J(\pi', a', b + s(a'|a)) A_1(a'|a) da'\} \\
s.t.\ c^y + m + \phi I(\pi', a) + wn + & \int q(a'|a) s(a'|a) da' = w\pi l^y(w\pi) - g(l^y(w\pi)) \\
\pi' &= H(a, m, n) \\
J(\pi', a', b + s(a'|a)) &= \max_{c^{o\prime}, b'} \{U(c^{o\prime}) + \theta V(\pi', a', b')\} \\
s.t.\ c^{o\prime} + \frac{b'}{1+r} &= wl^{o\prime}(w) + s(a'|a) + b - g(l^{o\prime}(w)).
\end{aligned}
\tag{CP2}
$$

If we assume an actuarially fair price for the contingent claims, $q(a'|a) = \frac{A_1}{1+r}$ and a perfect-pooled steady-state (Lucas, 1982)[3], then first-order necessary conditions, jointly with Benveniste-Scheinkman and envelope theorems, imply (see Appendix A):

$$\begin{aligned}
(1+r)\beta &= \frac{1}{\theta} && (4)\\
U_1(c^y) &= (1+r)\beta U_1(c^{o\prime}) && (5)\\
1+r &= wH_2(a,m,n)l^y, \text{ when } m>0 && (6)\\
1+r &= H_3(a,m,n)l^y, \text{ when } n>0 && (7)\\
wn+m+\phi &< \frac{w[\pi' l^y(w\pi') - al^y(wa)]}{1+r} - \frac{[g(l^y(w\pi')) - g(l^y(wa))]}{1+r}, \text{ for } m,n>0 && (8)\\
wn+m+\phi &\geq \frac{w[\pi' l^y(w\pi') - al^y(wa)]}{1+r} - \frac{[g(l^y(w\pi')) - g(l^y(wa))]}{1+r}, \text{ for } m,n=0, && (9)
\end{aligned}$$

where $\pi' = H(a,m,n) > a$.

When $\theta = 1$ (pure altruism), equation (4) yields $1+r = \frac{1}{\beta}$, the standard result for the neoclassical growth model; (5) is a usual Euler equation for a simple two-period economy. (6) and (7) stand for the non-arbitrage conditions which an equilibrium allocation must obey. The derivation of equations (8) and (9) are not straightforward, but their economic interpretation is very simple. Initially notice that a typical agent in our economy will invest in the human capital of his children only if m and n are both strictly greater than zero: it will never be the case of $m=0$ and $n>0$, or $n=0$ and $m>0$. This occurs because, for example, $H(a,0,n) = a = H(a,0,0)$: since the same outcome in terms of future productivity of the offspring would be achieved if no investment is made at all, to make no investment will afford the father to consume more presently. But when are $m,n>0$? Investment in children is occurs only if the current costs of becoming skilled $(wn+m+\phi)$ falls below the present value of the associated benefit net of the disutility of an extra burden in terms of labor $\left(\frac{w[\pi' l^y(w\pi')-al^y(wa)]}{1+r} - \frac{[g(l^y(w\pi'))-g(l^y(wa))]}{1+r}\right)$. As long as there are two forms to invest in the human capital of the offspring to enhance their skills, an efficiency condition always holds in this case:

$$wH_2(a,m,n) = H_3(a,m,n). \tag{10}$$

There is a distinguishing feature displayed by the efficient allocation: (8) and (9) imply decision rules $M(a)$ and $N(a)$ as well as a threshold, a^*, for the level of ability such that

[3]Every young and every old individual consume the same amount: c^y and $c^{o'}$ respectively.

$M(a), N(a) > 0$ if $a > a^*$ and $M(a), N(a) = 0$ otherwise. As we shall stress later on, this condition is not necessarily valid in an incomplete markets environment, giving rise to misinvestment.

An efficient allocation in a stationary equilibrium is therefore described by equations (4) to (9), standard profit maximization conditions for a competitive firm,

$$\begin{aligned} r + \delta &= O_1(\mathbf{k}, \mathbf{l}) && (11) \\ w &= O_2(\mathbf{k}, \mathbf{l}), && (12) \end{aligned}$$

and the constraints

$$\mathbf{l} = \int \pi l^y d\mathbf{A}(a) + l^o(w) - \int_{\mathcal{S}} N(a) d\mathbf{A}(a) \quad (13)$$

$$\int_{\mathcal{S}} N(a) d\mathbf{A}(a) \leq \int_{\mathcal{U}_{-1}} a l^y d\mathbf{A}(a) + l^o \quad (14)$$

$$\bar{\mathbf{c}} + \mathbf{i} + \int_{\mathcal{S}} [M(a) + \phi] d\mathbf{A}(a) \leq O(\mathbf{k}, \mathbf{l}), \quad (15)$$

where $\mathcal{S} = \{a : H(a, M(a), N(a)) > a\}$ is the set of skilled agents in steady-state, $\mathcal{U} = \mathcal{S}^{\,c}$ and $\mathbf{A}(a)$ is the stationary distribution associated with the Markov process for ability. Condition (13) is the total labor supply for goods production and (14) represents market-clearing in chid-care market.

1.2 Incomplete Markets Structure

Suppose now that bequests are required to be non-negative and there is no private insurance market, so that we have an incomplete markets structure. Assume in addition there is a government which taxes people's labor income uniformly by a tax rate $\tau \geq 0$ and returns the proceeds via identical lump-sum transfers (χ) to young agents. It is also assumed that the budget constraint of government does not need to be balanced every period and it may run a public debt (d_t). Let x_t denote total taxes revenue and d_t be the amount of debt in period t; the government budget equation is given by

$$d_{t+1} = (1 + r)(d_t + \chi - x_t). \quad (16)$$

Similarly to Flodén (2001), we examine only the choice of the average level of debt: $\bar{d}$. Since labor choices are still independent of consumption-savings decision, the dynamic programming problem facing a young agent is:

$$V(\pi, a, b) = \max_{c^y, m, n, s \geq -b} \{U(c^y) + \beta \int J(\pi', a', b+s) A_1(a'|a) da'\}$$
$$s.t.\ c^y + m + \phi I(\pi', a) + wn + \frac{s}{1+r} = (1-\tau)w\pi l^y((1-\tau)w\pi) - g(l^y((1-\tau)w\pi)) + \chi$$
$$\pi' = H(a, m, n) \tag{CP3}$$
$$J(\pi', a', b+s) = \max_{c^{o\prime}, b' \geq 0} \{U(c^{o\prime}) + \theta V(\pi', a', b')\}$$
$$s.t.\ c^{o\prime} + \frac{b'}{1+r} = (1-\tau)wl^{o\prime}((1-\tau)w) - g(l^{o\prime}((1-\tau)w)) + s + b.$$

Definition 1 *A stationary recursive competitive equilibrium consists of*

(i) Decision rules for $l^y, l^{o\prime}, s, m, n$ and b, respectively $l^y = l^y((1-\tau)w\pi)$, $l^{o\prime} = l^{o\prime}((1-\tau)w)$, $s = S(\pi, a, b)$, $m = M(\pi, a, b)$, $n = N(\pi, a, b)$ and $b' = B(\pi', a', s+b)$, such that, given r, w, χ and τ they solve (CP3).

(ii) Invariant distributions, D^y and D^o satisfying:

$$D^y(\pi', a', b') = \int T^y(\pi', a', b'|\pi, a, b) dD^y(\pi, a, b) \tag{17}$$
$$D^o(\pi', a', s+b) = \int T^o(\pi', a', s+b|\pi, a, b) dD^y(\pi, a, b), \tag{18}$$

where T^y and T^o stand for transition functions induced by $\mathbf{A}(a'|a)$ and decision rules in (i).

(iii) Real numbers w and r such that: $r = O_1(\mathbf{k}, \mathbf{l}) - \delta$, $w = O_2(\mathbf{k}, \mathbf{l})$, $\mathbf{k} = E\left(\frac{s}{1+r} + \frac{b}{1+r}\right) - \frac{\bar{d}}{1+r}$, and $\mathbf{l} = E(\pi l^y) + l^o(w) - E(n)$.

(iv) Government's budget constraint is satisfied:

$$\chi = \tau \int w\pi l^y dD^y(\pi, a, b) + \tau w l^o - \frac{r}{1+r}\bar{d} \tag{19}$$

2 Calibration

Time unity was set to 20 years, and $U(c) = \frac{c^{1-\mu}-1}{1-\mu}$. We pick $\mu = 2$, which is contained in the interval analyzed in Cunha and Ferreira (2003) for Brazil, and $\beta_{year} = 0.88 = \beta^{\frac{1}{20}}$

was selected so that incomplete markets economy without government has an equilibrium interest rate near 9% observed in Brazil. Only the case of pure altruism is considered: $\theta = 1$. Labor supply elasticity, $\frac{1}{\theta}$, was set to 0.1, as in Gomes et al. (2001).

Production function has the usual specification as for the US economy, except for a lower TFP: $\mathbf{o} = O(\mathbf{k}, \mathbf{l}) = z\mathbf{k}^{\nu}\mathbf{l}^{1-\nu}$, where $z = 0.7$ and $\nu = 0.36$. Capital depreciates at a rate of 6.5% each year (Val and Ferreira, 2001), implying $\delta = 1 - (1 - 0.065)^{20} \cong 0.74$.

The stochastic process driving ability satisfies $\log a' = \iota(1 - \omega) + \omega \log a + \sigma\sqrt{1 - \omega^2}\zeta$, $\zeta \sim N(0, 1)$. It is well-known this process is amenable to an approximation by a discrete Markov process, which will be useful in simulations. Productivity function is $H(a, m, n) = \eta a^{\chi}[\xi n^{\varepsilon} + (1 - \xi)m^{\varepsilon}]^{\frac{\rho}{\varepsilon}} + a$, $\varepsilon \leq 1$, as in Aiyagari et al. (2002). The fixed cost associated with m is arbitrarily selected: $\phi = 0.01$. ι, ω, σ, χ, η, ξ, ε and ρ are chosen so that the incomplete markets economy with $\tau = 0$ replicates income Gini index in Brazil (0.6). A summary of the calibration procedures in contained in Table 1.

3 Results

3.1 Incomplete Markets without government

Income Gini is 0.61, which is in line with recent empirical evidence in Brazil (Menezes-Filho, 2001), and interest rate is 7.7%. In efficient markets, there is a threshold level of ability, $a^* = 2.71$, such that every agent becomes skilled if his ability is strictly higher than a^* and remains unskilled otherwise. In an incomplete markets structure, 39% of agents with ability below 2.71 become skilled, characterizing a misinvestment phenomenon.

There is also an overinvestment phenomenon, in both human and physical capital, in incomplete markets, because agents are subject to idiosyncratic shocks and insurance markets are missing, and it is well-known, since at least Aiyagari (1994), that such setting gives rise to a overaccumulation behavior. Physical capital stock is 355% higher, investment in human capital measured by $\mathbf{m}$ rises 35% and investment in child-care (wn) increases 1.5%. The fraction of skilled agents consequently increases to 56% (as opposed to 43% in efficient markets).

In the set-up developed here, one may evaluate the "quality" of labor force by looking at the correlation between ability and labor supply. In complete markets, the correlation is 0.79; it decreases to 0.7 in incomplete markets. See tables 2 and 3 for a summary of the statistics presented in text and more.

Average consumption net of disutility of working is 44% higher in incomplete markets, and steady-state production is 48% higher at the same time. Therefore, agents are better off in terms of utility. However, when we perform a transition from incomplete to complete markets, we find that a typical would prefer to live in the last world ($\lambda_{tr} = -0.62$).[4]

3.2 Incomplete Markets with government

Suppose first there is no public debt and the government taxes agent's labor income. Solving (CP3) for $\tau \in [0, 1]$, we found $\tau(\lambda)$ to be a concave function which is maximized at $\tau^* = 0.55$.[5] At this tax level, standard inequality indexes are reduced. For example, income Gini is 22% and correlation of (log) income across generations decreases from 0.90 to 0.84. Moreover, consumption smoothing is more efficient: coefficient of variation of consumption is 60% lower.[6] If the correlation between labor supply and ability is a good proxy of the quality of labor supply, it must be noticed that such dimension is restored when government imposes an optimal tax: it is now 0.78, contrasting with 0.79 in efficient markets.

With respect to efficient investment in children, now 5% of agents with ability $a \leq a^*$ receive positive amounts of investment in human capital. However, if $\tau = \tau^*$ only 60% of high-ability agents become skilled, contrasting with 80% when $\tau = 0$.

Finally, if we keep tax rate at 0 or 0.55 and let $\frac{\bar{d}}{GDP}$ to vary, it is clearly noticed that welfare is reduced. Figure 3 illustrates the case when $\tau = 0.55$.

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[4]See Appendix A for a description of the algorithm which computes transition path.

[5]The upshot of the decomposition of welfare gains (as of Proposition 1 in Nascimento (2004)) is shown in figure 1.

[6]See tables 2 and 3 for most statistics presented in this section.

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A Appendix

A.1 Complete Markets Stationary Equilibrium

Conditions characterizing efficient steady-state are derived similarly to Aiyagari et al. (2002) framework with exogenous labor supply. (4) to (7) are straightforward. To find (8) and (9), firstly notice that, assuming a stationary setting and an actuarially fair price for insurance, budget constraints in (CP2) yield:

$$c^y + m + \phi I(\pi', a) + wn + \int q(a'|a)\left(c^{o'} + \frac{b'}{1+r}\right) da' = w\pi l^y(w\pi) - g(l^y(w\pi)) + \frac{wl^{o'}(w) - g(l^{o'}(w))}{1+r} + \frac{b}{1+r}. \quad \text{(A1)}$$

Thus, a young agent cares only about the present value of his income net of disutility of working, not how it splits between total wages net of disutility of working and bequests. An innocuous algebra in the right-hand side of (A1) implies it can be rewritten as a function of $w\left(\pi l^y(w\pi) - a_{-1}l^y(wa_{-1})\right) - \left(g(l^y(w\pi)) - g(l^y(wa_{-1}))\right) + \frac{b}{1+r}$, a, w and r. Therefore, omitting w and r, the value function of a young agent simplifies to

$$V(\pi, a, b) = W\left(w\left(\pi l^y(w\pi) - a_{-1}l^y(wa_{-1})\right) - \left(g(l^y(w\pi)) - g(l^y(wa_{-1}))\right) + \frac{b}{1+r}, a\right). \quad \text{(A2)}$$

Suppose the young agent solves its maximization problem subject to the constraint $m, n > 0$. To provide his offspring an extra premium in terms of labor income net of disutility of working, $w\left(\pi' l^y(w\pi') - al^y(wa)\right) - \left(g(l^y(w\pi')\right)$, costs him $m + \phi + wn$ today. Given the form of the value function, (A2), this amount is worth the same as

$$(1+r)\left(w\left(\pi l^y(w\pi') - al^y(wa)\right) - \left(g(l^y(w\pi')\right)\right)$$

in bequests, which costs

$$\int q(a'|a)\left(w\left(\pi' l^y(w\pi') - al^y(wa)\right) - \left(g(l^y(w\pi')) - g(l^y(wa))\right)\right) da' = \frac{\left(w\left(\pi' l^y(w\pi') - al^y(wa)\right) - \left(g(l^y(w\pi')) - g(l^y(wa))\right)\right)}{1+r}$$

in terms of current resources. Hence, it is true that, in order to skill the child, the costs associated with investment in human capital must fall below the costs of leaving the labor income premium in bequests, that is: $m + \phi + wn < \frac{\left(w\left(\pi' l^y(w\pi') - al^y(wa)\right) - \left(g(l^y(w\pi')) - g(l^y(wa))\right)\right)}{1+r}$.

A.2 Transitional Dynamics Algorithm

Suppose that, given $\mathbf{k}_0$ and Π_0 (productivity stationary distribution in incomplete markets), efficient markets are restored in $t = 1$. Our task is to find sequences $(\mathbf{k}_t)_{t=1}^T$, $(\mathbf{l}_t)_{t=1}^T$ and $(\mathbf{m}_t)_{t=1}^T$ such that complete markets steady-state is reached at $T + 1$ sufficiently large. The algorithm consists of the following steps:

1^{st}) Make an initial guess for transitional paths denoted by $(\mathbf{k}_t^0)_{t=1}^T$, $(\mathbf{l}_t^0)_{t=1}^T$ and $(\mathbf{m}_t^0)_{t=1}^T$, implying a guess for wages, $(w_t^0)_{t=1}^T$, according to (12).

2^{nd}) At $t = 0$, $\mathbf{k}_0$ and Π_0 are the givens. Given w_1^0, $\mathbf{m}_1^0$ and $\mathbf{k}_2^0$ solve for a_0^*, $M_0(\cdot)$, $N_0(\cdot)$, $\mathbf{l}_0$ and $\mathbf{k}_1$ using:

$$w_0 N_0(a_0^*) + M_0(a_0^*) + \phi = \frac{w_1^0[\pi' l^y(w_1^0\pi') - a_0^* l^y(w_1^0 a_0^*)]}{1 + r_0} - \frac{[g(l^y(w_1^0\pi')) - g(l^y(w_1^0 a_0^*))]}{1 + r_0}, \quad \text{(A3)}$$

where $\pi' = H(a_0^*, M_0(a_0^*), N_0(a_0^*))$, $w_0 = O_2(\mathbf{k}_0, \mathbf{l}_0)$ and $r_0 = O_1(\mathbf{k}_1, \mathbf{l}_1^0) - \delta$,

$$w_1^0 H_2(a_0^*, M_0(a_0^*), N_0(a_0^*)) = 1 + r_0, \quad \text{(A4)}$$

$$w_1^0 H_3(a_0^*, M_0(a_0^*), N_0(a_0^*)) = (1 + r_0)w_0, \quad \text{(A5)}$$

and

$$\left[O(\mathbf{k}_0, \mathbf{l}_0) + (1-\delta)\mathbf{k}_0 - \mathbf{m}_0 - \mathbf{k}_1 - \int g(l^y(w_0\pi_0))d\Pi_0 - g(l^o(w_0))\right]^{-\mu}$$
$$= \beta\theta(1 + r_0)\left[O(\mathbf{k}_1, \mathbf{l}_1^0) + (1-\delta)\mathbf{k}_1 - \mathbf{m}_1^0 - \mathbf{k}_2^0 - \int g(l^y(w_1^0\pi'))dA - g(l^o(w_1^0))\right]^{-\mu} \quad \text{(A6)}$$

Notice that $\mathbf{l}_0 = \int l^y(w_0\pi_0)d\Pi_0 + l^o(w_0) - \int N_0(a)dA$. Equations (A3) to (A5) are derived from first-order conditions in complete markets (section 2.1). It can be shown that (20) holds using (4), (5) and the perfectly-pooled equilibrium hypothesis.

In order to solve the above system, make an initial guess for $\mathbf{l}_0$ and $\mathbf{k}_1$, and then solve for a_0^*, $M_0(\cdot)$ and $N_0(\cdot)$ using (A3) to (A5). Compute a revised guess for $\mathbf{l}_0$ and $\mathbf{k}_1$ using $\mathbf{l}_0 = \int l^y(w_0\pi_0)d\Pi_0 + l^o(w_0) - \int N_0^{new}(a)dA$ and (20). Iterate until convergence is achieved in a_0^*, $M_0(\cdot)$, $N_0(\cdot)$, $\mathbf{l}_0$ and $\mathbf{k}_1$.

3^{rd}) Given $\mathbf{k}_1$ and Π_0 previously found, conduct a similar procedure as in 2^{nd}) and solve for a_1^*, $M_1(\cdot)$, $N_1(\cdot)$, $\mathbf{l}_1$ and $\mathbf{k}_2$ given w_2^0, $\mathbf{m}_2^0$ and $\mathbf{k}_3^0$. Travel down the entire path this way. Update initial guesses with $(\mathbf{k}_t^1)_{t=1}^T$, $(\mathbf{l}_t^1)_{t=1}^T$ and $(\mathbf{m}_t^1)_{t=1}^T$ given by the solution in each period.

4^{th}) Repeat until convergence is achieved in $(\mathbf{k}_t^j)_{t=1}^T$, $(\mathbf{l}_t^j)_{t=1}^T$ and $(\mathbf{m}_t^j)_{t=1}^T$. Finally, check whether child-care market clearing condition, (14), holds along the equilibrium path.

B Appendix

Table 1

Parameter	Value	Source or Target
Time Unity	20 years	AGS
	Tastes	
β	$0.88^{\frac{1}{20}}$	interest rate $\cong 9\%$
μ	2	Cunha and Ferreira (2001)
θ	1	pure altruism
$\tilde{\theta}$	10	Gomes et al. (2001)
	Production	
z	0.7	
ν	0.36	AGS
δ	0.74	Val and Ferreira (2001)
	Ability and Productivity	
ι	1	income Gini index
ω	0.7	"
σ	0.45	"
χ	2.92	"
η	0.4	"
ξ	0.31	"
ε	0.5	"
ρ	0.2	"
ϕ	0.01	

Table 2

Inequality Statistics for Income			
	Gini	$\frac{\text{Top 10\%}}{\text{Bottom 40\%}}$	$\frac{\text{Top 20\%}}{\text{Bottom 20\%}}$
Data*	0.6	24	27.4
Model (incomplete markets with $\tau = 0$)	0.61	5.9	20.01

*PNAD (1981-2001)

Table 3

Summary of Statistics			
Statistics	Complete Markets	Incomplete Markets with $\tau = 0$	Incomplete Markets with $\tau = 0.55$
Gini of Inc.	-	0.61	0.48
Gini of Lab. Inc.	0.69	0.62	0.64
Gini of Wealth	-	0.67	0.25
Coeff. of Var. of Cons.	-	2.09	0.84
$corr(\log a, \log a_{-1})$	0.70	0.70	0.70
$corr(\log lab.inc., \log lab.inc._{-1})$	0.63	0.80	0.78
$corr(\log inc, \log inc_{-1})$	-	0.90	0.84
$corr(lab., a)$	0.79	0.70	0.78
Annual Interest Rate	13.6%	7.7%	8.1%
Physical Capital	0.032	0.145	0.102
Skilled Agents	43%	56%	30%
Constrained	-	53%	94%

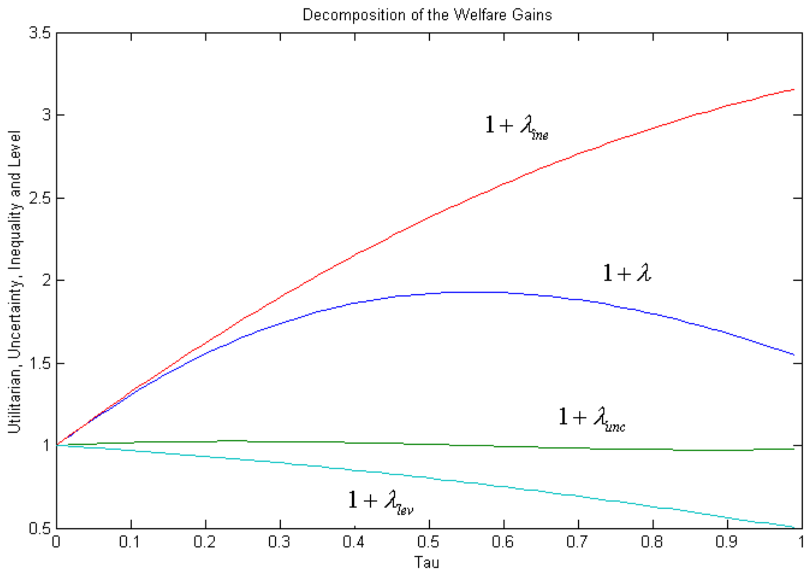

Figure 1

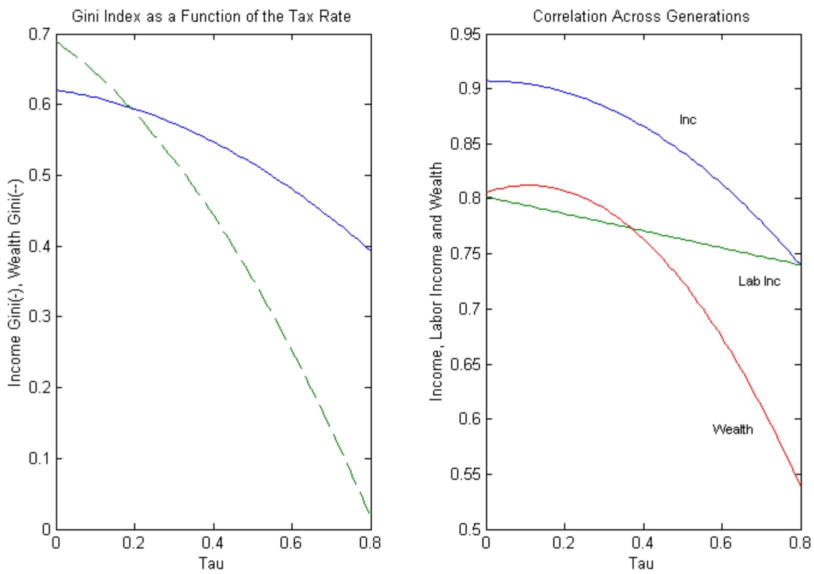

Figure 2

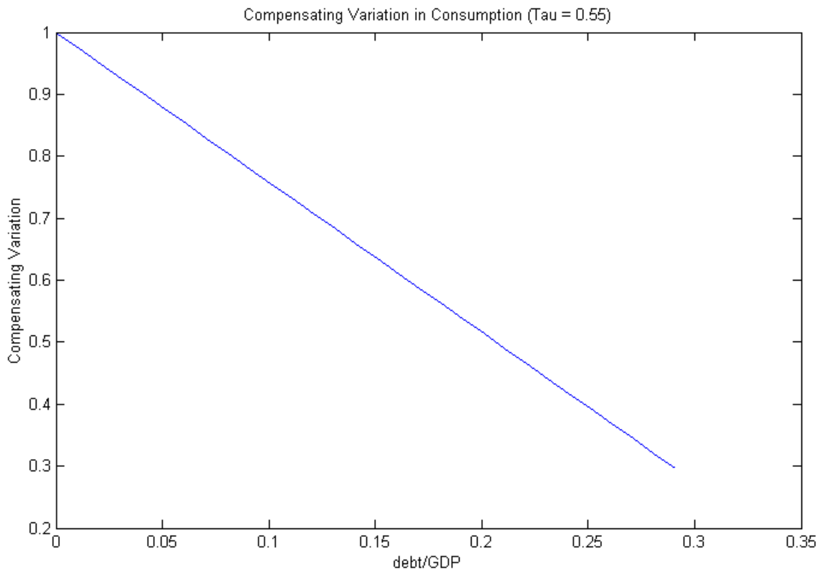
Compensating Variation in Consumption (Tau = 0.55)
Compensating Variation
debt/GDP
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35

Figure 3