CHAPTER 7

An Application: The Commons Problem

In this chapter we will discuss yet another application of Nash equilibrium, the usage of a commonly owned resource (such as deep-sea fisheries or the environment). One conclusion that we will draw from our study is that such resources are often overused. On account of this overuse, the phenomenon goes by the name of the *tragedy of the commons*.

Section 7.1 will provide a background to the commons problem. In section 7.2 we will introduce a simple model and characterize its Nash equilibrium. Section 7.3 will contrast the equilibrium overuse with the socially desirable outcome, and the subsequent section will show that this contrast is even more stark in a large population. Section 7.5 will discuss several case studies: buffalo, global warming, and the Internet. Possible resolutions to the tragedy will be the topic of section 7.6.

7.1 BACKGROUND: WHAT IS THE COMMONS?

The terminology—and background—to this problem can be traced to 16th-century England. Villages were designed in such a fashion that in the center of a village there would be a piece of green land that everybody could use. In addition to its serving as a site for village celebrations, this land—the *commons*—was also used by villagers as pastureland for their cattle.[1] There was, of course, pastureland that was privately owned—and in those cases only the owner had access to that land. However, everybody had access to the commons.

Indeed a common property resource is more than just a historical curiosity. In today's world, international waters (and the fish in them) form a common. The waters on the high seas, outside national boundaries, are commonly owned by all nations of the world, and so each one of these nations—or strictly speaking, fishing companies in

[1] *Indeed, even towns in the United States that were settled early—and presumably by Englishmen—have* village commons. *For instance, Morristown, New Jersey, has one. Only these days it is surrounded by Macy's and Wendy's with no cattle in sight!*

these nations—can fish freely in these waters. Perhaps the most important example of a common natural resource is the environment. Every one of us "owns" the environment (jointly) with everyone else. Usage of this common resource, by individuals or firms, adds pollution and greenhouse gases. For example, the burning of fossil fuels such as coal, oil, and natural gas adds carbon dioxide to the earth's atmosphere.

There are other examples that are not as immediate but that have the feature that everybody has access to the resource or commodity in question. Library books are commonly owned by all the students at any one college. Another (New York City) example is fire hydrants. Although these are meant to be used only by the fire department and only in an emergency, anybody who has spent a summer afternoon in the city knows that such an image is merely a myth perpetuated by the fire department. In actual fact, anybody can use the hydrants for some impromptu cooling off!

As the preceding discussion suggests, there are two critical characteristics of a commons:

- **Access to (nearly) everyone**. It is infeasible (think of the environment) or undesirable (think of National Parks) to restrict access.

- **Resource depletability**. The more people use the resource—or the more intensively each person uses the resource—the less there is of the resource in the future.

The second characteristic is worth a comment. The reason why we worry about commonly owned resources is precisely because they can be used over a long, long period of time—and by generations of our descendents—provided they are used in such a manner that they regenerate from time to time. The more fish are in the waters today, the more there will be a year from now. Conversely, the smaller is the buildup of carbon dioxide in the earth's atmosphere today, the less there will be 50 years from now. In other words, the result of our usage today bears a stamp well into the future. Of course this is not to say that the resource should not be used today at all; rather, the question is how much constitutes the "best" usage. And that brings us to the tragic part.

On account of the commonality of access, there are two sources of *externality* in the usage of such a common property resource or commodity: (1) a *current externality*—one person's usage may decrease the benefits to usage for other people—if you borrow a library book then I cannot use it today; and (2) a *future externality*—the condition in which you return the book determines how long the library will be able to keep it in circulation. Indeed, in many interesting problems—such as the environmental ones—the important externality is really the future externality. In this sense, this problem is different from the duopoly application of the previous chapter. In that application, overproduction by one firm generates an immediate externality for the other because it reduces the market price. However, this externality does not carry over into future market interactions.

The consequence of the fact that the resource is commonly owned is that it might get overused. This conjecture—or phenomenon, if indeed it is a correct conjecture—is

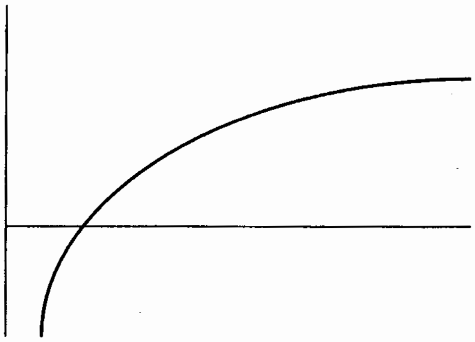

FIGURE 7.1

called the *tragedy of the commons*. The intuition for this conjecture is straightforward. Consider pollution. If your firm produces a chemical pollutant that it releases into the local river rather than treating it to remove the pollutant, you save yourself the costs of treatment. You—and everybody else in town—pay a cost for the polluted river, but this cost is shared among everyone. Hence, unless checked by regulation or your exemplary conscience, you have every incentive to dump that pollutant into the river waters. What is worse is that every other firm in town has exactly the same incentive to pollute. If you all ended up dumping your wastes in the river, the townspeople would have a very polluted river. They would have a tragedy of the commons.

7.2 A SIMPLE MODEL

Suppose that we have a common property resource of size $y > 0$. Each of two players can withdraw a nonnegative amount—c_1 or c_2—for consumption, provided of course that $c_1 + c_2 \leq y$. In the event that they attempt to consume in excess of what is available, suppose that the total amount is simply split between them; that is, each player ends up consuming $\frac{y}{2}$. When total consumption is less than y, then the leftover amount, $y - (c_1 + c_2)$, forms the future resource base, out of which comes future consumption. To keep matters simple, let us collapse the future into just one more period of consumption; that is, there are two time periods in this model.

In period 2, each player has to decide how much to consume out of the available quantity $y - (c_1 + c_2)$. Since there are no more periods left, there is no reason to save any fraction of the amount that is available in period 2. Each player would therefore like to consume as much as possible. Hence, in period 2, the total amount is divided among them—each gets $\frac{y-(c_1+c_2)}{2}$.

Now let us turn to period one. If player 1 were to consume an amount c_1, his utility from doing so will be taken to be $\log c_1$; that is, his utility of consumption looks like Figure 7.1.[2]

[2] *We assume that the Von Neumann–Morgenstern utility function is of this logarithmic form for computational simplicity alone. Much of what follows is true for any concave utility function.*

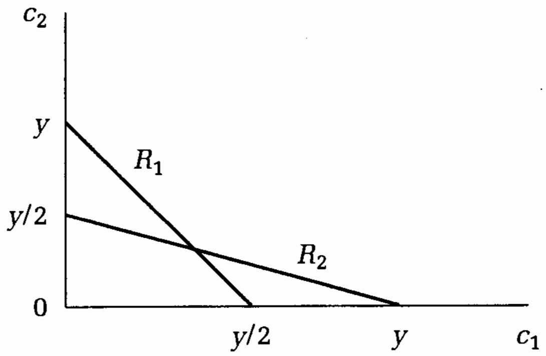

FIGURE 7.2

Player 1 has to determine how much he would like to consume out of the available stock of the resource. His utility depends on the amount that player 2 chooses to consume because the latter (partly) determines the size of the resource base that will be left over. As always, player 1 has a conjecture about player 2's consumption and determines his consumption level on that basis. In other words, Player 1's best response problem is

$$\text{Max}_{c_1} \ \log c_1 + \log \frac{y - (c_1 + \bar{c}_2)}{2}$$

where $\bar{c}_2$ is the amount that player 1 conjectures player 2 is going to consume in the first period. From the first-order conditions, player 1's best response consumption is[3]

$$\frac{1}{c_1} = \frac{1}{y - (c_1 + \bar{c}_2)}$$

or

$$c_1 = y - (c_1 + \bar{c}_2)$$

$$R_1(c_2) = \frac{y - c_2}{2}$$

CONCEPT CHECK

By similar logic, carefully derive the best response function of player 2 and show it to be

$$R_2(c_1) = \frac{y - c_1}{2}$$

The two reaction functions are pictured in Figure 7.2. The Nash equilibrium is therefore given by consumption levels c_1^* and c_2^* such that $R_1(c_2^*) = c_2^*$ and $R_2(c_1^*) = c_1^*$. Substituting into the reaction functions, we can compute the Nash equilibrium to be

$$c_1^* = c_2^* = \frac{y}{3}$$

[3] *The derivative of* $\log x$ *is* $\frac{1}{x}$. *The derivative of* $\log(ax + b)$ *is* $\frac{a}{ax+b}$. *(See Chapter 25.)*

In the first period, each player consumes $\frac{y}{3}$, leaving a total of $\frac{y}{3}$ for the second period, which is then split in half to yield a consumption of $\frac{y}{6}$ each in the second period. Each player's utility from this consumption pattern is $\log \frac{y}{3} + \log \frac{y}{6}$.

7.3 SOCIAL OPTIMALITY

In order to determine whether or not this equilibrium pattern constitutes a tragedy, let us see what pattern of resource use would constitute socially optimal usage. Let us define social optimality as follows: suppose that these two players constituted society and got together to decide how much they should each consume for the "common good." The common good is equivalent to making sure that the aggregate utility is maximized:

Definition. A pattern of consumption, $\widehat{c}_1, \widehat{c}_2$, is socially optimal if it maximizes the sum of the two players' utility, that is, if it solves the following problem:

$$\text{Max}_{c_1,c_2} \log c_1 + \log c_2 + 2 \log \frac{y - (c_1 + c_2)}{2}$$

This procedure leads to a socially optimal solution of

$$\widehat{c}_1 = \widehat{c}_2 = \frac{y}{4} \tag{7.1}$$

CONCEPT CHECK

Write down the first-order conditions, one for $\widehat{c}_1$ and another for $\widehat{c}_2$, of the social optimality problem. Solve those conditions to derive the socially optimal levels of consumption, equation 7.1.

Note that in this case, exactly half of the resource is used in the first period—in contrast to the Nash equilibrium where as much as two-thirds of the resource is used up in the first period.[4] In this sense we have an overextraction of the resource in the Nash equilibrium; that is, we have a tragedy of the commons.

The reason for this overextraction is precisely the externalities discussed previously. Notice that if player 1 were to cut back his consumption in the first period by one unit, he makes this one unit available for consumption by both players in the second period. He is, however, able to recover only half of this unit as additional consumption in the second period, because the other half accrues to player 2. Consequently, player 1—and by extension both players—tends to overconsume. This phenomenon does not arise in the socially optimal solution, however, because in that solution we

[4] *It should be clear that in the socially optimal solution, each player then consumes a quarter of the resource in the second period as well.*

care about the utilities of both players. A unit of consumption set aside for tomorrow—by either player—continues to be a unit of consumption for society as a whole in the second period. Hence, there is no externality that distorts first-period consumption.

7.4 THE PROBLEM WORSENS IN A LARGE POPULATION

To fix these ideas even more firmly, let us see what happens in this example as the number of players increases. The intuition would be that this increase makes the tragedy even more acute. The reason is that if player 1 forgoes a unit of consumption in the first period, he is able to retrieve only $\frac{1}{N}$ of that unit as consumption in the next period (where N is the number of players). That should make player 1 even less willing to set aside any resource for the future.

If player 1 conjectures that the others will consume an amount $\bar{c}$ in the first period, then his utility-maximizing consumption is determined from the following optimization problem:

$$\text{Max } \log c_1 + \log \frac{y - [c_1 + (N-1)\bar{c}]}{N} \tag{7.2}$$

From the first-order conditions, player 1's best response consumption is

$$\frac{1}{c_1} = \frac{1}{y - [c_1 + (N-1)\bar{c}]} \tag{7.3}$$

In the Nash equilibrium in which each player consumes the same amount, that is, $c_1 = \bar{c}$, it follows from equation 7.3 that the equilibrium consumption level is

$$c_1 = c_2 = \cdots = \frac{y}{N+1} \tag{7.4}$$

CONCEPT CHECK

Derive equations 7.2, 7.3, and 7.4.

The total consumption in a Nash equilibrium is therefore $\frac{N}{N+1}y$. Consequently, the amount that remains after the first period is $\frac{y}{N+1}$. As N becomes large, a vanishingly small amount of the resource reaches the second period. *The tragedy of the commons is exacerbated in large populations.*

Concept Check
Social Optimality

Show that in order to maximize the total utility of all players, consumption should be

$$c_1 = c_2 = \cdots = \frac{y}{2N}$$

In other words, society would always like half the resource to be saved for the future, no matter how many members there are in society. This result should also be intuitive. A unit of consumption withheld from social consumption today is always available as a unit of consumption in the future, regardless of how many members there are in society. So the conclusion that half of the total resource should be withheld for social consumption in the future is a result that is independent of the number of players.

7.5 CASE STUDIES: BUFFALO, GLOBAL WARMING, AND THE INTERNET

A very popular historical example of the tragedy of the commons is that of the *buffalo in the American West*. Till the middle of the 19th century great herds of American buffalo roamed the Great Plains. They were a commons in that any hunter could hunt them down if he so desired. Starting in the middle of the century, the hunting grew increasingly widespread and effective, and the buffalo herds had a more and more difficult time regenerating. By the end of the century many of these sprawling herds had disappeared. Here is a commentary on their usage:

> *The Plains Indians very eagerly accepted modern technology in the form of horses, guns and steel tools. . . . By 1840 the Indian had driven the buffalo from portions of the Great Plains area and there is evidence of concern about this problem. . . . Given multiple tribes and the fact of no real cooperation, the Indians were incapable of managing the buffalo as a common pool resource. . . . The benefits from harvesting one more buffalo accrue to the individual hunter, while the costs of depletion of the herd are spread among all hunters. The true costs of hunting are not borne by the hunter and overuse is predictable.* (American Indians: A Test of the Conservation Ethic, *by John Baden, Rick Stroup and Walter Thurman).*[5]

Here is a more recent example, *global warming*. Much of the energy that is produced to heat our homes, drive our cars, or produce our economy's goods and services

[5] *As quoted in* Costs of Innovation in the Commons, *September 1996, by Douglas Noonan, mimeo, Foundation for Research on the Economics and the Environment.*

comes from carbon sources. It comes from burning coal, oil, or natural gas. This combustion releases carbon dioxide into the earth's atmosphere and (although some of it is broken down by plant photosynthesis) much of it remains in the atmosphere. Carbon dioxide (and certain other "greenhouse gases") traps heat, in much the same way that a greenhouse does. Imagine then the total amount of carbon dioxide in the atmosphere as a common pool, since it is this total that determines temperatures all over the world. Each individual or firm—or simply, each nation—adds to this stock of carbon dioxide by its energy consumption. Whereas the benefits to this energy consumption—a higher national output, or a better standard of living—are entirely experienced by the energy user, the costs—hotter temperatures and consequent climatic and economic dislocation—are shared by everyone. Scientists say that average surface temperatures have risen between 1 and 2°F in the last 200 years and, more alarmingly, can arise another 4 to 8°F in the next 100 years if present trends of carbon dioxide buildup continue.[6]

Here is another recent example; let us call it the *Internet jam*. Starting in the summer of 1996, a number of online companies started offering a fixed-monthly-rate service; you pay a fixed rate, say $19.99, for the month, and you can gain access to the Internet as many times and for as long as you want that month. Under the pricing practice that existed before, consumers were charged an hourly rate for Internet access.[7] Note that since customers use phone lines to access the Internet, the resources of the phone system—the switches, the lines, and so on—can be thought of as a commons for all Internet users.[8] Once the monthly fee has been paid, it is a sunk cost; that is, it is as if Internet access is a free good. It is not of course free for society, since there is a fixed capacity for the phone lines. Unsurprisingly, phone systems were completely overloaded, systems broke down several times, and some customers complained of having to wait up to an hour before they could get through to the Internet.[9]

7.6 AVERTING A TRAGEDY

The question that many economists and other social scientists grapple with is how to balance the private desire for utility or profits against the social imperative of sustainable resource use. Many (incomplete) solutions have been proposed to deal with the tragedy of the commons. Some economists favor a privatization of the resource ownership. Privatization is clearly a solution that has been employed, up to a point, for land. There is no longer a common pasture in villages, nor are there common town centers in suburban developments.[10] Privatization would certainly remove the externality inherent in the common ownership of the resource. (Why?) The problem with privatization is that it simultaneously takes away the common access to usage that may be desirable for a variety of other reasons. Furthermore, for some resources—such as the environment—it is not even a practical solution.

Other economists have proposed a tax or fee for usage. This is the solution that is most widely used when dealing with pollution. It is also the solution that is used in

[6] *More details on global warming can be found in an authoritative report of the Inter-Governmental Panel on Climate Change,* Climate Change 95: The IPCC Scientific Assessment *New York: Cambridge University Press, 1995.*

[7] *The most significant switch came when the biggest access provider, America Online, started offering the new service in October 1996. Details on this story can be found in the* New York Times, *December 17, 1996.*

[8] *In the United States, some estimates put the number of people accessing the Internet at 50 million—and growing.*

[9] *In a development that only makes the tragedy of the commons worse, there is now software that will keep redialing for you and, once you get access, will keep you logged on while you sleep, go to class, or attend to the other duties of your daily life! (See the* Times *article referenced in note 7 for further details.)*

[10] *There is, of course, the ubiquitous shopping mall.*

National Parks. The problem here is that assessing fees is an inefficient solution whenever the marginal cost of production is near zero. In other words, if the cost of allowing one more person into the park is virtually zero, then that person should not be charged a high fee. Put differently, a fee of $5 would simply discourage from coming to the park a person who is only able to pay $2 (or who only derives a utility of $2 from using the park). This approach would be inefficient if the actual extra cost of cleanup is 10 cents. A desirable fee would be one that varies with the number of users, charging more as the park becomes more and more crowded. A proxy to this scheme is one that charges different fees for different times of the year (or even different times of day). The problem with these schemes is that they are more complicated to implement.

A third solution that has been proposed is to simply put a ceiling on the number of users of the resource. The size of the ceiling could be designed in such a manner that the resource is able to regenerate itself. For instance, park rangers close off Yosemite National Park in the middle of the afternoon if they determine that too many people have already entered the park that day. The same inefficiency problem discussed in the previous paragraph also plagues this solution. Yet from a long-term standpoint either of these last two solutions can be better than an unregulated tragedy of the commons.

It should be noted that the model analyzed in this chapter can be generalized in a variety of ways without changing the qualitative flavor of the result. First, we only analyzed an *exhaustible resource* problem; the amount of the resource that remains at the end of the first period, say, x, is precisely the amount available for consumption in the second period. However, the analysis carries over to a *renewable resource* model, that is, a model in which the amount x becomes something larger, say, $f(x) > x$, by the start of the second period. It is still true that there is a future externality because sacrificing a unit of consumption yields a player only half of the consequent increase in second-period resource stock. Hence, she overconsumes in the first period.

A second direction in which the model can be generalized is to allow several future periods or more general utility functions. Again the basic externality is unchanged and so therefore is the tragedy. Indeed in Chapter 18 you will see just such a generalization.

SUMMARY

1. A common (resource or commodity) has two characteristics: it is accessible to everyone, and its amount depletes upon usage.

2. In the Nash equilibrium of a commons game, each player overuses the resource because he gets all of the immediate benefits from usage and bears only a fraction of the future costs of depletion.

3. In a socially optimal solution, there is enough of the resource set aside for desirable regeneration.

4. The tragedy of the commons is exacerbated in large populations.

5. There are many real-world illustrations of the tragedy of the commons; the historical extinction of the American buffalo and the current possibility of global warming are two such examples.

6. Various solutions have been proposed to avert a tragedy. They include privatization, taxes or user fees, and limits to accessibility.

EXERCISES

SECTION 7.1

7.1
Give an example of a resource, natural or otherwise, to which there is not common access, that is, a resource that only selected people can use.

7.2
Give an example of a resource, natural or otherwise, that is not depletable, that is, a resource that does not deplete in quantity (for all practical purposes) when more people use the resource.

7.3
Is there a tragedy of the commons in either of your examples? Does a resource have to be commonly accessible and depletable for it to be overused?

7.4
Are the resources devoted to public education an example of a common property resource? Explain.

7.5
From your daily life can you think of a tragedy of the commons phenomenon? Explain your answer.

SECTION 7.2

(Calculus problem) The resource that we analyzed in the text is an exhaustible resource. To see that the phenomenon of the tragedy of the commons can also arise if the resource is *renewable*, consider the following variant of the model in the text. Each player extracts an amount c_i in the first period, $i = 1, 2$. Whatever is not extracted, that is, the amount $y - c_1 - c_2$, regenerates and becomes an amount equal to $\sqrt{y - c_1 - c_2}$ in period 2.[11] The rest of the model will be identical to that in the text; in particular, the utility function will be $\log c$, and the allocation rule (if the total desired is more than what is available) will be to give half to each player.

7.6

Write down the best response problem for player 1.

7.7

Show that the best response function is given by[12]

$$R_1(c_2) = \frac{2(y - c_2)}{3}$$

7.8

Compute the Nash equilibrium.

(Calculus problem) In the next few questions, we will walk you through a *private property* version of the common property model that is analyzed in the chapter. Suppose that we have two (duopoly) firms and they each own a homogeneous resource, such as a forest or a fishing pool, of size 25. Each of them can withdraw any nonnegative amount, c or d, in the first period, provided of course that $c \leq 25$, $d \leq 25$. There are two periods, and whatever is not extracted in the first period is extracted in the second. Finally, the extracted amounts are sold in a (common) market whose demand curve is given by

$$p = 100 - Q$$

where Q is the total amount extracted in any period. Assume that each firm is a profit maximizer and that costs of production are zero.

7.9

Write down the strategic form of this game.

7.10

What is the best response for firm 1 if it thinks that firm 2 will produce 7.5 units of output? What is the best response function of firm 1? (In answering this question, you will need to use the facts that profits are maximized when the slope of the profit function is zero and that the slope of a quadratic function $a + bx - cx^2$ is $b - 2cx$).

7.11

What is the Nash equilibrium extraction levels in period 1? period 2? What about market price?

[11] *In order that the resource is seen to grow, that is, in order for $\sqrt{y - c_1 - c_2}$ to be greater than $y - c_1 - c_2$, we will make the assumption that $y \leq 1$. This is simply an accounting convention; if the actual maximum amount is 100 we would simply consider a regeneration function like $\sqrt{100} \times \sqrt{y - c_1 - c_2}$.*

[12] *Use the fact that $\log \sqrt{x}$ is the same as $\frac{1}{2} \log x$.*

SECTION 7.3

7.12

Set up and solve the social optimality problem in the renewable resource problem. Show that the socially optimal extraction is $\frac{y}{3}$ for each player.

7.13

Explain why exercise 7.12 differs from the exhaustible resource model in which the socially optimal extraction is $\frac{y}{4}$ for each player.

7.14

In the private property model, how would your answers change if the firms operated as a cartel? Explain your answer carefully.

SECTION 7.4

7.15

Analyze the renewable resource problem for *N* players. Is it true that all of the resource is extracted in the first period if *N* approaches infinity?

7.16

Analyze the private property resource problem for *N* players. Is it true that the price of the resource is driven to zero as the number of firms increases without bound? Explain very carefully any assumptions that you make in order to do your analysis.

SECTION 7.6

7.17

Explain why privatizing the common resource would solve the overextraction problem.

7.18

Consider the *Internet jam* problem. Suggest a solution that would avert a tragedy in that case. Is the solution you propose in the interests of the online companies to implement? If so, speculate on why they have not done something similar already.