Elasticity

1. Economists use elasticity to avoid having to deal with units.

2. Elasticity is *responsiveness.* It is how one variable responds or changes with respect to another.

3. It is much easier to think of a percent change in price, rather than just a change in price which would necessarily involve some units of measure, both for the good itself and for currency in which the price of the good is expressed.

4. With elasticities, units problems melt away. They are also good for seeing how growth rates affect each other.

5. Elasticity is just percent change of the dependent variable with respect to the percent change in the independent variable. Hence if we have

$$Y = mX + b$$

as the expression for some economic quantity, demand, supply or really anything

$$\epsilon_{YX} = \frac{dY/Y}{dX/X}$$

or ϵ_{YX} the elasticity of Y with respect to X. When you have a continuous function, this work fine. But when no function is given, use the discrete form[1]

$$\epsilon_{YX} = \frac{\Delta Y/Y}{\Delta X/X}$$

6. Here is it easy to see that we are just working with a ratio of percent changes. Elasticity usually changes from point to point so we have to know where to start. Let's say we have a point X_0, Y_0. We want to calculate the elasticity from this point to X_1, Y_1. We know that by definition

$$\Delta X = X_1 - X_0 \text{ and } \Delta Y = Y_1 - Y_0$$

Keep in mind that ΔY is always measured as the second point minus the first point even if the second point is smaller. This just gives a negative elasticity.

7. Let's say the demand curve is

$$q = -4p + 6$$

We wish to calculate the elasticity of quantity demanded with respect to price at $p = 1$. The slope $dq/dp = -4$ and $p = 1$ gives $q = 2$.Hence the elasticity is $\epsilon = -4(1/2) = -2$. It will be different at a different p.

8. Elasticity can be visualized as the slope of the tangent to a function divided by the slope of the *chord line.* That is, the slope of a line from the origin to the point at which the elasticity is to be determined.

9. This is shown in Figure 1 for the function $y = f(x)$

10. The slope of the chord line is 1 and the slope of the function is $1/2$ so the elasticity at the point $x = 1$ is $\epsilon = 1/2$. Here are some extreme examples:

11. It makes a big difference which variable is on which axis.

12. Note that we have consistently defined at the elasticity of the y variable with respect to the x variable.

[1] The calculus form is when the ΔX goes to zero; that is:

$$lim_{\Delta X \to 0} \frac{\Delta Y/Y}{\Delta X/X} = \frac{dY/Y}{dX/X}$$

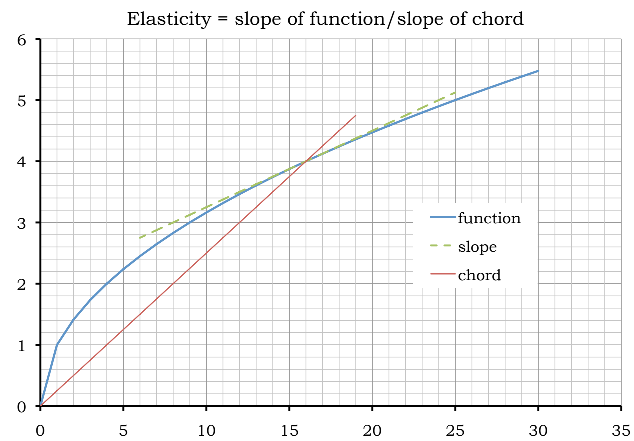

Figure 1: Elasticity is the slope of the function, $f'(x)$, divided by the slope of the chord line, y/x.

Figure 2: Perfectly or infinitely elastic function

Figure 3: Perfectly or infinitely inelastic function

13. It is easiest to stick with this convention. It is the same for slopes: it is always the rise (difference in the y-axis variable) over the run (difference in the x-axis variable.) Think of an elasticity as a slope on steroids.

14. When we talk about supply and demand the elasticity is always about quantity with respect to price. When the quantity is on the x-axis, the elasticity is the change in quantity with respect to price divided by the ratio of quantity to price.

15. Flat supply or demand curves are thus infinitely elastic while vertical supply or demand curves are perfectly inelastic.

16. A *straight line that goes through the origin* is unitary elastic, i.e., it has an elasticity of one.

17. A hyperbola is also unitary elastic but with a negative sign. The equation is

$$y = A/x$$

The slope is $dy/dx = -A/x^2$. Divide this by the average y/x to get

$$\frac{dy}{dx}\frac{x}{y} = \frac{-A}{x^2}\frac{x}{y} = -1$$

Exercises:

(a) Show mathematically that elasticity can be visualized as the slope of the tangent to a function divided by the slope of the chord line.

(b) Show that the elasticity of Y wrt to X can be thought of as answering the question of how the *growth rate of* X affects the *growth rate* Y.

(c) Show that elasticity of Y wrt X can also be expressed as

$$\epsilon = \frac{d \ln Y}{d \ln X}$$

What advantages of interpretation does this give?

(d) Show that the elasticity of Y wrt X cannot be constant for a linear function, except for a special case.

(e) Show that if Y is a linear function of X and the *intercept is negative,* the elasticity is *greater than one.* If the intercept is zero, the elasticity is constant and equal to 1. If the intercept is positive, the elasticity is less than one.

(f) Given two examples of constant elasticity functions. (*Advanced*: o.k, now give a third.)

(g) A consumption function has a negative intercept. Does this strike you as reasonable? Why or why not?