Opportunity Costs, PPF and Utility Maximization

Crusoe is shipwrecked on an island with only his labor at his disposal. He needs to consume food to survive. He finds that it takes l_c hours to fell one coconut and l_f hours to catch one fish. He is willing to work $\bar{L}$ hours per period. A basic question asked in economics is how much fish and how many cocos does Crusoe consume if he likes them according to the utility function:U

We use a *mathematical model*: defined as set of equations such that number of variables is equal to the number of equations. Each symbol in the model is either parameter or a variable. We use the model to solve for the variables in terms of the parameters. In this *neoclassical model of resource allocation* we have the following division between parameters and variables.

- parameters

1. *endowment* of resource labor is $\bar{L}$
2. *technology (labor coefficient*-hours of labor per unit of output) l_f, l_c
3. *Tastes characterized by the utility function,*$U = F^a C^{(1-\alpha)}$ where for simplicity we will write: $U = \sqrt{FC}$: 14.142

- Symbols $(F, C, U, \alpha, l_c, l_f, \bar{L})$,
- Variables: (F, C, U)
- Equations: utility function

$$U = \sqrt{FC}$$

1. Production possibility frontier (PPF),

$$\bar{L} = l_c C + l_f F$$

- Opportunity cost of cocos in terms of fish $= \frac{l_c}{l_f}$ is the *negative* of the slope of the PPF

$$-l_c/l_f$$

- To find the maximum level of utility, we must set the slope of the indifference curve equal to the slope of the budget constraint, that is, the PPF.
- The slope of the indifference curve is equal to the rise over the run. To make things easy, square both sides of the utility function:

$$U^2 = FX$$

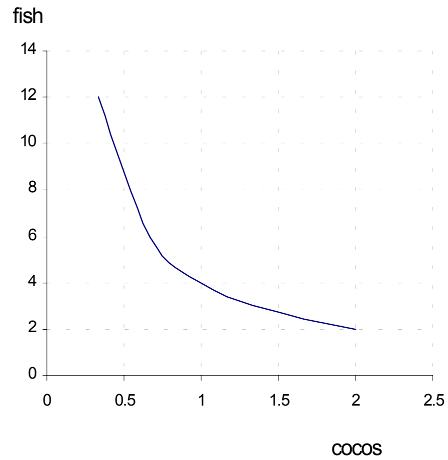

Figure 1:

- The indifference curve is defined as having the same level of utility. Thus on *two neighboring points, 1 and 2,*on the same indifference curve, we have:

$$U_1^2 = F_1 C_1$$
$$U_2^2 = F_2 C_2$$

Now subtract the two levels of utility to find zero on the left hand side:

$$0 = F_1 C_1 - F_2 C_2$$

Now add and subtract $F_1 C_2$

$$\begin{aligned} F_1 C_1 - F_1 C_2 - F_2 C_2 + F_1 C_2 &= 0 \\ F_1 (C_1 - C_2) - C_2 (F_2 - F_1) &= 0 \end{aligned}$$

and note that the slope of the indifference curve is defined as:

$$slope = -\frac{\Delta F}{\Delta C} = \frac{F_2 - F_1}{C_2 - C_1}$$

Thus we have

$$F_1 (C_1 - C_2) + C_2 \frac{\Delta F}{\Delta C} (C_2 - C_1) = 0$$
$$F_1 = -C_2 \frac{\Delta F}{\Delta C}$$

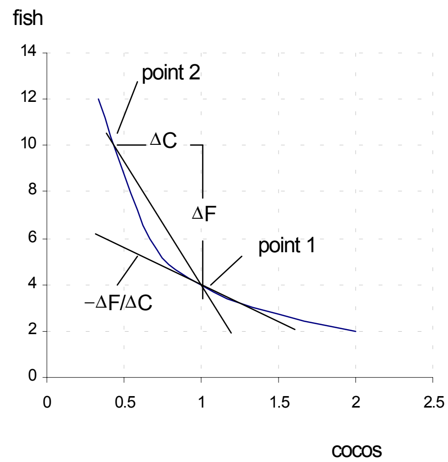

Figure 2:

Now let the two neighboring points come close together, that is let point 2 approach point 1. As the points come together we are no longer taking an average slope of the curve between two points, but the slope at one point. This is called the *instantaneous slope of the indifference curve:*

$$-\frac{F}{C} = \frac{\Delta F}{\Delta C}$$

and is different for each specification of the utility function.

- The slope of the indifference curve is called the *marginal rate of substitution* here of fish for cocos. This is how many fish one must substitute to get the same utility when the number of cocos goes down by one unit.
- To find the maximum utility, set the slope of the indifference curve equal to the slope of the budget constraint, or PPF. This is the *tangency condition*
- Thus the marginal rate of substitution of fish for cocos is equal to the opportunity cost of cocos in terms of fish.
- To solve for the optimal combination, use the tangency condition and the PPF and solve simultaneously.

$$\begin{aligned} \frac{l_c}{l_f} &= \frac{F}{C} \\ \bar{L} &= l_c C + l_f F \end{aligned}$$

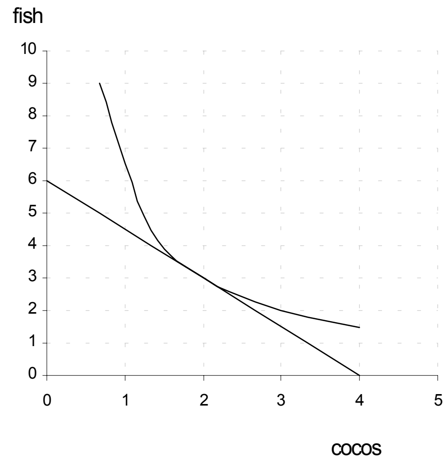

Figure 3:

The solution is :

$$F = \frac{1}{2}\frac{\bar{L}}{l_f}, C = \frac{1}{2}\frac{\bar{L}}{l_c}$$

- *Example*: let $l_f = 3$ and $l_c = 2$ and $\bar{L} = 12$. Crusoe will consume:

$$F = \frac{1}{2}\frac{\bar{L}}{l_f},$$
$$C = \frac{1}{2}\frac{\bar{L}}{l_c}$$

solution: $F = 2 : C = 3$