Shadow Values and Complementary Slackness

Shadow values are zero if the associated constraint is unbinding. Consider labor and capital in the figure below. Inside the both PPFs is the feasible region. At A only the labor constraint it binding; at B both constraints are binding. At C the capital constraint is binding. At A the shadow value of capital is *zero*. At C the shadow value of labor is zero. At B neither shadow value is zero. To actually calculate the shadow values (if not zero) more advanced methods are necessary. This is called *complementary slackness.* That is, if a constraint doesn't bind, it is slack and its complementary variable, the shadow price, is zero.

Tangency Condition for Rational Consumer

With $U = \sqrt{X_1 X_2}$ we have the slope as the rise over the run; take two points on the indifference curve and calculate the slope.

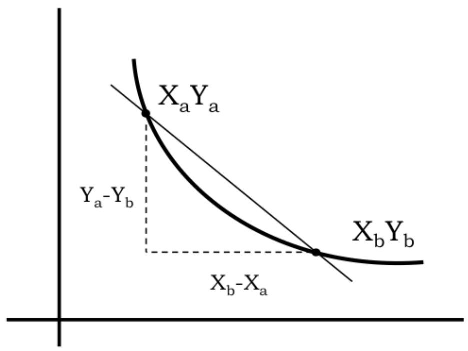

Figure 1:

So, squaring both sides, we have: $U_a^2 - U_b^2 = 0 = (X_1X_2)_a - (X_1X_2)_b$. Now add and subtract $X_{1a}X_{2b}$

$$0 = (X_{1a}X_{2a} - X_{1a}X_{2b}) - (X_{1b}X_{2b} - X_{1a}X_{2b})$$

Rearrange by factoring:

$$0 = X_{1a}(X_{2a} - X_{2b}) - X_{2b}(X_{1b} - X_{1a})$$

The rise is $-(X_{2a} - X_{2b})$ and the run is $(X_{1b} - X_{1a})$ so the slope, or rise over run, is:

$$\frac{rise}{run} = \frac{-(X_{2a} - X_{2b})}{(X_{1b} - X_{1a})} = -\frac{X_{2b}}{X_{1a}}$$

Now as a approaches b we have an instantaneous slope or *derivative.* If cocos is on the horizontal axis and fish on the vertical we have the slope of the indifference curve

$$-F/C$$

The slope of the indifference curve shows the *rate of substitution*) which shows how much of one good must be substituted for the other *to maintain the same utility.* Thus, if we have a point with a low F and a high C then we have a low rate of substitution. That is, if coco consumption is reduced by one, then just a small increase in fish is necessary to maintain the same level of utility. This is because the last coco gives a

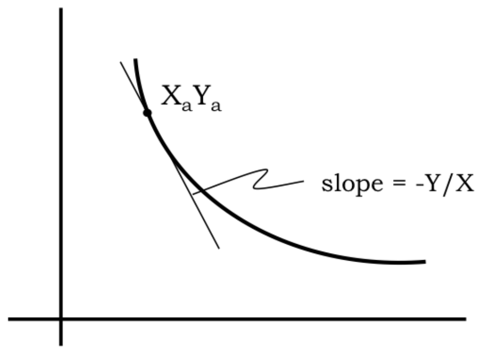

Figure 2:

small amount of added utility, the so-called *diminishing marginal utility.* Hence to maintain the same total utility, we need only have a small increase in fish.

This is also the slope of the PPF. It has an equation:

$$Labor(hours) = (hours/coco) * C + (hours/fish) * F$$

where C is the amount of cocos and F is the amount of fish. Thus, the slope of the PPF is $-(hours/coco)/(hours/fish)$ or $-(fish/coco)$. With 2 hours to catch a fish and 1 hour to fell a coco, the slope would be -1/2. Setting these two slopes equal is the *tangency condition.*