1 Consumer's problem

The consumer's problem is to maximize utility, $U = x\alpha y^{1-\alpha}$, subject to a budget constraint, $B = px + y$. It takes the budget, B, and the relative price of good x in terms of the numeraire good y. Tastes, given by α are also taken as given. The derivation starts with the givens and generates a demand curve for the first good. It also determines the demand for the second good, but since the price is fixed, it is not quite right to call that a demand curve. Finally it determines the total utility of the market basket of the quantities of the two goods selected by the consumer.

The basic idea is to solve two equations and two unknowns, x and y and then plug these values into the utility function to determine U. The two equations are first the budget constraint and second the **tangency condition**

$$p = [\frac{\alpha}{1-\alpha}]\frac{y}{x}$$

which is derived from setting the slope of the indifference curve (on the right) equal to the slope of the budget constraint (on the left) and canceling both negative signs.

Let's see how this is done:

steps:

1. multiply through the tangency condition by x to ge

$$px = [\frac{\alpha}{1-\alpha}]y$$

2. Substitute this px into the budget constraint

$$B = px + y = [\frac{\alpha}{1-\alpha}]y + y$$

3. clear the fraction by multiplying through by $1 - \alpha$

$$(1-\alpha)B = \alpha y + (1-\alpha)y$$

4. simplify to find

$$(1-\alpha)B = y$$

5. Next find x by substituting the y expression into the budget constraint

$$px = [\frac{\alpha}{1-\alpha}]y = \alpha B$$

6. So that

$$x = \frac{\alpha B}{p}$$

7. This is a non-linear demand function

8. Finally plug both x and y into the utility function

$$U = [\frac{\alpha B}{p}]^{\alpha}[(1-\alpha)B]^{1-\alpha}$$

2 Some examples

Example 1. Solve the following consumer's problem with $\alpha = 0.4$, $p = 1$ and $B = 2$

$$\begin{aligned} x + y &= 2 \\ 1 &= \frac{\alpha}{(1-\alpha)} \frac{y}{x} \end{aligned}$$

Solution is $[y = 1.0\,2, x = 0.8]$.

3 Tangency condition explained

This tangency condition can also be expressed in words as setting the **marginal utility per dollar** to be the same for all goods.

T