Does the marginal product of labor rise or fall with an increase in the share of capital (β)?[1]

June 3, 2019

1

The problem

$$M = (1 - \beta)k^{\beta} \tag{1}$$

Where M is the marginal product of labor and k is the capital labor ratio. Now differentiate with respect to β First take ln of both sides

$$\ln M = \ln(1 - \beta) + \beta \ln\ k \tag{2}$$

and differentiate

$$\frac{dM}{d\beta}(1/M) = -1/(1 - \beta) + \ln k \tag{3}$$

simplify

$$dM/d\beta = -k^{\beta} + M \ln k \tag{4}$$

$$dM/d\beta = -k^{\beta} + (1 - \beta)k^{\beta} \ln k \tag{5}$$

$$dM/d\beta = k^{\beta}[(1 - \beta) \ln k - 1)] \tag{6}$$

So the sign depends on the sign of $[(1 - \beta) \ln k - 1)]$. If $\beta = 0.25$, then $\ln k > 4/3$, which is very likely. TheN $\ln 4 = 1.4$, for example. So as long as the capital-labor ratio is 4, then the marginal product increases with β. Figure 2 shows that when the the $\ln k$ is low, the marginal product first increases and then decreases. Increasing k sharply shifts the function to having a positive first derivative in the relevant range.

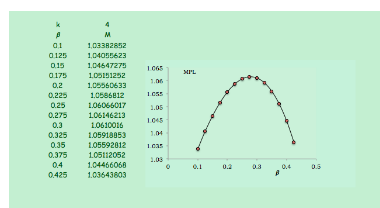

Figure 1: A graph of the marginal product of labor as a function of β with $k = 4$

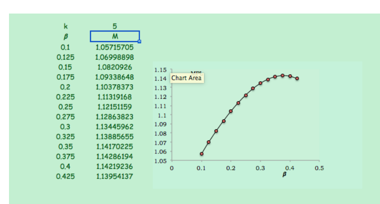

Figure 2: A graph of the marginal product of labor as a function of β with $k = 5$

What about informal sector? If the informal sector wage is

$$w = Q_n/(\bar{L} - L) \tag{7}$$

where Q_n is informal output and $\bar{L}$ is the labor supply. Now differentiate using the chain rule

$$w(\bar{L} - L) = Q_n \tag{8}$$

Or:

$$w\bar{L} - wL = Q_n \tag{9}$$

Now differentiating

$$dw\bar{L} - wdL - Ldw = 0 \tag{10}$$

Simplifying

$$dw/dL\bar{L} = w - Ldw/dL \tag{11}$$

$$dw/dL == w/(\bar{L} + L) \tag{12}$$

Of course