Check your understanding: Exponent rules[1]

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[1] Thanks to Farzad Ashouri

Elasticity

Economists use elasticity to avoid having to deal with units. It is much easier to think of a percent change in price, rather than just a change in price which would necessarily involve some units of measure, both for the good itself and for currency in which the price of the good is expressed. With elasticities, units problems melt away. They are also good for seeing how growth rates affect each other.

Elasticity is just percent change of the dependent variable with respect to the percent change in the independent variable. Hence if we have

$$Y = mX + b$$

as the expression for some economic quantity, demand, supply or really anything

$$\epsilon_{YX} = \frac{dY/Y}{dX/X}$$

or ϵ_{YX} the elasticity of Y with respect to X. When you have a continuous function, this work fine. But when no function is given, use the discrete form[2]

$$\epsilon_{YX} = \frac{\Delta Y/Y}{\Delta X/X}$$

[2] The calculus form is when the ΔX goes to zero; that is:

$$lim_{\Delta X \to 0} \frac{\Delta Y/Y}{\Delta X/X} = \frac{dY/Y}{dX/X}$$

Here is it easy to see that we are just working with a ratio of percent changes. Elasticity usually changes from point to point so we have to know where to start. Let's say we have a point X_0, Y_0. We want to calculate the elasticity from this point to X_1, Y_1. We know that by definition

$$\Delta X = X_1 - X_0 \text{ and } \Delta Y = Y_1 - Y_0$$

Keep in mind that ΔY is always measured as the second point minus the first point even if the second point is smaller. This just gives a negative elasticity.

Example. Let's say the demand curve is

$$q = -4p + 6$$

We wish to calculate the elasticity of quantity demanded with respect to price at $p = 1$. The slope $dq/dp = -4$ and $p = 1$ gives $q = 2$.Hence the elasticity is $\epsilon = -4(1/2) = -2$. It will be different at a different p.

Elasticity can be visualized as the slope of the tangent to a function divided by the slope of the *chord line.* That is, the slope of a line from the origin to the point at which the elasticity is to be determined. This is shown in Figure 1 for the function $y = f(x)$

graphics/elas.pdf

Figure 1: Elasticity is the slope of the function, $f'(x)$, divided by the slope of the chord line, y/x.

The slope of the chord line is 1 and the slope of the function is $1/2$ so the elasticity at the point $x = 1$ is $\epsilon = 1/2$.

Here are some extreme examples:

It makes a big difference which variable is on which axis. Note that we have consistently defined at the elasticity of the y variable with respect to the x variable. It is easiest to stick with this convention. It is the same for slopes: it is always the rise (difference in the y-axis variable) over the run (difference in the x-axis variable.) Think of an elasticity as a slope on steroids.

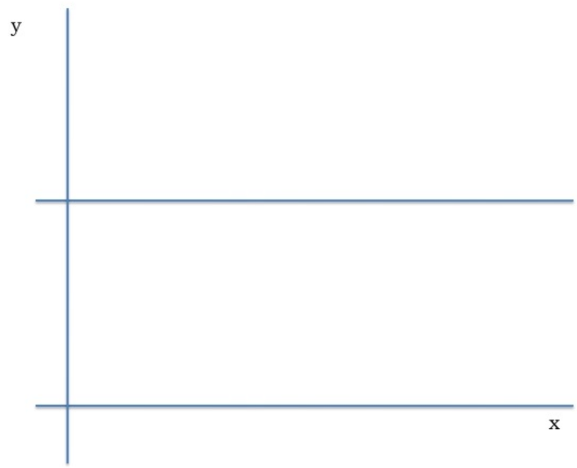

Figure 2: Perfectly or infinitely elastic function

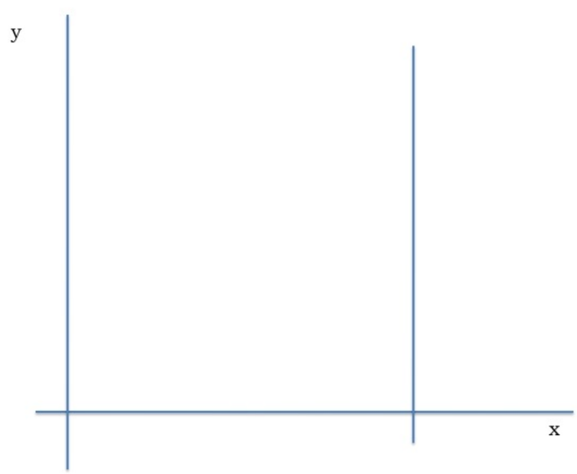

Figure 3: Perfectly or infinitely inelastic function