Utility maximization and trade

Since the consumer utility problem requires solving two equations and two unknowns simultaneously, lets first get some practice:

Example 1 *Solve these two equations*

$$
\begin{aligned}
4x + y &= 9 \\
3x &= 6
\end{aligned}
$$

Solution is: $[x = 2, y = 1]$.

Example 2 *Solve these two equations*

$$
\begin{aligned}
4x + y &= 9 \\
3y &= 6
\end{aligned}
$$

Solution is: $[y = 2.0, x = 1.75]$.

Example 3 *Solve these two equations*

$$
\begin{aligned}
4x + 2y &= 20 \\
2x + 3y &= 13
\end{aligned}
$$

Solution is: $[x = 4.25, y = 1.5]$.

Example 4 *Solve these two equations*

$$
\begin{aligned}
4x + 2y &= 20 \\
2x + 3y &= 16
\end{aligned}
$$

Solution is: $[x = 3.5, y = 3.0]$.

Example 5 *Solve these two equations*

$$
\begin{aligned}
x + 2y &= 10 \\
2x - 3y &= 6
\end{aligned}
$$

Solution is: $[x = 6.0, y = 2.0]$.

Now let solve the consumer's problem. It always involves solving the tangency condition (slope of the indifference curve equal slope of the budget constraint, p) with the budget constraint itself. Note that the tangency condition is

$$p = \frac{\alpha}{1 - \alpha}\frac{y}{x}$$

the budget constraint is

$$B = px + y$$

After getting the solution for the demand for both goods, one can calculate the level of utility to see how well off the consumer is.

Example 6 *Let the price of good 1 be 4 and the price of good 2 be 1, as it always is in the problems. The solution for budget constraint with $B = 10$ is*

$$4x + y = 10$$

Example 7 *Solve the following consumer's problem with $p = 3$. The first is the budget constraint and the second is the tangency condition*

$$\begin{aligned} 3x + y &= 12 \\ 3 &= \frac{y}{x} \end{aligned}$$

Solution is $[y = 6.0, x = 2.0]$.

Example 8 *Solve the following consumer's problem*

$$\begin{aligned} \frac{3}{4}x + y &= 3 \\ \frac{3}{4} &= \frac{y}{x} \end{aligned}$$

Solution is $[y = 1.5, x = 2.0]$.

Example 9 *Solve the following consumer's problem with $\alpha = 0.4$, $p = 1$ and $B = 2$*

$$\begin{aligned} x + y &= 2 \\ 1 &= \frac{\alpha}{(1 - \alpha)}\frac{y}{x} \end{aligned}$$

Solution is $[y = 1.2, x = 0.8]$.

Example 10 *Let the budget be $B = 10$ and the relative price be $p = 0.5$. With a square root utility function, solve the demand for x and y*

$$\begin{aligned} 0.5 &= \frac{y}{x} \\ 10 &= 0.5x + y \end{aligned}$$

Solution is $[x = 10.0, y = 5.0]$.

Example 11 *From the previous problem solve for utility. Solution: Plug the solution in for x and y*

$$U = \sqrt{xy} = \sqrt{10(5)}$$

to find $U = 7.0711$.

Example 12 *With a budget of $B = 10$ and the relative price be $p = 0.5$ solve the consumer's problem with a Cobb-Douglas for $\alpha = 0.5$.*

$$
\begin{aligned}
0.5 &= \frac{y}{x} \\
10 &= 0.5x + y
\end{aligned}
$$

Solution is: $[1, x = 10.0, y = 5.0]$

Example 13 *From the previous problem solve for utility. Solution: Plug the solution in for x and y*

$$U = \sqrt{xy}$$

to find $U = 7.071$. Note that this is the same as the square-root utility function!

Example 14 *Solve the system with a Cobb-Douglas with $\alpha = 0.3$, $B = 10$ and $p = 0.5$*

$$
\begin{aligned}
0.5 &= \frac{0.3}{0.7}\frac{y}{x} \\
10 &= px + y
\end{aligned}
$$

Solution is: $[x = 6.0, y = 7.0]$.

Example 15 *From the previous problem solve for utility. Solution: Plug the solution in for x and y*

$$U = 6^{0.3}7^{0.7}$$

to find $U = 6.6837$.

Example 16 *Solve the same system with a Cobb-Douglas with $\alpha = 0.7$ with $p = 0.5$. The solution is*

$$
\begin{aligned}
0.5 &= \frac{0.7}{0.3}\frac{y}{x} \\
10 &= px + y
\end{aligned}
$$

Solution is $[x = 14.0, y = 3.0]$.

Example 17 *From the previous problem solve for utility. Solution: Plug the solution in for x and y*

$$U = 14^{0.7}3^{0.3}$$

to find $U = 8.8191$.

Example 18 *Solve the system with a Cobb-Douglas with $\alpha = 0.7$ and $p = 1$. The solution is*

$$
\begin{aligned}
1 &= \frac{0.7}{0.3}\frac{y}{x} \\
10 &= x + y
\end{aligned}
$$

Solution is $x = 7.0, y = 3.0$

Example 19 *From the previous problem solve for utility. Solution: Plug the solution in for x and y*

$$
U = 7^{0.7}3^{0.3}
$$

or $U = 5.4288$. The higher price makes the consumer worse off.

Example 20 *If the last example is for a full economy, what is the level of output for each good? Solution: Since demand must equal supply and demand is just the solution for the utility maximization problem, output must be $[x = 7.0, y = 3.0]$. Exports and imports are both zero.*

Example 21 *Assume that the international terms of trade are equal to 1.5. Which good will the country specialize in? Solution: The price of good one is 1. Hence the country can produce good 1 more cheaply than it goes for in the world market. The country will specialize in good 1.*

Example 22 *If the country specializes in good 1, what is the GDP? Solution: The budget constraint was $10 = px + y$. If y were zero, then x would be 10 since $p = 1$ in autarky. But now the price is 1.5 so the new GDP (with trade) is 15. (Note that it would be wrong to say that the GDP is $1.5(7) = 10.5$ since that forgets that the country does not have to produce the 3 units of good 2.)*

Example 23 *What is the new level of demand? Solution solve the system of equations with $pp = 1.5$*

$$
\begin{aligned}
1 &= \frac{0.7}{0.3}\frac{y}{x} \\
15 &= 1.5x + y
\end{aligned}
$$

Solution: $[x = 7.0, p = 1.5, y = 4.5]$. . Utility is

$$
U = 7^{0.7}4.5^{0.3}
$$

or $U = 6.1317$.

Example 24 *What is the gain from trade in the last example? Solution: the gain from trade is simply the improvement in utility, $\Delta U = 6.131 - 5.4288 = 0.7022$ or $0.7022/5.4288 = 0.12935$ or 13%.*

Example 25 *What are exports?   Solution: We produce* 10 *of good one but only consume* 7. *Hence exports are* $E = 3$.

Example 26 *What are imports?   Solution: We produce 0 of good two but consume* 4.5. *Hence imports are* $M = 4.5$. *Note that trade is balanced:* $pE = M$ *is* $1.5(3) = 4.5$.