Check your understanding: The Sawtooth Diagram[1]

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[1] Thanks to Farzad Ashouri

Adjustment in the Keynesian cross model

The the sawtooth diagram shows how income adjusts to a change in aggregate demand. When demand changes and before production has time to adjust, there is **unintended inventory accumulation**, $\Delta\eta$, which can be positive or negative. The basic assumption of the model is that production increases when there is unintended inventory loss, $\Delta eta < 0$ and decreases when there is unintended inventory accumulation, $\Delta\eta > 0$.

In other words, *if there is excess demand, then inventories must fall*. This is the Keynesian adjustment mechanism. To begin assume that the MPC is $c = 1/2$ and $\bar{C} = 1$ with $I = 1$. Total aggregate expenditure, A_e, is then

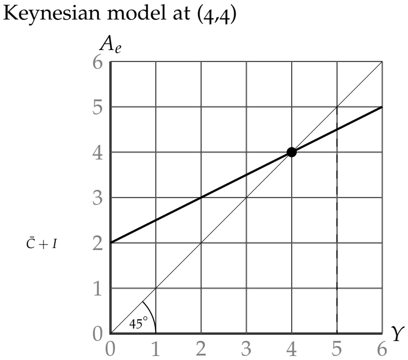
$$A_e = C + I = 1 + Y/2 + 1 = 2 + Y/2$$

where Y is the level of income. In this highly unrealistic model (mpc is too low) the equilibrium is obtained by setting $A_e = Y$

$$Y = 2 + Y/2$$

which gives: $Y = 4$. This equilibrium is shown in figure 1. Note that the slope of the aggregate expenditure line is $1/2$.

Figure 1: Initial equilibrium in the Keynesian model at (4,4)



At this equilibrium there is, presumably, a level of inventories desired by the producers. This is an equilibrium in the sense that producers neither want to increase or decrease their inventory. Think of the number of cars at dealership. There is a certain number of cars that the dealership needs to have on hand so that people can drive them and see which one they want to buy. This is the desired level of inventories.

Now assume there is an exogenous increase in demand, say because $\bar{C}$ increases by 1. For some reason, consumers may decide that now is the time to buy a new car. The *instant* that the consumers drive away with their new car, inventories must fall. In the model, $A_e = 3$, with the increase in $\bar{C}$. Production remains at 4 and so income is still $y = 4$. There is *unintended inventory loss* of $\Delta\eta = -1$.

Replacing the lost inventory

There must now be an *increase* in production to replace this lost inventory. After all, the dealership still needs the same number of cars on the lot to maintain sales. This implies that ΔY = +1. Since production rises by 1, new workers must be hired (or some workers must work overtime). Production must rise from 4 to 5 and so does *income*. The new level of demand is $Y = 3 + Y/2 = 3 + 5/2 = 5.5$. Thus there is, again, is *unintended inventory loss* of $\Delta\eta = 5.5 - 5 = 0.5$. This is loss comes from the fact that newly employed workers will themselves buy cars (or in general, they will buy more of everthing).

A convergent process

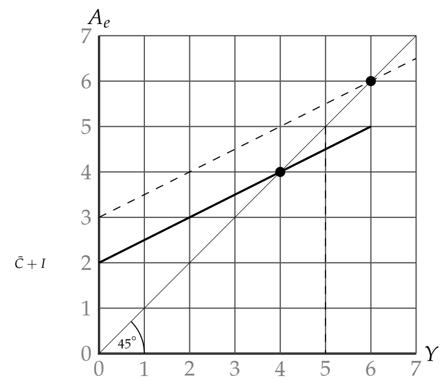
After the first round following the adjustment to an initial rise in demand, inventory losses were equal to 1. In the second round, the *induced* rise in demand caused by replacing the lost inventory, inventories again will fall, but by less, 0.5. Now, again the dealership will want to replace this loss in inventories and will again order new cars.

Production will rise 5.5, which will give income at 5.5. This means aggregate demand will rise to $Y = 3 + Y/2 = 3 + 5.5/2 = 5.75$. Now the unintended change in inventories has fallen to 0.25, $A_e - Y$. This process converges to a new equilibrium of

$$Y = 3 + Y/2 = 6 \tag{1}$$

as shown in figure 2.

Figure 2: Final equilibrium in the Keynesian model at (6,6)



The sawtooth diagram

Figure 3 shows the sawtooth adjustment. For simplicity, the initial aggregate expenditure line is *not* shown. The initial inventory loss is given by η_1 in the diagram. This is made up by an increase in production and, thus, income in the horizontal line labeled Y_1. Not that the length of η_1 *is exactly the same as* Y_1 and so it forms two legs of a triangle that must have a 45 degree hypotenuse. The process goes on *ad infinitum* until the new equilibrium is reached at point (6,6). The convergence is assured be fact that $\eta_1 < \eta_2 ... < \eta_n$

Check your understanding

The SAM is given in table 1. Show the patten of inventory change for $\Delta I = 10$. The marginal propensity to consumer is 0.7.

Solution

The $\bar{C} = 10$ is calibrated from SAM to given $C = 80$.

Figure 3: The sawtooth diagram

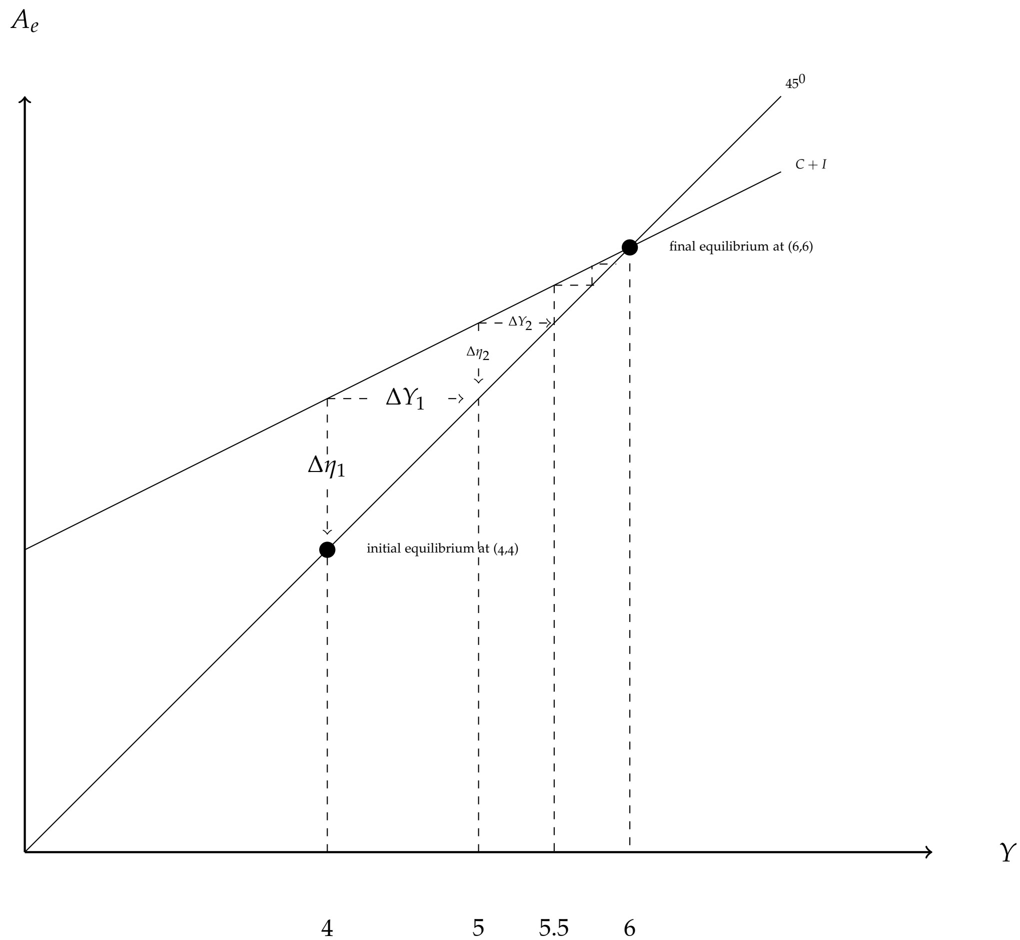


Table 1: SAM

	Firms	HH	Inv	Total
Firms		80	20	100
VA	100			100
Labor	50			50
Capital	50			50
Savings		20		20
Total	100	20	100	

Table 2: Solution

Period	Y	C	I	A_e	η	ΔY
0	100	80	30	110	-10	10
1	110.00	87.00	30	117.00	-7.00	7.00
2	117.00	91.90	30	121.90	-4.90	4.90
3	121.90	95.33	30	125.33	-3.43	3.43
4	125.33	97.73	30	127.73	-2.40	2.40
5	127.73	99.41	30	129.41	-1.68	1.68
6	129.41	100.59	30	130.59	-1.18	1.18
7	130.59	101.41	30	131.41	-0.82	0.82
8	131.41	101.99	30	131.99	-0.58	0.58
9	131.99	102.39	30	132.39	-0.40	0.40
10	132.39	102.67	30	132.67	-0.28	0.28
11	132.67	102.87	30	132.87	-0.20	0.20
12	132.87	103.01	30	133.01	-0.14	0.14
13	133.01	103.11	30	133.11	-0.10	0.10
14	133.11	103.18	30	133.18	-0.07	0.07
15	133.18	103.22	30	133.22	-0.05	0.05
16	133.22	103.26	30	133.26	-0.03	0.03
17	133.26	103.28	30	133.28	-0.02	0.02
18	133.28	103.30	30	133.30	-0.02	0.02
19	133.30	103.31	30	133.31	-0.01	0.01
20	133.31	103.31	30	133.31	-0.01	0.01

Source: made up numbers.