Check your understanding: Solow 2: Growth in the Transient Solution

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How does one use the Solow model if not in the steady-state.

Transient v. Steady-State

- The transient solution in the Solow model can last for a very long time, 100-200 years.
- It follows that one should know more about the transient solution and not simply rely on the steady state as "the" solution to the Solow model.
- The steady state k_{ss} can then be written as

$$k_{ss} = (\frac{s\mathcal{A}}{\delta + n})^{1/(1-\beta)}$$

  where s is the savings-investment rate, $\mathcal{A}$ is the exogenously determined technical change paramater, δ is the rate of depreciation and β is the share of capital in total value added or GDP.
- The steady state income per worker as

$$y_{ss} = (\frac{s\mathcal{A}}{\delta + n})^{\beta/(1-\beta)}$$

- Consider the question of the rate of growth of the capital stock per worker, *before* the steady state, say at the level of $k_{ss}/2$.
- This is a definite point on the Solow path that leads to the steady state.
- To calculate the growth rate of the capital stock at that point, use the Solow equation:

$$\Delta k = sk^{\beta} - (\delta + n)k$$

  Convert this to a "hat" equation by dividing by

$$\Delta k/k = sk^{\beta-1} - (\delta + n)$$

  where the rule of exponents is use to simplify k^{β}/k to what is shown in the first term on the right. All the variables are known in this equation with exception of k itself. If the problem asks to evaluate k at $k_{ss}/2$, for example, this too is known.

- To get the rate of growth of output per worker

$$y = k^{\beta}$$

where $\mathcal{A}$ is set to 1 for simplicity.[1] Using the hat rule for exponents gives

$$\hat{y} = \beta\hat{k}$$

[1] It must be given anyway since it is exogenous and so we might as well be able to set it to one.

Example

Consider an economy with $\beta = 0.3$, $n = 0.015$, $s = 0.25$, and $\delta = 0.04$[2]. How fast is output per worker growing at one third of the way to the steady state? What about half way and three quarters of the way? As the economy approaches the steady state is growth likely to speed up or slow down?

[2] $\mathcal{A} = 1$

Solution: First compute the steady state value of k for the parameters given above.

$$k_{ss} = (\frac{0.25}{0.04 + 0.015})^{1/(1-0.3)} = 8.698$$

with $k_{ss}/3 = 2.9, k_{ss}/2 = 4.349$ and $k_{ss}(3/4) = 6.523$ This gives, from equation ,

$$\hat{k} = (0.25)(2.9)^{(0.3-1)} - (0.04 + 0.015) = 0.0637$$

Income per worker then grows at $\beta\times$ this rate, $\hat{y} = 0.0637(0.3) = 0.0191$. Half way to the steady state, growth slows dramatically to $\hat{k} = 0.0343$ or $\hat{y} = 0.0343(0.3) = 0.0103$ and by three-quarters of the way, $\hat{k} = 0.0123$ and $\hat{y} = 0.0123(0.3) = 0.0037$.

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