Check your understanding: The Keynesian model[1]

Bill Gibson

March 27, 2017

[1] Thanks to Farzad Ashouri

The Keynesian model

The model can be written many ways. Here is a fully worked out version. See table 1 for the definitions of the symbols.

$$Y = C + I + G + E - mY \tag{1}$$

$$C = \bar{C} + cY_d \tag{2}$$

$$Y_d = Y - T \tag{3}$$

$$T = tY \tag{4}$$

Since there are only 4 equations in this version of the Keynesian model, there can only be 4 unknowns. The same model could be written differently with more equations and more variables (or fewer) and the variable and parameter lists would be different. For this model the variable list is $V(Y, C, Y_d, T)$ and the parameter list $P(I, G, E, m, \bar{C}, c, t)$ There are then 28 different multipliers.

To solve the Keynesian model, first get a *reduced form*. This is obtained by substituting *upward*, bottom equation into the next to the bottom, then that equation into the one above and so on. The result is

$$Y = \bar{C} + c(1-t)Y + I + G + E - mY \tag{5}$$

This expression has all the *information of the entire model contained it*, yet it is expressed as one equation in one unknown. This makes solving the model getting the multipliers very straightforward.

To simplify the computation of the solution to the Keynesian model, combine all the constants into on *portmanteau* parameter, A_0

$$A_0 = \bar{C} + I + G + E \tag{6}$$

These are all *autonomous* components of aggregate expenditure. [2] Equation would then expressed as

[2] The word "autonomous" means independent and in this case the expenditure is independent of *income*. It would not be proper to include m here since the term "mY" is *not* independent of income.

$$Y = A_0 + c(1-t)Y - mY$$

Next collect terms on Y and factor

$$[1 - c(1-t) + m]Y = A_0$$

so that the solution for Y is

$$Y = \frac{A_0}{[1 - c(1-t) + m]}$$

Table 1: Notation

Y	income
Y_d	household disposable income
t	tax rate
T_r	transfers from government to households
$\bar{C}$	autonomous consumption
c	marginal propensity to consume
I	investment
G	government expenditure[1]
T	government income (taxes)
E	exports
M	imports
m	marginal propensity to import
A_0	autonomous expenditure

1. May or may not include government wages, which are part of GDP.

Once Y is computed, it is easy to use the rest of the equations of the model to determine, C, Y_d, and T. In fact, model can generate the an entire balanced SAM as will be seen in the exercises below.

Multipliers

Calculate the *multiplier*, defined as the change in the equilibrium value of Y with respect to a change in one and only one parameter of the model. There are several ways to do this. An easy way is to go back to the reduced form equation and "take the deltas", that is, put a Δ in front of each term that is going to change. This includes all the variables *and the one parameter chosen to vary*.

Consider a change in *investment*, which is inside the parameter A_0.

$$\Delta Y = c(1-t)\Delta Y + \Delta I - m\Delta Y$$

Again, combine terms with ΔY

$$\Delta Y - c(1-t)\Delta Y + m\Delta Y = \Delta I$$

$$[1 - c(1-t) + m]\Delta Y = \Delta I$$

Divide by ΔI and then solve for the multiplier

$$\frac{\Delta Y}{\Delta I} = \frac{1}{1 - c(1-t) + m}$$

Some practice problems

Here are some practice problems using the Keynesian model. As noted, $\bar{C}$ is autonomous consumption and c is the marginal propensity to consume. The tax rate is t, so that total taxes are tY. Exports are given by the parameter E. The marginal propensity to import is m and transfers to households are T_r, all as in the table above. If there is no mention of government or foreign, assume that there are only firms and households.

1. Let $I = 20, \bar{C} = 10$ and the $c = 0.8$. Compute Y and C.[3]

2. Let $I = 100, \bar{C} = 34$ and the $c = 0.75$ (marginal propensity to consume). Compute Y and C.[4]

3. In the previous problem calculate household savings.[5]

4. Let $I = 150, \bar{C} = 34$ and the $c = 0.75$ (marginal propensity to consume). Compute Y and C.[6]

5. In the previous problem, compute income using the multiplier, μ.[7]

6. In the previous problem, if consumption increases to 600, while production remains at 736, compute the change in inventories as the difference between expenditure and income before the change in expenditure. If this is not clear, think of it this way. A producer has a certain inventory. Demand increases. The only way to satisfy this demand is to run down current inventories. *In the next period* income increases to make up for the lost inventories. The additional amount of income causes inventories to fall again, but not by as much. Therefore the answer to this question is based on the previous problem's income and this problem's expenditure. [8]

7. Autonomous expenditure in the economy is 4. The marginal propensity $c = 0.5$ and the tax rate is $t = 0.20$. Calculate the equilibrium level of income. [9]

8. For the SAM given in table 2, let the $mpc = 0.67$. Compute $\bar{C}$ to calibrate the consumption function to the SAM.[10] Autonomous consumption is then $\bar{C} = 40.5 - 0.67(50) = 7$.

9. Using the $\bar{C} = 7$ from the previous problem, show that the solution for the Keynesian model is given in the SAM.[11]

10. Let $I = 20, \bar{C} = 10$, $G = 15$ with $c = 0.8$ and the tax rate $t = 0.2$ Compute Y and C and government savings, S_g, (the negative of the deficit). [12]

[3] *Solution:* Solve $Y = C + I$ and $C = \bar{C} + cY$ simultaneously (isolate and substitute) to obtain $Y = 150, C = 130$.

[4] *Solution:* $Y = 536, C = 436$.

[5] *Solution:* $S = Y - C = 536 - 436 = 100$, or from $S = I, S = 100$. Note that S was not in the variable list in the model above. It could have been.

[6] *Solution:* $Y = 736, C = 586$.

[7] *Solution:* $Y = \mu \Delta I$, where $\mu = 1/(1-c) = 4$ so the change in Y is the multiplier times the change in I. $\Delta Y = 4(50) = 200$. The new level of $Y = 536 + 200 = 736$.

[8] *Solution:* $Y = 736 - 600 - 150 = -14$. Inventories *fall* by 14. This will cause output to rise in the next period.

[9] *Solution:*

$$\frac{4}{1 - .5(1 - .2)} = 6.6667$$

$$Y = 4 + 0.5(1 - .2)Y$$

[10] *Solution:* Household income is 50 with consumption at 40.5.

[11] *Solution:* Solve the system

$$Y = C + I$$
$$Y = \bar{C} + cY$$

simultaneously for Y, C. This should give

$$Y = C + 9.5$$
$$C = 7 + 0.67Y$$

or, substituting the second equation into the first $Y = 7 + 0.67Y + 9.5$ or $Y = 50$. Thus, $C = 7 + 0.67(50) = 40.5$. Savings is $Y - C = 50 - 40.5 = 9.5$

[12] *Solution:* $Y = 125, C = 90, S_g = 10$.

Table 2: A Social Accounting Matrix

	Firms	Household	Invest	Total
Firms		40.5	9.5	50
HH	40.5			40.5
Savings		9.5		9.5
Total	50	50	9.5	

Source: made-up numbers.

11. Let $I = 20, \bar{C} = 10$. $G = 25$ and $c = 0.8$ and the tax rate $t = 0.2$ Compute Y and C and the new government savings. [13]

12. In the previous problem government spending went up by 10. Compute the change in GDP using the multiplier $1/[1-(1-t)c]$.*Note that the multiplier is different with taxes in the model.*[14]

13. The level of government spending went up by 10, but government savings fell by only 4.4. Why is this?[15]

14. Let imports be mY with the marginal propensity to import $m = 0.14$. Exports are $E = 12$, $I = 20, \bar{C} = 10$. $G = 25$ and $c = 0.8$ and the tax rate $t = 0.2$. Compute Y and C, S_g and foreign savings (the current account deficit), S^*.[16]

15. In the previous problem raise exports to 15. Compute Y and C and the new government savings and foreign savings.[17]

16. In the previous problem exports went up by 3. Compute the change in GDP using the multiplier $\mu = 1/[1-(1-t)c+m]$. *Note that the multiplier is different with taxes and imports in the model.*[18]

17. If $c = 0.5$ and the tax rate is 0.2 the *the exogenous spending multiplier is?* [19]

18. If the marginal propensity to import is

$$m = 0.1$$

the exogenous spending multiplier is?[20]

19. Write out a new SAM for the previous problem.[21].

[13] *Solution:* $Y = 152.8, C = 107.8, S_g = 5.6$.

[14] *Solution:*Here $\mu = 2.78$, so $\Delta Y = 2.778(25-15) = 27.8$. The new level of $Y = 138.9 + 27.8 = 152.8$, the same as calculated directly.

[15] *Solution:* Think about what has happened to total tax revenues.

[16] *Solution:* $Y = 134, C = 95.8, S_g = 1.8, S^* = 6.8$.

[17] *Solution:* $Y = 140, C = 99.6, S_g = 3, S^* = 4.6$.

[18] *Solution:* $\mu = 2$, so $\Delta Y = 2(3) = 6$. The new level of $Y = 134 + 6 = 140$, the same as calculated directly. Note that you can also use the multiplier to calculate the change in consumption, government savings and the level of foreign savings.

[19] *Solution:*

$$\frac{1}{1-0.5(1-0.2)} = 1.6667$$

If E_0 is raised by one dollar, equilibrium income Y will increase by 1.667 dollars.

[20]

$$\begin{aligned}\frac{\Delta Y}{\Delta AE_0} &= \frac{1}{[1-c(1-t)+m]} \\ &= \frac{1}{1-0.5(1-.2)+0.1} = 1.4286\end{aligned}$$

This is a *lower* multiplier.

[21] *Solution:* The SAM is table 3

Table 3: A Social Accounting Matrix

	Firms	Household	Invest	Govt	Net Exports	Total
Firms		99.6	20	25	−4.6	140
HHolds	140					140
Savings		12.4		3	4.6	20
Govt		28				28
Foreign						0
Total	140	140	20	28	0	

Source: made-up numbers.

Table 4: A Social Accounting Matrix

	Firms	Household	Invest	Govt	Net Exports	Total
Firms		800	200	150	-30	1120
HHolds	1120			80		1200
Savings		220		-50	30	200
Govt		180				180
Foreign						0
Total	1120	1200	200	180	0	

Source: made-up numbers.

20. In the SAM of table 4, let the marginal propensity to consume be 0.7 and the marginal propensity to import be 0.1. Note that transfers to households are 80. Compute the level of Y, C, S_g, S^* and show that they agree with the SAM. [22]To solve this model, write

$$Y = C + I + G + E - mY$$
$$C = \bar{C} + (1-t)(Y + T_r)$$

Substituting the second equation into the first and then putting in the data from the SAM.

$$Y = 86 + 0.7(1-.15)(Y+80) + 200 + 150 + 82 - 0.1Y$$
$$0.505Y = 86 + 47.6 + 432$$
$$Y = 1120$$
$$C = 86 + 0.7(1-0.15)(1200) = 800$$

[22] *Solution:* We must figure out the tax rate, which is 180/1200 =0.15. Net exports are -30 so this must mean $Nx = E - mY$ or $E - 0.1(1120) = -30$ so that E = 82. Disposable income is $Y_d = (1-t)(Y+T_r) = (1-0.15)(1200) = 1020$. Thus $\bar{C} = C - cY_d = 800 - 0.7(1020) = 86$.

21. Now calibrate the Solow model to this SAM. The capital output ratio is 3 and $\beta = 0.4$. Let $A = 1$. Compute the capacity utilization ratio. [23]

[23] *Solution:* The capital stock is 3(1120) = 3360. The share of labor is 0.6 so that labor is 0.6(1120) = 672. The capital-labor ratio, $k = 3360/672 = 5$. Per worker income is then $y = Ak^{\beta}$=$5^{0.4} = 1.9$, but the actual per worker income of the SAM is $1120/672 = 1.67$. Thus $u = [1.67(672)]/[1.9(672)](1.67/1.9) = 0.875$.

22. Now increase transfers by 10 and recalculate the new levels of Y, C, S_g and S^*.[24]

23. In the last question, what happens to capacity utilization?[25]

[24] *Solution:* $Y = 1131.8, C = 813, S_g = -56.7, S^* = 31.2$.

[25] *Solution:* Capacity remains the same but demand has increased. So capacity utilization increases. Income per worker rises to $1131.8/672 = 1.68/1.9 = 0.885$.