Consumer and Producer Surplus and Deadweight Loss

The *deadweight loss*, *value of lost time* or *quantity waste* problem requires several steps. **A ceiling or floor price must be given**. We call that price the fixed price, $\bar{p}$. It is usually imposed by government. The steps are

1. First solve for the supply and demand equilibrium, p^*, q^*.
2. It really makes no difference how the two equations are expressed. The most general form is **implicit** or $F(p, q) = 0$.
3. How do we know whether $F(p, q) = 0$ is a supply or demand curve when in implicit form? We don't! It depends on the slope.
4. Slope of demand curve is negative–slope of the supply curve is positive
5. y-intercepts are also different. The demand curve must have a y-intercept that is greater than the supply curve for a solution to exist.
6. If the demand curve is expresses as: $p = f(q)$ or what is the same thing $p = D(q)$ this is **most common or "classic" form** how Alfred Marshall first described the demand curve in the 1920s. In this form p is really a "willingness to pay" on the part of consumers **given the quantity** q. This is what the books calls: reading the demand curve "horizontally".
7. Demand curves are often inverted in economy theory writing $q = f(p)$ which is sometimes more useful. In this form the demand curve is "really quantity response to a price signal", that is, it answers the question: taking price as given, how much will the consumer demand. This is what the books calls: reading the demand curve "vertically".
8. It is really easier to remember that either p or q can be on either axis in a graph. When we write $q = f(p)$ it is assumed that p is on the horizontal axis and q is on the vertical. It is the other way around when we write $p = f(q)$.
9. The same information is contained in the demand curve whether expressed as $p = f(q)$ or $q = f(p)$.
10. Which way the demand curve is expressed depends on the question in which the economist is interested. When calculating the elasticity, for example, note that the word elasticity really means "response". How does the consumer respond to a given price. In this case $q = f(p)$ is the obvious choice.
11. Consumer surplus problems, however, are best solved the other way around with $p = f(q)$ since we are asking, "what is the marginal benefit" of a given consumer at a given quantity.

12. All of these remarks apply to the supply curve as well.

13. When we are working with both supply and demand we often use the notation $q = D(p)$ instead of $q = f(p)$. This just make it easier to know which curve is which

14. If we write $p = S(q)$ we are saying the for a given quantity q the firm will have to charge at least p or go out of business. We say that p is then as **supply price**

15. If we write $q = S(p)$ the we are asking how much the firm (or market) will supply if the market price is p. Again, which way we write the supply curve depends on the question of interest. To calculate supply elasticity, $q = S(p)$ is the obvious choice.

16. In they can even be mixed, with the supply curve stated one way and the demand curve the other.

17. Demand and supply curves can be either **linear** or **nonlinear**

18. When one wants to plot linear demand and supply equations should be in **standard slope-intercept form**

$$\begin{aligned} \text{supply: } p &= \textit{intercept} + \textit{slope} \times q \\ \text{demand: } p &= \textit{intercept} + \textit{slope} \times q \end{aligned}$$

19. We use elementary algebraic operations to transform any pair of supply and demand equations into this form.

20. If the demand or supply curve is non linear then the slope changes and the intercept may be **asymptotic**, that is, never crossing the vertical axis.

21. Since nonlinear demand and supply curves are more difficult to work with economists often take a **linear approximation** in a small region around the equilibrium.

22. If the fixed price is *above* the equilibrium, this is a price floor and you will be solving for the value of *wasted quality*, as when the Civil Aeronautics Board fixed a price floor and airlines had to compete by making the sandwiches bigger and tastier. Since consumers did not necessarily want big, tasty sandwiches, the extra quality was a waste, yet the price floor forces them to buy it anyway.

23. So if $p^* < \bar{p}$ there is a price floor and the quantity transacted is determined by the demand curve.

24. If $p^* > \bar{p}$ then there is a price ceiling and quantity transacted is determined by the supply curve. A price ceiling is like the maximum rent that landlords are able to charge. This occurs in New York City with "rent controlled" apartments.

25. In both cases, we say the "short side" of market determines the quantity transacted.

26. Call the quantity transacted $\bar{q}$ which corresponds to the controlled price $\bar{p}$.

27. At the quantity transacted their will be a vertical *gap* between the supply price and demand price. If there is price floor, then the gap is the fixed price minus the supply price. With a price ceiling the gap is the demand price minus the fixed price.

28. To calculate deadweight loss with a *price floor* we write

$$W_d = \frac{(q^* - \bar{q})}{2}(\bar{p} - p_s)$$

where p_s is the supply price, the price on the supply curve at $q = \bar{q}$. The price gap in this case is $\bar{p} - p_s$.

29. If there is a price floor, then we calculate the *value of wasted quality*, V_{wq}. This is price gap times the quantity transacted.

$$V_{wq} = q^*(\bar{p} - p_s)$$

30. To calculate deadweight loss with a price ceiling, we write

$$W_d = \frac{(q^* - \bar{q})}{2}(p_d - \bar{p})$$

where p_d is the demand price, the price on the demand curve at $q = \bar{q}$. The demand price is the willingness to pay. The price gap in this case is $p_d - \bar{p}$.

31. If there is a price ceiling, then we calculate the *value of lost time*, V_{lt}. This is price gap times the quantity transacted.

$$V_{lt} = q^*(p_d - \bar{p})$$

32. The maximum gain from trade sum of the consumer surplus and consumer surplus. And is easy to calculate. Compute the price gap at quantity of zero. This is just the intercept of the demand curve less the intercept of the supply curve. Now multiply the price gap times the equilibrium quantity.

$$G_{max} = q^*[p_d(0) - p_s(0)]$$

where $p_d(0)$ is the demand price at zero quantity and $p_s(0)$ is the supply price at zero quantity. These are just the intercepts as explained above.

33. Consumer surplus, S_c, is the area under the demand curve and above the price. This can be calculated as the area of the triangle defined by the intercept of the demand curve, the equilibrium price and the equilibrium quantity.

$$S_c = \frac{q^*[p_d(0) - p^*]}{2}$$

34. Producer surplus, S_p, is the area under the price and above the supply curve. This can be calculated as the area of the triangle defined by the intercept of the supply curve, the equilibrium price and the equilibrium quantity.

$$S_p = \frac{q^*[p^* - p_s(0)]}{2}$$

Example 1

$$\begin{aligned} p &= 1 + 2q \textit{ supply} \\ p &= 10 - q \textit{ demand} \end{aligned}$$

Solution: $[p^* = 7, q^* = 3]$. Let $\bar{p} = 3$ and solve for the deadweight loss. At this price, $q = 1$ from the supply curve and 13 from the demand curve. If we let the "short side" determine the quantity transacted, we have the supply curve determining the quantity transacted at $\bar{q} = 1$. The demand price at this quantity is willingness to pay is $p = 10 - 1 = 9$. The price gap is then $9 - 3 = 6$. Hence the DWL is the one half the price gap times the difference between the transacted and equilibrium quantities or $(6)/2(3 - 1) = 6$. The value of lost time is the price gap times the quantity transacted: $(6)1 = 6$.

Using the same demand and supply curves lets say there was a price floor of 8. At this price the quantity transacted is $\bar{q} = 2$, which is obtained by putting at price of 8 into the demand curve. At this quantity the supply price is 5 which is plugging the $\bar{q} = 2$ into the supply curve. The price gap is then $8 - 5 = 3$.

Hence the DWL is the one half the price gap times the difference between the transacted and equilibrium quantities or $(3)/2(3 - 2) = 1.5$. The quality waste is the price gap times the quantity transacted: $(3)2 = 6$.

The maximum gain from trade is intercept of the demand 10 less the intercept of 1 times the equilibrium quantity $q^* = 3$ over 2 or $27/2 = 13.5$. The consumer surplus is $(10-7)3/2 = 4.5$ and the producer surplus is $(7-1)3/2 = 9$.

Example 2 *Solve the supply and demand curves for the equilibrium price and quantity*

$$\begin{aligned} q - p + 5 &= 0 \textit{ supply} \\ q + p - 15 &= 0 \textit{ demand} \end{aligned}$$

Solution: First put them in standard slope-intercept form

$$p = 5 + q \text{ supply}$$
$$p = 15 - q \text{ demand}$$

Solving: $[p^* = 10, q^* = 5]$. At $\bar{p} = 8$, the DWL is 4 and the value of lost time is 12. At $\bar{p} = 13$, the DWL is 9 and the quality waste is 12. The maximum gain from trade is 25. The consumer surplus is 12.5 and so is the producer surplus.

Example 3 *Solve these two equations for the equilibrium price and quantity*

$$-2q + 3p = 16 \text{ } \textit{supply}$$
$$4q + 2p = 20 \text{ } \textit{demand}$$

Solution: In standard slope-intercept form these equations are

$$p = 16/3 + 2q/3 \text{ supply}$$
$$p = 10 - 2q \text{ demand}$$

Solving: $[p^* = 6.5, q^* = 1.75]$. At price ceiling of 6, the DWL 0.75. Willingness to pay is 8. The quantity transacted is 1 So the price gap is $8 - 6 = 2$. The value of lost time is $2(1) = 2$. The maximum gain from trade is 4.08. The consumer surplus is 3.06 and the producer surplus is 1.02.

Example 4 *Solve these two equations for the equilibrium price and quantity.*

$$3p - 2q = 5$$
$$2p + 3q = 28$$

Solution: In standard slope-intercept form these equations are

$$p = 5/3 + 2q/3 \text{ supply}$$
$$p = 14 - 3q/2 \text{ demand}$$

Solving: $[p =^* 5.46, q^* = 5.69]$ At a price floor $= \bar{p} = 8$, the quantity transacted is determined by the demand curve is 4. The supply price is 4.33 and the price gap is $8 - 4.33 = 3.67$. The deadweight loss is $(3.67/2)(5.69 - 4) = 3.1$. The maximum gains from trade is $(14 - 5/3)/2(5.69) = 35.1$. The consumer surplus is 24.3 and the producer surplus is 10.8.