# Roc Model and Density Dependence, Part 2

Terri Donovan

recorded: February, 2012

Let's think about this model a little bit more. We don't know what the slope actually should be, but we know that the number of nesting sites is fixed at 3,000. That means that when we hit a population size of 3,000, we want the clutch size to be 1. Each breeding individual produces one offspring, and so the entire cohort is replaced exactly with 3,000 new individuals.

So how do we figure out what slope that has to be so that this cell, cell J19, ends up being 1?

Well we could click on cell I10 and try some different entries, but we can also solve it analytically. Here's the equation of the line. We know that Y is going to be 1. So we can say 1 is equal to the slope. That's M. We don't know what that is. How about X? Well we want X to be 3,000 individuals. And our intercept is the same, 1.06.

And we can solve this equation analytically for M. We're just going to move the 1.06 over. So we'll have 1 minus 1.06. And then we'll divide the entire thing by 3,000. And if we put a equal sign in front of this, that should give us the slope we desire. And I actually have a typo up here, 1.06. So if we make our slope negative 0.12342, there we go, we see that at 3,000 individuals, the clutch size is 1.

Now what do you expect will happen then? The population should stabilize because, as I said, every single Roc in that time step is reproducing exactly one offspring. There's no mortality here. That one offspring will persist, and those will become the breeders in the future time steps.

Now I just want to show you one other way of finding the slope. I'll set this back to zero. And that's using a spreadsheet tool called 'Goal Seek'. This is a handy little function to get to know. It's in the Data tab under the 'What If' analysis. And it's called Goal Seek.

Here you have three boxes to fill. The first is Set Cell. So here you say which cell do you want to set to a particular value? Well we want to set cell J19 to a value of 1. That's the per-capita birth rate of 1. The last argument is by changing cell. Which cell can you change? Well in our case, we want to change the slope, so we'll enter I10, and then press OK. And the spreadsheet will find the answer for you. It's changed the formatting, but if you click on it, you can see that it ends up with a number very, very close to what we had found with an analytical method.

So we're going to use this linear function and develop our linear model. So we'll be computing the number of Rocs with a linear model, and we'll also be calculating the per-capita birth rate for the linear model as well. And just like we did before, we'll start with a population size of 2,000. In this case though, we are going to calculate what the per-capita birth rate should be based on 2,000 Rocs.

So if we even look at our graphic here, we find 2,000 Rocs, and we should find that the per-capita birth rate is 1.02. That's if our slope is what we have in this particular cell. Let's go ahead and enter that up here, four zeros and a 2, and it's negative.

How do we calculate the birth rate? Just as we've done before. We're calculating the birth rate, Y. It's equal to the slope. Anchor that. Times X; that's the number to the left of it. Plus B; that's the Y intercept, and we'll anchor that as well. So these 2,000 Rocs are breeding. And we know that they should have a per-capita birth rate of 1.02.

Now when we compute the Rocs in the next generation, we're going to compute it as the number of Rocs in the previous generation times their per-capita birth rate. And we can drag both of these equations all the way through, and then we'll have our linear model.

Now we're in a position to add in the linear model to our previous graphs. Here's the population-size projection. So we'll select this model, and then we'll select the data, and we'll add a new series. The series' name will be Rocs Linear Model, or LM for short. And the values will be the values in cells F8 through F23.

So here we have three different models. The first is no density dependence, ignoring the nest site. The second is the despotic model, where we paid strict attention to this nest site. And the third is our linear model.

Let's go ahead also now and look at the per-capita birth rates as well.

We'll right-click on this, select the data, and add the series. This is going to be the birth rate for the linear model. The X values in this particular model, which I've sort of hidden, are down column F. And the Y values are in column G. And we click OK.

Now why does this graph stop here abruptly? Well the reason is, that the linear model was very slowly chugging, chugging, chugging up, and it didn't even reach 2,500 animals. So it stops right here. This is the most we've depicted in our model. We could easily grab these cells, pull them down and really see how these dynamics play out.

What is different about these models? These models are hugely different in terms of how the modeling problem was established. And you can make an argument of whether or not they reflect the problem at hand. Clearly, the Roc model in red doesn't reflect the model at hand, the problem at hand, the real world, because it ignores the fact that there are 3,000 nesting sites.

The despotic model handled the problem of the 3,000 nesting sites, but it did it in a particular way. It let everybody reproduce at this rate, 1.06. However, it limited the number of breeders to 3,000 maximum. And we saw that the model leveled off, and it leveled off a little bit higher than 3,000. That's because each individual is more than replacing itself.

The linear model, in contrast, is very different. What this assumes is that for each and every time step in the model, what is being adjusted is not the nesting sites but the clutch size. That's a hugely different way of thinking about this problem. When we started off the population, as soon as we added some individuals, the moment we went above 2,000, that population per-capita birth rate started to drop. And it will drop, drop, drop, drop, drop until it reaches 3,000 individuals, at which point the per-capita rate would be 1.

What do you think would happen once it hits the number 3,000? It should go on and level off and stay at 3,000 because when the per-capita rate reaches 1, then the population is perfectly replacing itself.

Now I want to return back to this kind of curve because I promised we would start and get to this problem here and talk about how to model that. And we said, "Well in order for us to model this red curve, let's start by modeling this linear equation. And then we'll build from there." So let's do that now.

We've just looked at the case where we have a graph that looks roughly like this. We have population size on the X axis, and we have the per-capita birth rate, B, on the Y axis, this way. And we did a linear function to begin with. And we said that that equation was: Y is equal to M, the slope, times X, the population size, plus B. So if we know the slope, we know the population size, and we know the intercept, then we can use this equation to find what the per-capita birth rate should be.

To get the line that looks like this, what we'll do is use what's called a 'polynomial function'. And let me just define it because this, in fact, is a polynomial function. I'm looking at Wikipedia, and it says, "In mathematics, a polynomial is an expression of finite length constructed from variables and constants, and it uses only the operations of addition, subtraction, multiplication, and non-negative integer exponents."

That does satisfy the case. There's an exponent here on the 1. This is a piece. This is a polynomials constructed of two parts here. We use addition in this particular case. It's a finite length; it's constructed with some constants; and it doesn't have an exponent that is not negative. It's an integer.

Well we can expand this to a second-order polynomial by writing Y is equal to A times X squared plus MX plus B. This is now a function that will allow us to draw something that looks along these lines.

So here, Y is, again--we're predicting the birth rate, but we have three things we need to know. We need to know what this constant is, called A, and that gets multiplied by X-squared. We need to know what the slope is. And we know from our last model that it is negative 00002, .00002, and we know the intercept is still 1.06. What we don't know is A. And we can find and use this kind of polynomial equation, find A, and see if we can get and generate a graphic that looks this way.

Now the tricky part about using these kinds of equations is you have to estimate what these things are. Now, instead of a simple linear model, we've got three things we have to estimate. Think how different that is from the despotic model where the only rule we had was we needed to know the number of nesting sites was limited. We know that that was limited to 3,000. And we knew the per-capita birth rate, 1.06.

When we start dealing with these more complicated models, we find that we have to add new parameters to the model. And those are things that have to be estimated. And of course, there's often a huge amount of uncertainty associated with those estimates. So sensitivity analysis is the rule of the day here.

Let's see how this would play out on the spreadsheet. Let's add that polynomial term to this portion of our spreadsheet. We called that first parameter the "A" parameter. And we had set that, hypothetically, to a very small number, negative 0.one-two-three-four-five-six 1s, followed by the number 1.

Now we can model clutch size, Y, as a function of this, not linear model, but a polynomial model that includes this squared term. Remember, here's the number of Rocs, and here are three parameters. So we're trying to predict clutch size, Y. That's a function of this A parameter, anchored, times the Rocs squared. And we do that by adding a caret symbol and then the number 2. So we take the Roc, square it, then to that, we're going to add the slope, M--anchor that--times the Roc population size. And then to that, again, now we'll be adding the intercept, and we'll anchor that as well. And then we'll pull this through. And now we can add this graph to our chart here. We'll just select it and then pull the boxes over. And here, now, you begin to see a function that starts to decline.

These kind of models--it's worth repeating--they're very flexible. They add a lot of flexibility to your modeling. And people tend to use them when they need to model something as a function of something else. In this case, we've really focused on how birth rate changes as a function of population size. But you'll see these kind of equations used in all kind of models. You know, death rate is a function of temperature, or birth rate is a function of elevation. There could be any number of different scenarios.

Let's take a second just to recap what we've gone through. In this session, the Rocs were limited by a particular resource. In this case, it was nest sites. Our simple despotic model handled this by allowing only 3,000 breeders, if the population size actually exceeded this nest site number. And in this model, clutch size remained constant.

This is in sharp contrast to the linear model. The linear model assumed that the clutch size is a linear function of population size so that the birth rate decreased when the population size increased. And we selected a slope that resulted in a birth rate of 1 when the population reached 3,000 individuals.

So there you have it; three very different models with very different assumptions. And your job, as a modeler, is to try to, once again, think about the model world and how that matches up with the real world. We'll see you next time.

< 00:18:46 END >