Roc Model and Density Dependence, Part 1

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You've now completed several modeling exercises dealing with the Roc population. So far, the caliph of Baghdad has been very happy with your work. The year is now 2000. Roc flu has been eradicated, and silver has been completely mined from the earth. So silver bullets and Roc flu are no longer threats to Roc survival. The number of Rocs surveyed by Sinbad's great great great great grandson is 2,000.

The Rocs, however, still threaten sailors worldwide. The clutch size has remained constant at a stable 1.06. However, oil has been discovered in this area of the world, and new extraction operations have reduced the number of nesting sites by 25 percent, from 4,000 nesting sites to 3,000 nesting sites. So we know the following information. The clutch size remains 1.06, and there are just 3,000 nesting sites present. Now remember, these nests can only be used once per century.

While the caliph is pleased with the new riches brought forth by the oil, he wants to ensure that the Roc population persists, lest he lose his kingdom. So once again, he's calling on you and your consulting firm to make a projection for the Roc population to the year 3000. Your challenge: build a model that projects population size to the year 3000, but keep in mind the number of nesting sites is limited to 3,000.

Put your video on pause and take some time to work out a prototype of this problem now, and then we'll come back, and we'll look at this problem from a few different angles.

<u>PAUSE the video</u>: Develop a prototype of this problem.

We're going to build this model a few different ways. And we're going to start with a base model that ignores the fact that there are only 3,000 nesting sites.

So we'll start off by just entering our key parameters in the upper-left corner. We have a clutch size that remains constant at 1.06. The number of nesting sites was fixed to be 3,000 for this particular model. And the initial population size that was surveyed suggested the population size was 2,000 animals. These are all model inputs, so we'll shade those green.

Now let's set up our model to run down column A. First we'll type in the year, and we'll begin at the year 2000. That's the present year. And once again, we'll be modeling the population in 100-year time steps. We'll take this down to 3000 because that was what the request was, but just for demonstration, I'm going to pull this all the way down to 3500.

We'll be modeling the Roc populations, and we're interested in the number of Rocs within each and every time step. And we'll start with the population at its initial size of 2,000.

Now before I go further, I'm going to just clean up the spreadsheet a bit, and that will help us keep some things straight.

We need an equation in cell B9 that would project the Roc population to the next century. What equation would you use? There's no silver bullets. There's no Roc flu. It's a pretty big Roc population size. We don't need to worry about a demographic, stochasticity model. Our model is just simply the number of Rocs in the next time step is the number of Rocs in the previous time step times the clutch size. And we'll anchor that reference to the clutch size and then pull this through.

So remember, all the Rocs here, these 2,000 Rocs, they reproduce, and they reproduce at a rate of 1.06. Then these 2,000 original Rocs have all passed on. And the number that's reflected in the next time step all are brand new Rocs. They're new Rocs for the century. So this is a non-overlapping-generations kind of model. These are numbers. So I'm going to turn those into numbers and then hide the decimal points. So we see this population's growing. No surprise there.

What we'd like to do next is to calculate the per-capita birth rate. And before we do that, we're going to head back over to our tablet and just remind ourselves what a per-capita birth rate actually is.

You've heard Tony talk about writing an equation that can be used in a modeling context. And in population dynamics, we often use a model that is a discreet time step model where we're interested in projecting, or documenting, the number of individuals at time step T plus 1. And we say that that is a function of the number of individuals at time step T, plus all of the new animals that are added by birth, but we subtract off all of the individuals that die or leave the population through death. In this population, we're not considering immigration and emigration.

Now the Rocs are a curious case because in each time step, the number of deaths is exactly equal to the number that entered the population. So these two terms cancel out. And what we're left with for the Rocs is NT plus 1 is equal to just the number of births.

Now how do we count the number of births? How many new Rocs were born? Well in our equations, that was made up of two pieces. It was made up of the number of individuals in the previous time step times the clutch size. And we're going to call the clutch size, from now on, B, a birth rate. So NT times the birth rate, B. This is a per-capita birth rate. So our equation is NT plus 1 is equal to NT times B.

Now if I were to ask how do you calculate the per-capita birth rate, well we can calculate that by dividing both sides by NT, and we end up with NT plus 1 divided by NT. And that will give us the per-capita birth rate. And this rate can be calculated for any single time step. If I know the number in the previous time step and the next time step, that ratio will give me the per-capita birth rate. Now remember this is for the Roc population problem. This is not generally how you would calculate a per-capita birth rate because it's not often the case that these two things cancel, and we often have to consider births and deaths separately.

So now let's go ahead and enter equations down column C that will calculate the per-capita birth rate for our population. That's simply going to be NT plus 1 divided by NT. And we can pull this equation down and see that we have a constant birth rate, and it matches our clutch size. No surprises there because our model was set up to be this way, and if we had not seen that this exactly matched, we'd know there was a problem.

Don't worry about this last 0 because this is a reflection of dividing one cell that has no entry in it by another.

We're going to call this our base model. You might've known, or might've noticed, that we did not even pay attention to this limiting factor here of nesting sites. So we assume that no matter how big the population got, every single individual bred and every single individual reproduced at this rate. So we've created a 'density-independent model'.

Now in contrast to that, let's try to set up a model that considers some density dependence. And we're going to call this our "Despotic Model." So this is an example of a density-dependent model.

And once again, we're going to be entering an equation to project Roc population through time, and we'll also be calculating the per-capita birth rate. Let me name this "Rocs Despotic Model" and "Birth Rate Despotic Model."

Once again, we'll start our population with the initial size in cell B3. And at this point, we need to enter an equation that will now consider the fact that we have only 3,000 nesting sites. Here's where the despotic model comes into play.

First of all, a despot is defined as a ruler or a tyrant. And we can think of this model as - there are 3,000 nesting sites, and if the population exceeds 3,000, then only the first 3,000, the despots, will actually find a breeding site and use it. The other ones will have to go off and not breed at all. So the despots are the ones that actually secure a breeding site when the population size is higher than 3,000. If the population's lower then 3,000, then of course, there's plenty of nesting sites for everybody, and everybody can breed.

So what kind of equation do we need in D9 to reflect this case? We're looking at another "if" function. Let's open the "if" dialog box. And we start off with the logical test. So the first argument is the logical test, and we're simply going to ask, "Is the population size in the previous time step less than 3,000?" So that's our logical test. Is the cell in the previous time step less than the value in cell B2? And we'll go ahead and anchor that reference. The logical test returns true or false. If this is true, then the spreadsheet will carry out what is embedded in the second argument. So now we need to say, "What calculation happens if the population's less than 3,000?" Well if it's less than 3,000, the number of Rocs is just the number of Rocs in the previous time step times the clutch size. And we'll anchor that clutch size.

What if this is false, though? What if the population actually exceeds 3,000? Well in that case, then, the only individuals that can reproduce are the despots, and there are 3,000 of them. So we'll click on cell B2. We'll let 3,000 of these despots reproduce, and each of them will reproduce with the clutch size given in cell B1. Now both of those need to be anchored. And at this point, we have our modeling equations in place, and we can copy these through.

We can calculate the birth rate the same way we did in the previous example. It's just going to be NT plus 1 divided by NT. And we see that, initially, the birth rate starts at 1.06, 1.06. But suddenly, around the year 2700, the birth rate, the per-capita birth rate drops, and then it hits the number 1. These are numbers, and I'd like to display three decimal places for these.

Question: Why is b (the per capita birth rate) equa to 1.00 when Nt equals 3180?

Answer: You are Correct! 3000 despots times 1.06 offspring per despot plus 180 losers (non-breeders) times 0 offspring per loser equals 3180 births divided by 3180 individuals, equals 1.0

Now the easiest way to compare these two models is to, first, graphically depict them, and that's a good practice, really, no matter what. We'll go ahead and highlight the Rocs population from the base model. Press the Ctrl-key, and then select the Rocs from the Despotic Model, and we'll insert a line graph. We don't need to display the year. And here we have our Roc population through time for the two different models.

I'll go ahead and add the axes titles. The horizontal axis, in this case, is going to be the year. And the vertical axis will be Roc population size. Let's click on the model again. This time, go up to the Design tab. We'll select the data. And we need to tell the spreadsheet that the horizontal axis is going to be represented by the years. So we'll choose Edit, and then select our data range, and then choose OK.

Now what you notice here is that, in the base model that ignored this nesting site problem, the population was starting to grow, grow, grow, grow, grow. In the model that included the despots, the population grew, and it reached some level, and then it leveled straight off. And the level it reached was 3,180. Now if you've seen graphs like this before, you would be tempted to call this a carrying capacity because the population doesn't exceed that. It hits this level, and it stays flat at that level.

Where did this 3,180 come from? Well it's just the number of Rocs that could maximally reproduce, 3,000, each of them reproducing at this average. So that's an average population level rate. Of course, a Roc cannot have 1.06 eggs, but as a population on a whole, that's the average, and that gives you your number.

What happens if we have more than 3,000 individuals? Well, according to this model, 3,000 individuals would get the nesting site, and they would reproduce at this rate. And then the remainder, remaining individuals--let's say there are 400 of those--they're left without a place to breed. They reproduce zero offspring, and we're still left with 3,180.

Now let's take a look at these birth rates. In this case, we're going to graph the Roc population size as a function of birth rates, and we'll do this in two different ways.

First, we'll select those, and we'll do a scatter graph, and we'll connect them with lines.

So this is our base model, and what you can see is this is population size and this is the per-capita birth rate. And for the base model, where we ignored nesting site limitation, everybody had the same birth rate. The birth rate was constant from year to year.

Let me add these axes titles. The bottom axis is "Roc Population Size." And the vertical axis is the "Per Capita Birth Rate."

Now let's click on this and add the next model. And we'll add the Despot Model, and let's see what happens there. We'll choose the chart, and then choose Select Data, and we're going to add a new series. The series name can just be "Roc Despotic Model." The X values, the X values along this axis, is the Roc population size. And the Y values is going to be the per-capita birth rates.

And here now, we see a very big difference between our two models. I'm noticing that we should start our horizontal axis at the number 2,000. So let's change that by choosing Format Axis.

So here's a case now where the birth rate, in blue, is from our base model. But the birth rate from the Despotic Model starts off at 1.06 and then plummets and heads straight down to 1. And if we ran this model out for many more centuries, it would stay at 1, and this whole pattern would continue to repeat. The 3,000 individuals would continue to have a clutch size of 1.06, so they would over-produce. They'd produce 180 more birds than the population can sustain. But only 3,000 individuals are potential breeders, and so that brings us back down to the carrying capacity, and the cycle just continues.

Now let's take a look at this kind of graphic in a little bit more detail. What we've seen is a graphic that looked like this. We have, on the horizontal axis, population size. And on the Y axis, we have birth rate. This is the per-capita birth rate. And we notice that it plummeted down to 1, but we are interested in seeing how it changed over from--as a function of the population size.

So here would be 1.01. This is 1.02, 1.03, 1.04, 1.05, and there's 1.06. Now this is the initial clutch size, the birth rate that we would expect when we had no problem with nesting sites. So that's a 1.06. And we saw a graphic that looked roughly like this. It started off at 1.06, and then it plummeted down to 1.

Now this line can be represented in your model by an equation. But we didn't actually program this equation in. Instead, this is an emergent property of this model. This pattern emerges from our model, just based on that simple "if" function that controls how many breeders there can be.

However, there is a way to include these kinds of equations directly in your model. And if you choose to do a model that includes this, then you're definitely making some different kinds of assumptions. **But we want to show you how you include a function, birth rate, or a rate as a function of something else.** In this case, we're dealing with population size.

Now before we can start to talk about this kind of function, we're going to start with a simpler function, which you may have seen before, and that's just going to be a linear function that assumes that the birth rate will decline as the population size increases.

How would we do that? Well you might remember from earlier math classes that the line has the formula Y equals MX plus B. In this case, B does not mean birth rate. This is the intercept, or the Y intercept. M is called the slope. X is the population size, and Y is the birth rate. So in our context, the birth rate is Y. M is the slope. X is the Roc pop size. And B is the Y intercept. And we know that that is the clutch size. That's 1.06. It's the place where this line crosses the Y axis.

Now we can use this equation to predict what the birth rate should be if we know what the population size is, if we know what the slope is, and if we know what B is. So let's try that out. We'll do this for a few different cases of X.

So we'll let X start with zero. And we'll consider what's the case when the population size is 100 animals or 200 animals. And let's start off by assuming that the slope is zero. So we're interested in calculating the birth rate, and we're going to put a little subscript here indicating that the slope is zero. So how would we do that? Well we just follow through our formula. We're interested in calculating the birth rate, so Y, the birth rate, equals M, is the slope--we know that's zero--times X--that's zero--plus B. That's 1.06. So when the population size is zero, the clutch size, or the birth rate is 1.06.

How about when the population size is 100? Well here we have Y equals M, the slope, is zero times 100 Rocs plus 1.06. This is zero plus 1.06, and the birth rate is 1.06.

Last one, Y is equal to--what's the birth rate when the population size is 200 individuals? Well the slope is zero, X is equal to 200, and the Y intercept is 1.06.

So what we've done here is use this model here, and we've set the slope to zero, and what we've done is re-create our original Roc model. The birth rate is constant no matter what the population size is.

Now let's try a different case. Let's calculate the birth rate when the slope is negative 0.001. We use the same method. We're interested in calculating the per-capita birth rate when we start off with X is equal to zero. When the Roc population size is zero, the per-capita birth rate is M, the slope, in this case, zero, negative 0.001 times zero plus 1.06. Well that's 1.06. So these two are equivalent.

What about when X is 100? Then we have Y is equal to negative 0.001 times 100 plus 1.06. Well we just multiply by 100. So we move the decimal place over two times, one, two, and we're left with negative .1 plus 1.06, and that gives us .96. With me so far?

Let's try this last one. Y is equal to negative 0.001 times 200 Rocs plus the Y intercept, 1.06. And if we carry this through, multiply this through, move the decimal place over. We're left with .1 times 2--that's .2--plus 1.06, and that gives us 0.86.

So in this simple model, we've built a model where the birth rate is dropping as a function of the number of animals in the population. So this is a density-dependent model.

Let's take another look at this on the spreadsheet since it's easier to do these calculations there. Let's add these equations to our spreadsheet. And we'll use a little bit of our spreadsheet to just demonstrate how that linear model will work. We're going to now add this to our spreadsheet, the linear model.

Before we do that though, I just want to set up a little bit of a portion of our spreadsheet and actually just build the model as a linear model. And, of course, we need this slope, and we need the intercept. And we know that the intercept, the Y intercept, is 1.06. We don't really know what the slope is going to be, but we know that that's an input.

We know that Rocs is X in our equation, Y equals MX plus B. And, again, Y is what we're trying to predict, and this is the clutch size, or the birth rate. And I'll just designate that with a Y.

So we can begin by filling in a number here. Let's let the slope be represented by this number. And we'll look at a number of different cases for Rocs' population size.

Let’s start it off with zero, 500, 1,000, and we’ll carry this through to, say, 4,500, 5,000. Now we need to just enter our equation here. We’re predicting Y, the clutch size, as a function of M, the slope, X, the number of Rocs, and B, the Y intercept, 1.06. So we’ll just enter it that way directly. Y is equal to M--anchor that--times X, the population size, plus B. Anchor that. And now we can pull this all the way through.

The best way to study this is to make a quick graph of it. So let’s do that now. Here’s an example of clutch size as a function of population size, based on the parameters that we have entered in these green boxes. I’m going to pause my video and add the axes labels right now.

Now what you can see is when the slope is positive, we see an increasing function. So as the population size goes up, the per-capita birth rate goes up. So every single Roc that is breeding is producing more and more offspring. If we make that a negative number, then we see that the function reverses its direction. As the population size goes up, the per-capita birth rate goes down.

What happens if it’s zero? Well if it’s zero, we’re back to the situation we looked at before. There’s no effect of population size on per-capita birth rate. And so, no matter what the population size is, we’re stuck with a per-capita birth rate of 1.06. This is very much like our base model. So you can see here that we can add in a slope parameter. We’ll call it “M.” And we’ll be able to enter a number here. And if we start off with a zero, then we will basically be invoking the base model.

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