

DECISION ANALYSIS

Linear Programming Using Solver

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Well, now you've seen how to solve the problem in Erewhon, using a spreadsheet and the **Solver** routine. It's really neat, isn't it? It's actually more than neat, because one of the things you get out of the Solver solution is something called an 'Answer Report'. And if you have not already done so, click on the Answer Report and see what it tells you. And what you will see is, it tells you what the maximum number of species is that you've saved, the 390. It tells you the 'binding constraints', and it tells you what the 'slack' is in the nonbinding constraints.

Well, what does binding constraints and slack signify? The **binding constraints** are those constraints that determine the solution. They are the constraints that intersect at the solution point. You cannot change those without changing the solution. The other constraints, the **nonbinding** ones, can be changed without changing the solution. And the **slack** tells you how much you could change them before they would affect the solution.

So for example, if you look at your very final constraint, which was that H had to be less than or equal to 80, you will see that it is nonbinding and that the value of the slack is 50. What does this mean? If we went back to our graphical solution, the constraint H, less than or equal to 80, is this line over here. The solution to the problem was here, where H was 30 and V was 60.

Now, if the slack refers to H, in other words if the constraint has been written with H on the left-hand side - in this case, H less than or equal to 80 - it means that you can change H by 50 units before it affects the solution. So here's H equal to 80. Move it down 50 units, and that line will move parallel to itself to the point where H = 30. And at that point, that constraint will become a binding constraint because it passes the solution point. Change H by anything more than 50 in that direction, and you will get a completely different solution to the problem.

If your constraint happens to be a line like that, then the slack could either refer to V or H, depending on how you wrote the constraint when you put it into Solver. If it has, for example, V on the left-hand side and if you find that the slack, again, for example, is 10, that means you could move the line parallel to itself down 10 units along the V axis. And at that point, for

example, the social upheaval index might pass through the solution point and become a binding constraint. So the answer report tells you quite a lot about the solution.

The next thing you can do with your Solver routine is, especially if you've set up the spreadsheet carefully, you can go and, for example, say, "What would happen if we raised another 10,000 pickles?" All you need to do is change your budget from 24,000 pickles to 34,000 pickles, go to Solver, press Solve, and you'll get the new solution. And, if you were to do that, you would find that the budget is no longer a binding constraint and that there is slack in the budget. And if you look carefully at your graph and think it through, you'll be able to figure out how much money you've got left over that you don't need to spend.

So notice again how you can use, with Solver and a spreadsheet, a linear programming or optimization solution as a tool to work with a group of stakeholders or to work with decision makers to explore the answer to the problem.

If you think about it, what we have really done here is taken a multi-objective problem and finessed it into a single objective problem. Why do I say this? If you step back to the real world behind this problem, it is a problem where there are social, political, financial and conservation objectives. So there's more than one objective.

What we have done is taken the political, the financial and the social objectives, and we have negotiated them into constraints so we're only left with one objective, which is to maximize the number of species. So in a sense, we have taken a multi-objective problem and negotiated it into a problem that, mathematically speaking, we solve as a single objective problem. And that can be fairly powerful, but it might not always go down well with a group of stakeholders.

For example, maybe the sociologists are concerned that saving as many species as possible is an objective, whereas you have handled the social upheaval merely as a constraint. If there were some concern about that in a workshop, you could actually perform a series of experiments on your spreadsheet. You could do this in just a few minutes, and the experiments could lead to a graph that looks something like this. What I do here is I change the limit on the social upheaval index. Remember we had had it as 400. I say, "What would happen if we made it 500? What would happen if we made it 350, 300, 250 and 200?" And, each time I change the social upheaval index, I solve the problem, and I get a completely different solution point to the problem. I also get the maximum number of species I can save. So what I've done here is I've graphed the number of species I can save versus the limitation that I've put on the

social upheaval index. And I would get a curve like that, that maxes out at 390 species, if we keep the budget fixed at 24,000 pickles.

Now that curve is a very interesting curve. And there are three points I want to make about it.

The first point is that every point on that curve represents the best that you can possibly do under those conditions. So each point on that graph is an optimum point. This is really valuable because suppose somebody comes up and argues that here is a solution which has a social upheaval index of 300 and saves 200 species, and says, "Why don't we do that?" And, as soon as I have a graph like this, I can look at that point and say, "You know that isn't an optimal solution. We can, in fact, do better."

If we want to improve the solution, we could have a win-win situation. For example, we could move from there to there. And there we have a solution where we've got a lower social upheaval index, and we've saved a lot more species. So creating an optimum envelope like this enables you to look at various suggestions and see which of those can be improved and where you have win-win situations.

The third point I want to make is that as you go up this graph, you can look at what you are gaining and what you are losing as you increase the limit in the social upheaval index. So for example, I don't have the exact numbers in here, but as I increase my social upheaval index from 200 to 250, I might save quite a lot of new plant species. This could lead to an interesting debate because we could have the sociologists talk about what the implications are of having an upheaval index of 250 versus 200, and we could have the conservation biologists argue what the advantages are of saving that many more species. And chances are you might come up with a very clear-cut situation where the tradeoff is valuable and falls to the favor of the species. In other words, you could argue that it is worth having that slightly more social upheaval in order to save quite a lot more species.

But notice what happens as you get to this part of the graph. Suppose your social upheaval index limit is changed from 350 to 400. Well, at that point, you might not save that many more species. And so again, if you had the sociologists explain what might be the difference between 350 and 400, you might actually decide from looking at this graph, that you are not going to save 390 species with a social upheaval of 400. But you are going to go to a plan where the social upheaval index is only 350 and you save slightly fewer species.

Notice again the way in which I'm using my linear programming solution. I am not using it to find 'the' optimum solution. In fact, I cringe very often when people talk about the optimum solution, because the optimum solution depends on the conditions that you apply. **The optimum solution is a model world concept.** The real world is always more complicated. So what I'm doing here is, I'm using an optimization technique to explore, to interpret back from the model world to the real world and try and come up with a really good solution. At each stage of my argument, I am getting the best answer that I can and that is extremely valuable.

So what have we done? If you like, you can think about a modeling toolkit that you are slowly accumulating in this class. And you have added to that toolkit linear programming and a whole lot of other algorithms that enable you to do constrained optimization. And as you add tools to your kit, you've got to start asking yourself which is the appropriate tool to use.

There are some people out there who are specialists in linear programming. There are people who are so good at linear programming, that present them with almost any problem, and they will figure out how to finesse that problem into a linear programming statement. And they will solve it using linear programming. In the process, it's possible that they will distort the problem and come up with an answer that's totally inappropriate.

Remember the adage "To a man with a hammer, every problem is a nail." To a person who is expert in linear programming, very often the only way to solve problems is linear programming.

So one has to be cautious with one's toolkit. Not to have the tool in the kit, not to be able to look at a problem and say, "Gee, you know, I think we can really do a lot with this problem if we use linear programming," - is a loss. You are not going to be as good a modeler as you could be if you knew about linear programming.

But to use linear programming indiscriminately and inappropriately is also a loss. Because you are potentially going to bludgeon problems into straightjackets that confuse the people you're working with and don't really help you solve the problem.

So ultimately linear programming is just another modeling paradigm, and you use it as you always use models: to change your assumptions, change your numbers, do a sensitivity analysis and try and interpret your answers back to get a good solution in the real world.

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