<u>DECISION ANALYSIS</u>

Linear Programming Optimization, Part 2

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Okay. We've solved the problem. What have we learned? Well, we've discovered a new technique, and that technique is called **linear programming**. Some of you might have heard the term before, and people very often call it LP for short. We have recognized or discovered that this is a problem that can be solved using the linear programming technique. And what you have seen on the graph is a graphical solution for a linear programming problem.

What characterizes a linear programming problem? The first is that we have 'variables'. In this case, we had two variables, and they were V and H. Then we have an 'objective function', which we either have to maximize or minimize, and in this case, our objective function was the number of species we saved, which was 5V + 3H. Notice that the objective function is mathematically speaking linear. It plots as a straight line. It has no terms like V x H, or the square root of V, or any other expression involving V in it. It's a number times V plus a number times H.

Then we have eight conditions, and the jargon for those conditions, which I've already used in talking about them, is 'constraints'. And each constraint plots as a straight line.

So a linear programming problem is one that has variables, an objective function and constraints, and both the objective function and the constraints are mathematically linear.

Okay. What happens if we have three variables? Well if we look back at our graph, if we had three variables, we could have another axis coming out of the page. And instead of having a two-dimensional graph, we would have a three-dimensional graph. Our constraints would plot on that as planes, and those planes would define an empty area in the middle that would look a bit like a diamond. And one of the points on that diamond would be the solution to the problem. If you had a holograph, you might be able to solve that problem graphically.

But what happens if you have four variables? Mathematician has no problem with this. You would have a four-dimensional space, and in the middle of that four-dimensional space, you would have a four-dimensional diamond, whatever that looks like. And somewhere there would be a corner on that four-dimensional diamond, and that would be the answer to the problem. And here I just cannot imagine solving the problem graphically.

Well it turns out that some people have thought hard about how to solve problems like this in conditions where you have more than two and maybe hundreds of variables. And there is an algorithm called the '**Simplex Method**', which is basically just a computer program that gets you to the correct answer as quickly as it can.

I've used a new term here. I've called that an **algorithm**. And I just want to take a moment out to talk about the difference between algorithms and a word we've heard before which is heuristics. Remember a heuristic is like a rule of thumb. It is often useful. It often helps you solve the problem, but it comes without guarantees. It doesn't always work. An algorithm, on the other hand, comes with guarantees. It has been mathematically shown that if you follow an algorithm, you will always get to the solution you want to get to, and moreover you will get there pretty darn efficiently.

If you think about stopping in the days before GPSs and asking somebody how to get to a particular address, 99 times out of 100, that person would give you a heuristic. You'd probably get lost following the instructions. If you got a really good set of instructions that would tell you precisely when you'd got to your end point, you would have got an algorithm.

Okay, I want to go back to the graph and show some of the things you can do with a solution like this. Imagine, for example, that you were a facilitator in a workshop, and you had various stakeholders in the workshop. You had conservation biologists who were anxious to save as many plant species as possible. You would have some political representatives from the tribes who are jockeying for position. You've got a manager who also wants to save as many species as possible, but is limited by how much money he or she can spend. And then you've got your sociologists who are concerned about social upheaval and so on. And suppose you have developed your conditions, you've negotiated the conditions you're prepared to live with, and you've come up with a solution, and the solution is this point. What this tells you is that the

budget is really limiting how many species you can buy. Notice that the social upheaval condition does not limit how many species you can save.

Now imagine a situation where the sociologists might have been arguing, and maybe there were two schools of sociology, and one school said, "You know it's quite permissible, there's not going to be much upheaval if you limit the social upheaval to 500 instead of 400." And if you think about, if you made that condition V less than or equal to 500, it would mean that you would move this line out parallel to itself some distance. Whereas another school of sociologists says, "No, 500 is going to cause problems, you really need to limit it to 400." Which is this line.

Now if you were just facilitating a workshop in which people were arguing without an algorithm or a technique like linear programming, the arguments between the sociologists could go on forever. Notice, however, that once you've drawn this graph, it makes no difference at all whether the limit is 400 or 500. So although that might be of great interest to the sociologists, it doesn't affect the solution. Notice, by the way, what we're doing here as we explore questions like this. It's a sensitivity analysis.

Well, let's ring the changes a bit. There's nowhere under present financial conditions that you're going to be able to get more than 25,000 pickles from the government in Erewhon. But suppose an enterprising person in the workshop says, "I'm going to start a committee, and we are going to go out and raise an additional 10,000 pickles to save the plants in the valley and the highland." Suppose that suggestion is made. What would it mean to save an extra 10,000 pickles? Well, you could actually change the 24,000 to 34,000, and this line would move up, and it might move out to somewhere like that. You could actually do it. And, if you were to move this line from there to there, notice that this area would no longer be hatched out, but now your solution point would be completely different. And it is likely that your solution point would probably - not definitely - but probably be there, or maybe even there. You'd be able to find out which of those it really is.

And now you have a situation where potentially the social upheaval index is limiting you, in which case, this debate between 400 and 500 for the social upheaval index becomes relevant. Or else you are maxing out on the number of species you save in the highland and you are being limited by the political arrangements. So it's possible that you could renegotiate the political arrangements.

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