EXERCISE 4: SINGLE-SPECIES, SINGLE-SEASON MODEL WITH SITE LEVEL COVARIATES

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SINGLE-SPECIES, SINGLE-SEASON MODEL WITH SITE LEVEL COVARIATES SPREADSHEET EXERCISE

OBJECTIVES

- To learn and understand how site-level covariates are evaluated in a basic occupancy model.
- To understand additive, polynomial, and linear models with interactions.
- To use Solver to find the maximum likelihood estimates for the probability of detection and the probability of site occupancy, given a set of covariates.
- To use model selection approaches to compare and rank models.
- To compute the observed data's modified Pearson Chi-Square value.
- To conduct a MacKenzie and Bailey Goodness of Fit test, in which bootstrap simulations are used determine if the observed Chi-Square value is larger than expected by chance.
- To learn how to simulate occupancy data with covariates.

BACKGROUND

Now that you have a handle on the general occupancy models, we can make them a bit more complex by adding covariates to the analysis. The information for this exercise roughly follows the materials presented in chapter 4 of the book, Occupancy Modeling and Estimation. Click on the worksheet labeled "Site Covariates." Adding site covariates to the general occupancy model is extremely useful for ecological studies - the analysis lets you assess whether occupancy of a site is a function of some covariate(s), such as patch size, habitat type, site condition, etc. After all, most research projects are set up to evaluate how

covariates affect occupancy probability, so you might as well learn how to analyze covariates correctly!

Let's step back for a second and answer the question, "what exactly is meant by a "covariate" effect?" Well, remember the parameters of the "general" occupancy model we analyzed in the previous exercise: p_1, p_2, p_3, and ψ. The only raw data we had to run a model were the encounter history frequencies. We found the combination of parameter estimates that maximized the multinomial log likelihood. But with covariate analyses, a lot of other data is collected at a site in addition to the encounter history frequencies. For example, you might collect data on the date the site was sampled, the time the site was sampled, weather conditions of the sampling period, as well as physical and biological characteristics of the site. The data can be collected remotely (with GIS) as well as on-site. The covariates you collect could help explain differences in ψ among the sites (site-level covariates) or could help explain differences in detection probabilities among the surveys (survey-specific covariates). In this exercise, we will focus on site-level covariates that explain differences in occupancy probability among the sites sampled. In the next exercise, we will focus on survey-specific covariates that explain differences in detection probability among the different replicate samples.

EXAMPLES OF LINEAR COVARIATE MODELS

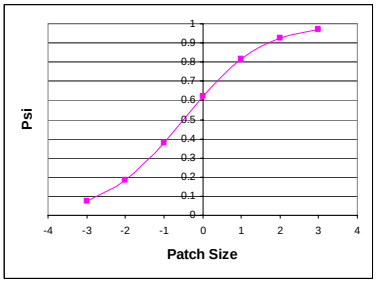
Our focus will be on two covariates that might influence the probability that a given site is occupied: patch size and habitat type. To begin, suppose you think that "patch size" could affect the probability that a site is occupied. In this case, the patch size is called a "site" covariate because whether a site is occupied or not is a function of the individual site's patch size. In this example, patch size is a

continuous covariate, where the values fall within a given range (e.g., 1 ha to 1000 ha patches). Because each site has its own patch size, each site will end up with a unique probability of occupancy that is directly linked to its corresponding patch size. As a second example, suppose that whether a site is occupied or not depends on the habitat type associated with that site. In this case, habitat type is a site-level covariate that is categorical. A categorical variable can take on one of many discrete values; e.g., a person's eye color can be blue, brown, green, or hazel. Habitat type is another is another example of a categorical variable, where values can be "grassland," "forest," and "wetland." A site is characterized as one of these types only; i.e., a site can't be characterized as both grassland and wetland.

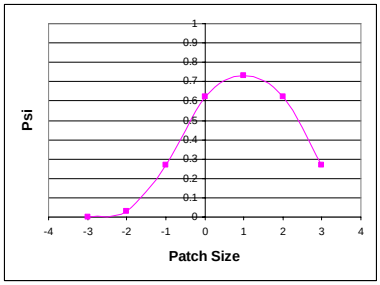
One important point to keep in mind is that, for a single season occupancy model, the sites are assumed to be closed – meaning that the occupancy status of the site cannot change over the course of sampling periods. As such, it makes sense that covariates thought to affect occupancy are relatively stable (or unchanging) over the season. Covariates such as patch size work well in this context because patch size will not likely change within a single season. However, covariates such as temperature or date make less sense because these values can change within a sampling season.

Here's what you can look forward to in this exercise -- we'll be exploring five different linear models where psi is a function of covariates:

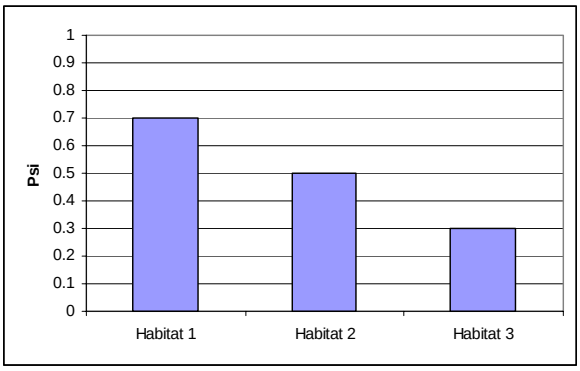
1. Psi is a linear function of patch size:



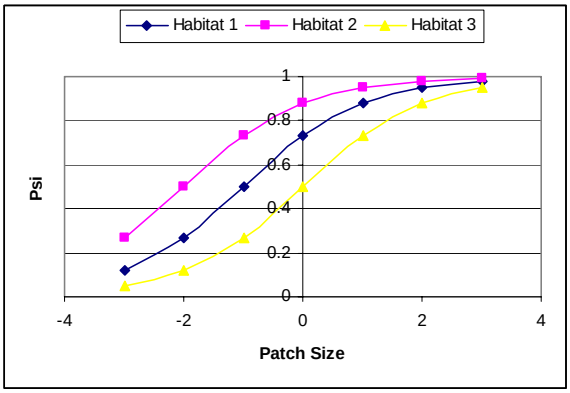
2. Psi is a polynomial function of patch size:



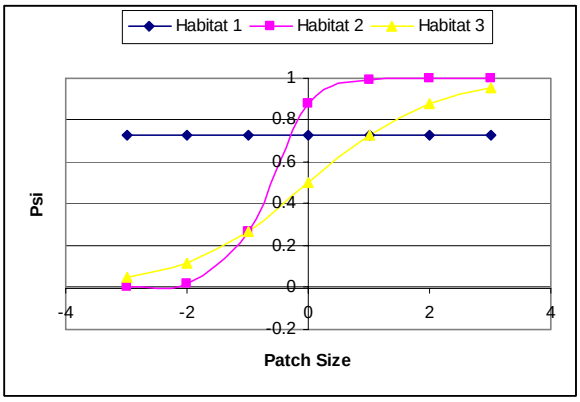
3. Psi is a function of habitat type:



4. Psi is a function of habitat <u>and</u> patch size.



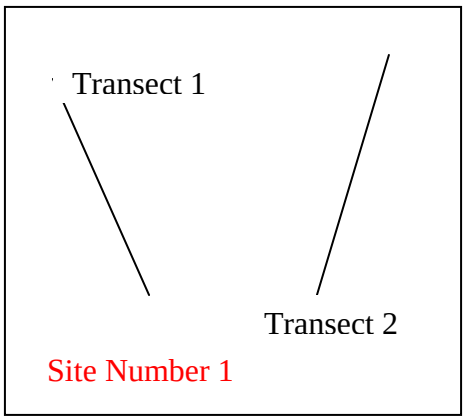
5. Psi is a function of an interaction between habitat and patch size.



All of these are examples of linear models in the statistical sense (even though the graphs don't look linear!). Psi might be also be a function of many other physical or biological properties of the site, such as food levels, number of predators, distance to other patches, etc. However, we will cover a LOT of ground (a crash course in linear modeling, in fact) just by focusing on the 5 linear models that include patch size (which is continuous), habitat (which is categorical), and combinations of patch size and habitat. Then, we'll use model selection procedures to compare the five different models. Finally, we'll run a parametric bootstrap to assure that at least one model in our model set "fits" the underlying occupancy model to an acceptable degree. This is a long exercise (sorry about that!), so let's get started.

SAMPLING DESIGN

In this spreadsheet, we'll assume that each site is surveyed by searching for the target species along two, spatially replicated transects (each sample was conducted in a different location within the site).



However, this example works just a well by assuming that each site was surveyed at two different times during the study (temporal replication), under the assumption that the population was closed to changes in occupancy between sample periods. At each site, we collect data on the patch size in which the site is located, and document the dominant habitat type associated with the site as a whole. For the sake of simplicity, we'll assume that there are only three habitat types in the study area.

SPREADSHEET SET-UP

This spreadsheet has a slightly different set up than the previous one. First, to simplify things, there are only 2 sampling sessions for each site. Second, the cells on this sheet are not named. The reason is that when you name a cell in Excel, the cell becomes a fixed reference (e.g., $A$1 instead of A1), and for reasons which should become apparent to you soon, we don't want this to happen. Third, instead of analyzing the multinomial log likelihood based on the summarized capture frequencies, we develop a log likelihood for EACH site. You'll see why this is so in a minute. For now, note that the sites are given in column B, the results of Survey 1 are given in column C, the results of Survey 2 are given in column D, and the history associated with each site is given in Column E. If the histories in column E do not

match the histories in column A, please copy the values from A16:A215 to E16:215 to restore the original site history values so that your analysis matches the exercise values.

Since there are only 2 capture sessions in this example, there are only $2^2 = 4$ possible histories: 11, 10, 00, and 01. These histories are summed across the sites in cells E5:H5 with a COUNTIF function. (Note: we won't actually analyze these summed histories...they're just there to summarize the data.)

	E	F	G	H	I
3	**Summarized Inputs**				
4	11	10	01	00	Total
5	50	18	22	110	200

Also note that for this spreadsheet example, there are 200 sites – a heck of a lot of sites!

DETECTION PROBABILITY

As we mentioned, in this exercise we are not concerned with survey-specific covariates that might influence p_1 and p_2. But we still have to model p_1 and p_2 because they are necessary components of the encounter history probabilities. Furthermore, we will assume that p is constant for both surveys ($p_1 = p_2$), so our underlying model will be p(.). Thus, in column F (shaded green), we enter a 1 for each site (which is the value that will be multiplied by the intercept for p; we'll explain this later).

	B	C	D	E	F
15	Site	Survey 1	Survey 2	History	P (Int)
16	1	0	0	00	1
17	2	0	0	00	1
18	3	0	0	00	1
19	4	0	0	00	1
20	5	0	1	01	1

SITE LEVEL COVARIATES

Now let's focus on the ψ side of the equation (where the cells are shaded blue). In addition to a capture history for each site, we record covariates associated with each site. Again we enter a 1 in column G which will ultimately be multiplied by the intercept for psi. Now, what factors might affect the occurrence of a species of interest? Ecological theory provides us with many potential examples: patch size, patch isolation, disturbance level, number of competitors, etc. These covariates can be continuous or categorical (though as a rule of thumb you should strive for measuring covariates on a continuous scale whenever possible). As we mentioned, we will focus on two factors that are potentially associated with occupancy (psi): patch size (which is continuous) and habitat type (which is categorical).

For the sake of argument, let's start with Cov 1 (column H), which is the patch size associated with each site. Then, in columns J and K we use 0 and 1 coding to identify habitat type…we'll describe this coding in a few minutes.

	G	H	J	K
13	Int	Cov 1	Cov 3	Cov 4
14	Occupancy Covariates			
15	Psi (Int)	Patch Size	Habitat 1	Habitat 2
16	1	-1.9936	1	0
17	1	0.9660	0	0
18	1	1.1794	0	1
19	1	-1.8955	1	0
20	1	1.1377	0	0

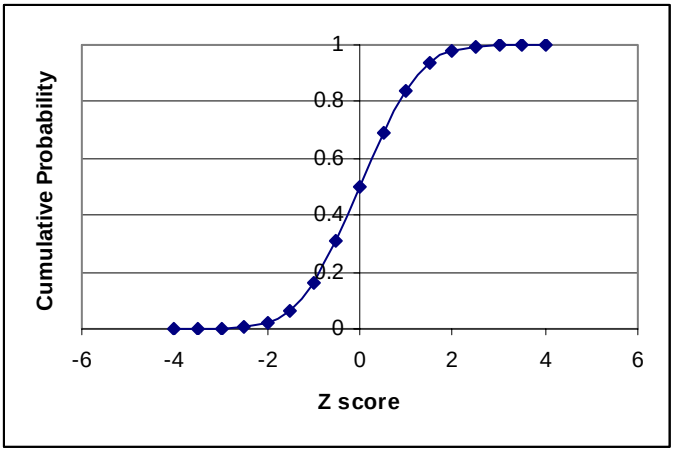
STANDARDIZED COVARIATE VALUES

Now that we know what the potential covariates are (patch size and habitat), let's look at the values that are entered for the first 5 sites (above). You should notice first off that the categorical covariates (habitat: Cov 3, 4) contain only 0's and 1's, whereas the continuous covariates (patch size: Cov 1) range from about +3 to -3. For example, site 1 had a patch size covariate of -1.9936; site 2 had a patch size covariate of 0.9660, and site 4 had a patch size covariate of -1.8955. But you might be scratching your head and thinking, "How can patch size have a value of -1.8955?" The answer is that the continuous variables have been standardized. That is, they've been converted to Z scores. You might remember from your introductory stats course that the Z transformation takes continuous data, and standardizes the data so that the mean for the population is 0 and the standard deviation is 1. This is done by computing the mean and the standard deviation for the raw data; each raw covariate score is then standardized by subtracting the mean from raw value, and then dividing the result by the standard deviation:

$$Z = (\text{raw score} - \text{mean score})/\text{standard deviation}.$$

If you have raw data, you can convert them to Z scores with the STANDARDIZE spreadsheet function. Z values range from about +3 to -3, and a 0 indicates the average. About 68% of the Z data should have values between +1 and -1, and about 95% of the Z data should have values between +2.5 and -2.5. This can be seen by studying the cumulative graph below, where the data were generated with the NORMSDIST function in Excel. The Z scores are on the X axis, and cumulative probability is on the Y axis. Note the symmetry of this graph: half of the data

occur below a Z score of 0, and half of the data occur above a Z score of 0. The graph shows cumulative probability: find a Z score of interest, and then find its corresponding cumulative probability. For instance, the probability of getting a Z score of at least 0 is 0.5. The probability of getting a Z score of at least -2.0 is 0.023. The probability of getting a Z score of at least 1.0 is 0.84. This means that if a site has a Z score of 1.0 for patch size, 84% of the sites had Z scores lower than the site and 100-84% = 16% of the sites had Z scores higher than the site.



So, getting back to our example, site 1's patch size covariate of -1.9936 indicates that this site was quite a bit smaller than the average site. Site 2's patch size covariate of 0.9660 indicates that this site was quite a bit larger than the average site, and site 6's patch size covariate of 0.0262 indicates that this site is situated within an average-sized patch for this study. Thus, Z scores provide you with a lot more information than the raw score. For example, a raw score of, say 510 might not mean much to you, but the graph above shows that a site with a standardized Z score of 1.21 indicates that about 90% of the sites had Z scores less than this site, and about 10% of the sites had Z scores greater than this site. (For more

information on Z scores, see http://www-stat.stanford.edu/~naras/jsm/FindProbability.html).

Why standardize? The major reason is provided by Gary White in the MARK helpfiles: "When the mean value of individual covariates is very large or small, or the range of the covariate is over several orders of magnitude, the numerical optimization algorithm may fail to find the correct parameter estimates." Aside from converting your raw data to Z scores, there are other ways to avoid this problem. For instance, if you are dealing with temperature or elevation data, you could simply divide the raw scores by some constant to reduce the range of the data itself. Here is a quote from the Program PRESENCE page about strandardizing/transforming covariates: "The best approach is to transform your data onto another scale which is still meaningful to you. You could divide the covariate values by some constant (i.e., rather than entering 80% humidity as 80.0, use 0.80); subtract the average of the covariates from each observed value (i.e., X* = X – average(X's)); or some combination of the two. Such transformations are not carried out by PRESENCE automatically, but can be done easily with a spreadsheet and the modified values pasted back into the Data Window."

Gary provides a warning about the use of Z scores in the MARK helpfiles:
"One caution is in order about using the z transformation on one or more individual covariates and another temporal or group covariate in the design matrix to predict a single real parameter. Situations can arise where the real parameter estimates and the model's AIC differ between runs using the standardized covariates and the unstandardized covariates. This situation arises because the z transformation

affects both the slope and intercept of the model." There is a very good description of this problem in Chapter 12 of Cooch and White's book, "MARK: A Gentle Introduction."

CODING TREATMENTS

OK, enough about standardization of patch size (for the moment). Now let's look at the occupancy habitat covariates (Cov 3 and Cov 4, in columns J and K). Cov 3 and Cov 4 represent three different kinds of habitat types. Three? Not two? Yes, three….to code for 3 habitat types, you need 2 covariates. Take a look again at the data associated with the first five sites:

	G	H	J	K
13	Int	Cov 1	Cov 3	Cov 4
14		Occupancy Covariates		
15	Psi (Int)	Patch Size	Habitat 1	Habitat 2
16	1	-1.9936	1	0
17	1	0.9660	0	0
18	1	1.1794	0	1
19	1	-1.8955	1	0
20	1	1.1377	0	0

There were three habitats surveyed, and the 0 and 1 coding for Cov 3 and Cov 4 reveals the habitat type associated with each site. If the site was surveyed in habitat 1, then Cov 3 = 1. If Cov 3 = 0, then the site was not characterized as habitat 1. If the site was surveyed in habitat type 2, then Cov 4 = 1. If Cov 4 = 0, the site was not located in habitat type 2. By this coding, if Cov 3 = 0 and Cov 4 = 0, the site is located in habitat 3. So, to summarize, Cov 3 = 1 and Cov 4 = 0 codes for habitat type 1, Cov 3 = 0 and Cov 4 = 1 codes for habitat type 2, Cov 3 = 0 and Cov 4 = 0 codes for habitat type 3. Because habitat 3 is coded as 0 0, it is called the "reference habitat" and the other habitat types are compared to it. You can

make the reference habitat any type you want (1, 2, or 3) by altering your coding system. You might be wondering, "what if Cov 3 = 1 and Cov 4 = 1? What does that code for?" The answer is it's not a valid coding for habitat type (in this case) because it suggests the site is both habitat 1 and habitat 2. Using this coding system, sites 1 and 4 were in habitat 1, sites 3 was in habitat 2, and sites 2 and 5 were in habitat 3. We'll come back to this in a bit. For now all you need to understand is that each site has its own history and its own set of covariates, how continuous covariates are standardized, and how categorical covariates are coded.

You've probably noticed that there are three other covariates in the dataset (Cov 2, Cov 5 and Cov 6). These covariates are computed from the patch size and habitat covariates. For example, Cov 2 is patch size squared, so the formula in cell I16 is =H16^2. We will visit this covariate when we run a polynomial model. Similarly, Cov 5 and Cov 6 are computed by multiplying patch size by the habitat coding. We will visit these covariates when we run the patch size by habitat interaction model. Thus, we have 6 site-level covariates available for modeling.

	G	H	I	J	K	L	M
13	Int	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
14	Occupancy Covariates						
15	Psi (Int)	Patch Size	Patch Size ^{2}	Habitat 1	Habitat 2	H1*PS	H2*PS
16	1	-1.9936	3.9743	1	0	-1.994	0.000
17	1	0.9660	0.9332	0	0	0.000	0.000
18	1	1.1794	1.3911	0	1	0.000	1.179
19	1	-1.8955	3.5929	1	0	-1.895	0.000
20	1	1.1377	1.2943	0	0	0.000	0.000

USING LINEAR MODELS TO ESTIMATE SITE-SPECIFIC PSI

OK, now given each site's covariate values, we need to determine what p and ψ are for each site. (Remember that our underlying model is p(.)psi(covariate), so we are

interested in estimating two real parameters, p and psi, and we'll do this on a site-by-site basis). Let's focus on only one covariate (patch size) to begin with….we'll add more covariates after you have a handle on the basics. Let's assume that as the standardized Z score for patch size increases or decreases, the probability of occupancy increases or decreases in a predictable fashion. We could use a regression model to do this:

psi = B_0 + B_1 * covariate, or in our case…
psi = B_0 + B_1 * standardized patch size.

OK, if you've taken an introductory stats course you'll notice that this is the equation of a line (y = mx+b, or y = B_0 + B_1x) and by knowing B_0 (the intercept) and B_1 (the slope), as well as a site's Z score for patch size, you can estimate psi with linear regression approaches. If B_1 is positive, the relationship is positive, where sites with high Z scores (i.e., those sites with larger patch sizes) will have a higher occupancy probability, and if B_1 is negative, the relationship is a negative relationship where sites with high Z scores will have a lower occupancy probability.

LN(ODDS) OR LOGIT MODELS

But, hang on! Psi is a probability, and is bounded between 0 and 1. We can't do a regression analysis for this model because regression analysis requires that the response variable (psi) be unbounded. What now? The way around this problem involves converting the probability, psi, to odds, and then taking the natural log of the odds, and then modeling the log odds (or logit) of psi instead of psi, and then back-transforming the logit of psi to get psi. Hmmmm, let's try that more slowly. You're all familiar with odds (e.g., "what are the odds that the Chicago Cubs will win

the World Series this year?"). Suppose the Cubs play a 10-game season and we record the number of possible wins and losses. The odds are computed as the ratio of wins:losses, or wins/losses. For example, if for every 10 games played there are 9 wins and 1 loss, the odds of winning are 9:1 = 9/1 = 9. The relationship between probability and odds is expressed with the following equation:

$$\text{probability} = \text{odds} / (1+\text{odds}).$$

Thus, if the odds of winning is 9:1, the probability of winning is 9/10 = 0.9.

wins	losses	odds	probability	ln (odds)
0	10	0.000	0	#NUM!
1	9	0.111	0.1	-2.19722
2	8	0.250	0.2	-1.38629
3	7	0.429	0.3	-0.8473
4	6	0.667	0.4	-0.40547
5	5	1.000	0.5	0
6	4	1.500	0.6	0.40547
7	3	2.333	0.7	0.8473
8	2	4.000	0.8	1.38629
9	1	9.000	0.9	2.19722
10	0	#DIV/0!	#DIV/0!	#DIV/0!

Take a good look at the table above. Notice anything special about "odds"? They range from 0 to positive infinity (in theory). So by using odds instead of probability in our linear equation, we take care of the probability bounding issue on the positive side (unlike probability, odds are not bounded to be less than 1). However, we still need to deal with the negative "boundedness". How? We take the natural log of the odds, or ln (odds). Look at the far right column in the table above and you should see that log odds (also called logits) are unbounded whereas probability is bounded between 0 and 1. So, instead of modeling psi (a probability

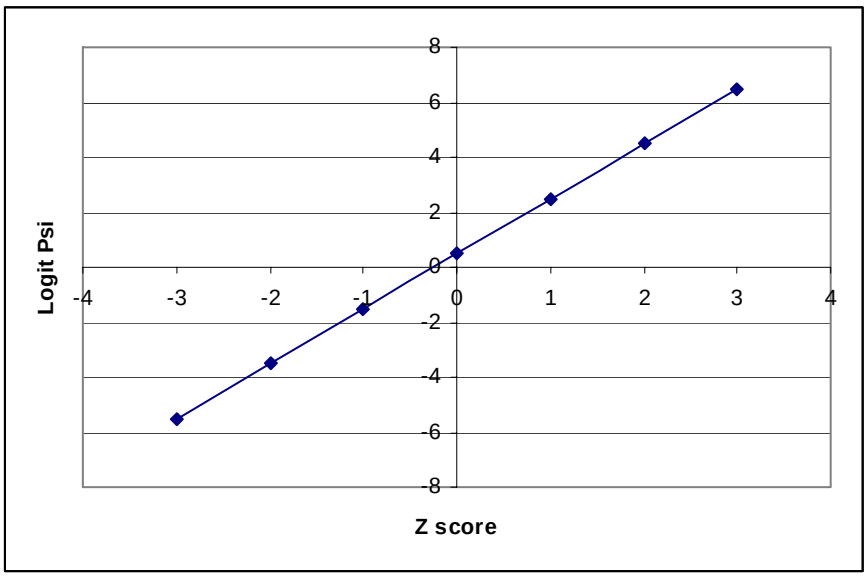
bounded between 0 and 1), we model the log-odds of psi (which is unbounded). The linear equation is now:

Logit psi = B_0 + B_1 * standardized patch size (which is a correctly specified linear model)

instead of

psi = B_0 + B_1 * standardized patch size (which is an incorrectly specified model)

The logit transformation of psi allows us to use standard linear modeling, and the goal of analysis now focuses on the estimation of B_0 and B_1 to derive an estimate of the logit of psi. If B_0 and B_1 are 0.5 and 2 respectively, the logit of occupancy probability (psi) can be pictured as shown below:



THE LOGIT LINK

But "logit psi" doesn't intuitively make sense because we're really interested in understanding how occupancy probability is associated with patch size. So how do

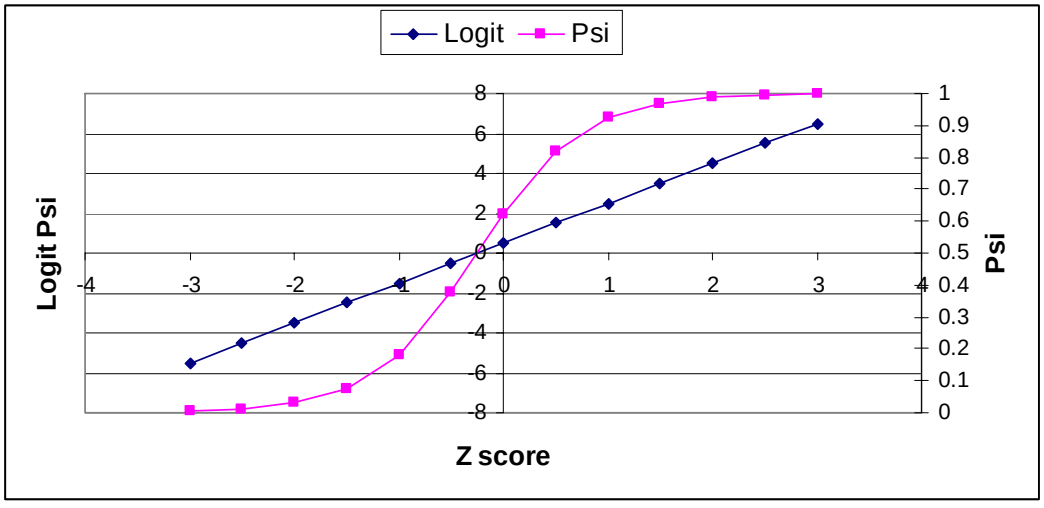
you transform the logit back to a probability? You take the anti-logit, which has the form:

$$psi = Exp(B_0 + B_1 * \text{standardized patch size}) / (1 + Exp(B_0 + B_1 * \text{standardized patch size}))$$

which back transforms the log and odds computations, or more generally

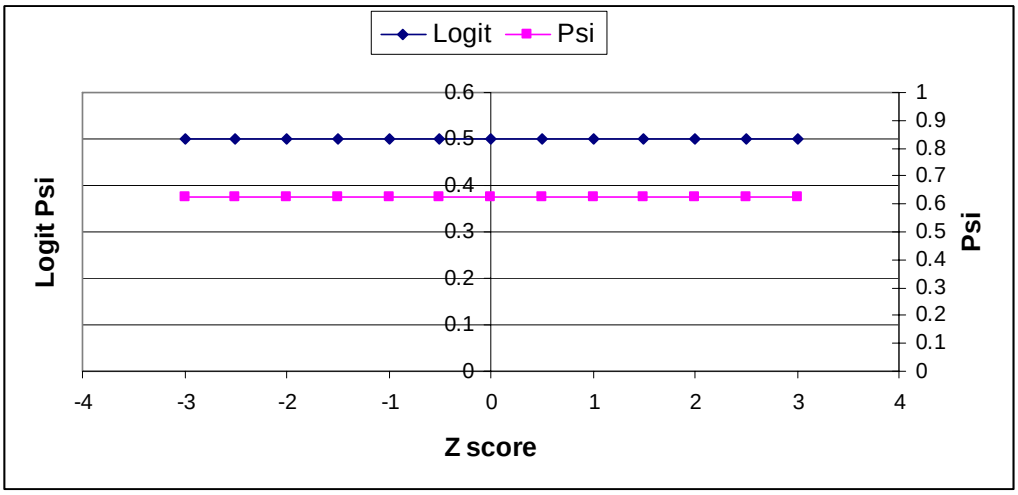
$$Exp(\text{linear equation})/(1+exp(\text{linear equation}))$$

This gets you back to the probability, psi, and the equation for converting logits to probabilities is called the logit link. In MARK and PRESENCE, the logit link is the default link when covariates are used in a model. Below is a graph of the logit as a function of standardized patch size (left hand scale, diamonds), where $B_0 = 0.5$ and $B_1 = 2$. A second series graphs the back-transformed psi as a function of standardized patch size (right hand scale, squares). Note the linear relationship between logit psi and standardized patch size, whereas the relationship between psi and standardized patch size is an s-shaped (logistic) function.



How does this apply to our occupancy model? In this case, the anti-logit gives us psi, and it does so for <u>each</u> site because we replace words "standardized patch size" in the equation above with the Z patch size of <u>each</u> specific site. An example might make this clearer. If $B_0 = 0.5$ and $B_1 = 2$, a site with a standardized patch size of Z = +0.75 will have psi =Exp(0.5 + 2*0.75) / (1+ Exp(0.5 + 2*0.75))=0.8808, whereas a site with a standardized patch size of Z = -0.75 would have psi =Exp(0.5 + 2*-0.75) / (1+ Exp(0.5 + 2*-0.75))=0.26894. That's quite a difference in psi between the two sites! In this case, the site with a smaller patch than average size (Z = -0.75) had a much lower occupancy probability ($\psi = 0.26894$) than a site with a larger than average patch size ($\psi = 0.8808$). If you were to give this model a name, you'd call it p(.)psi(patch size), indicating that psi is a function of patch size (and p is the dot model where $p_1 = p_2$).

What if $B_0 = 0.5$ and $B_1 = 0$? In this case, standardized patch size would have no effect on psi (because a site's Z patch size would be multiplied by B_1, which is 0), and the result would simply be the intercept effect ($B_0 = 0.5$). When $B_0 = 0.5$, and there are no other covariates, logit psi = 0.5 for all sites (regardless of their Z date), which corresponds to psi = 0.622 for all sites: exp(0.5)/(1+exp(0.5)) = 0.622. For this reason, some people refer to models with no covariate effects as "intercept models." If you were to give this model a name, it would be p(.)psi(.). In this exercise, we'll call the intercept for detection probability B_0, and the intercept for psi B_{00}.



Take some time with this material to make sure it really sinks in. There is also an excellent discussion of this material in MARK: A Gentle Introduction by Evan Cooch and Gary White. Darryl MacKenzie notes that you can also interpret things on the odds scale, which is sometimes more intuitive than trying to interpret the effect of a covariate on the standardized scale.

POLYNOMIAL LINEAR MODELS

OK! Now we are ready to move on to the second type of linear model that we will explore: polynomial models. These are simple extensions of the models we just discussed. A polynomial equation has the general form:

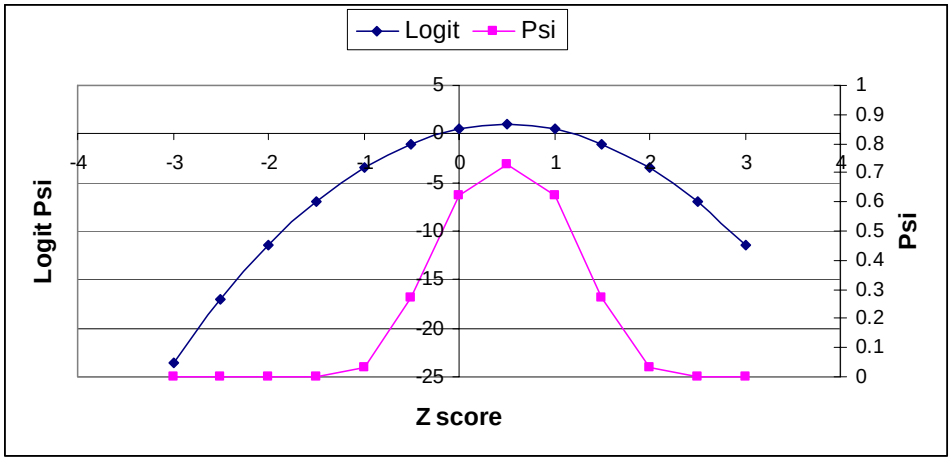
$$Y = B_0 + B_1X_1 + B_2X_1^2 + B_3X_1^3 + \ldots$$

The number of terms in a polynomial is defines its "order." A first order polynomial considers only $Y = B_0 + B_1X_1$. Thus, logit psi = B0 + B1 * standardized patch size is an example of first order polynomial. A second order polynomial adds

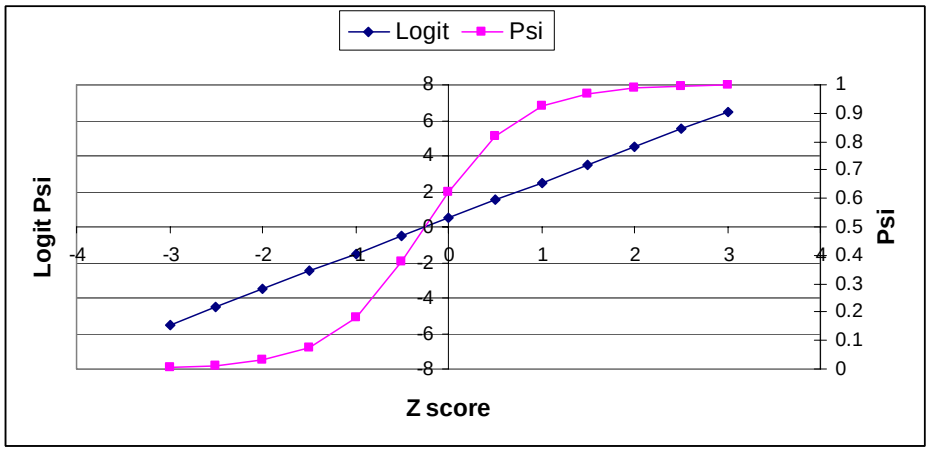
the X^2 term ($Y = B_0 + B_1X_1 + B_2X_1^2$); a third order polynomial adds the X^3 term, and so on. Using patch size for X, a second order polynomial model would have the following form:

$$\text{Logit psi} = B_0 + B_1 * \text{Patch Size} + B_2 * \text{Patch Size}^2$$

Using polynomial models opens up a whole new set of possibilities for modeling either occupancy or detection. And ecological theory provides us with many, many reasons to consider running polynomial models. For example, in many cases we can predict a "threshold response" to occupancy, i.e., we might suspect that occupancy probability will increase with increasing patch size up to a certain point, after which the probability remains constant. Or, in some cases we might suspect that occupancy will increase with increasing patch size up to a certain point, after which the probability declines. These relationships look non-linear (although you model them with a linear polynomial equation!). For example, suppose $B_0 = 0.5$, $B_1 = 2$, and $B_2 = -2$. The relationship between psi and standardized patch size would look like this:



If B_2 was 0 instead of -2.0, we'd be back to our first order polynomial, where the relationship between patch size and occupancy is quite different:



CATEGORICAL LINEAR MODELS

OK, so we've covered first and second order polynomial models where the explanatory variable, patch size, was continuous. This material might be familiar to you if you have taken a course in regression. Now let's switch gears and think about linear models where the variables are categorical rather than continuous. In our spreadsheet example, habitat is a categorical variable that has three levels (habitat 1, habitat 2, and habitat 3). Note that these habitats are not ordered in any way (e.g., habitat 1 is not better than habitat 2 or 3). We could have easily named these habitats A, B, and C, or Forest, Grassland, and Wetland. As a quick refresher, we identified each site's habitat type by coding them with two covariates (Cov 3 and Cov 4). If Cov 3 = 1 and Cov 4 = 0, the site was in habitat 1. If Cov 3 = 0 and Cov 4 = 1, the site was in habitat 2. If Cov 3 = 0 and Cov 4 = 0, the site was in habitat 3. These codings are given in columns J and K in the spreadsheet.

	G	H	I	J	K	L	M
13	Int	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
14	Occupancy Covariates						
15	Psi (Int)	Patch Size	Patch Size ^{2}	Habitat 1	Habitat 2	H1*PS	H2*PS
16	1	-1.9936	3.9743	1	0	-1.994	0.000
17	1	0.9660	0.9332	0	0	0.000	0.000
18	1	1.1794	1.3911	0	1	0.000	1.179
19	1	-1.8955	3.5929	1	0	-1.895	0.000
20	1	1.1377	1.2943	0	0	0.000	0.000

We model categorical data the same way we modeled the continuous patch size data. If we were to model logit psi as a function of habitat type, our linear model would look like this:

$$\text{Logit psi} = B_0 + B_3 * (\text{Habitat 1}) + B_4 * (\text{Habitat 2}).$$

Now, let's look at what happens when we plug in some values for B_0, B_3 and B_4. Suppose B_0 = 0.5, B_3 = 2, and B_4 = -2. From these betas, we can derive estimates of psi for each of the three habitat types by just plugging in the site-specific codings.

Habitat 1:

Logit psi = 0.5*(1) + 2*(1) + -2*(0) = 2.5.

Psi = exp(2.5)/(1+exp(2.5)) = 0.924.

Habitat 2:

Logit psi = 0.5*(1) + 2*(0) + -2*(1) = -1.5.

Psi = exp(-1.5)/(1+exp(-1.5)) = 0.182.

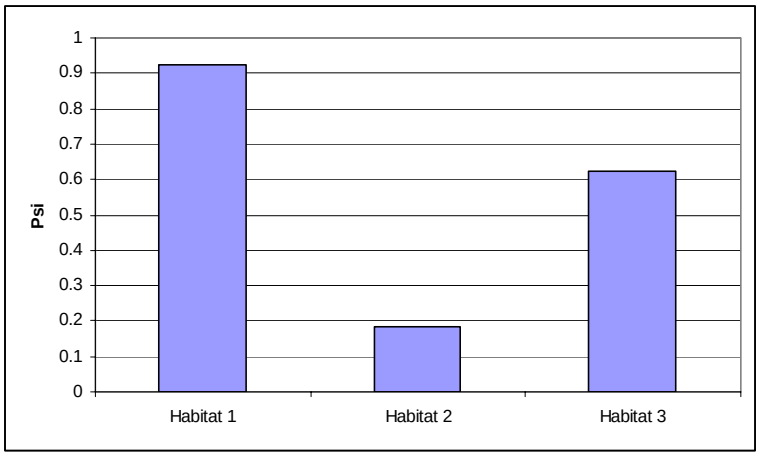
Habitat 3:

Logit psi = 0.5*(1) + 2*(0) + -2*(0) = 0.5.

Psi = exp(0.5)/(1+exp(0.5)) = 0.622.

Notice that the coding for habitat 3 (Cov 3 and Cov 4 = 0) renders just the intercept. So, the probability of occupancy in habitat 3 is determined by the value of the intercept of the linear model, which is true of any "reference" category.

We could graph these results as follows:



Of course, you would need to include the standard errors of these estimates before you can conclude whether habitat type truly influences occupancy probability. This material is probably familiar to you if you've taken a statistics course in Analysis of Variance.

ADDITIVE LINEAR MODELS

Now, what if psi was a function of both patch size and habitat? Well, we simply expand our linear model to include the additional effects:

$$\text{Logit psi} = B_0 + B_1 * \text{standardized patch size} + B_3 * \text{habitat1} + B_4 * \text{habitat2}.$$

And we can back-transform the logit to a probability with the logit link:

psi = exp(B_0 + B_1 * standardized patch size + B_3 * habitat1 + B_4 * habitat2)/(1+exp(B_0 + B_1 * standardized patch size + B_3 * habitat1 + B_4 * habitat2)).

This model would be called p(.)psi(patch size + habitat), and the focus of the analysis would be on estimating B_0, B_1, B_3 and B_4 to derive psi for each site, as well as the intercept for psi. This is called an additive model, because the effects are simply added together and each piece of additional information (patch size, habitat) simply builds on the other effects. A key assumption of additive models is that the covariates are not influencing each other in any way, i.e., if you know the Z score for patch size, it has no bearing on what the habitat is, other than a chance relationship.

Let's assume that the intercept (call it B_{00}) = 1, B_1 = 1, B_3 = 2, and B_4 = 0.1. (These betas are colored to help you track where they are located in the linear equation. Also, assume that B_2 = 0 and that B_5 and B_6 = 0 for the interaction model…we can ignore B_5 and B_6 for now). Now let's look at the first five sites and predict what psi is for each site:

	G	H	I	J	K
15	Psi (Int)	Patch Size	Patch Size ^{2}	Habitat 1	Habitat 2
16	1	-1.9936	3.9743	1	0
17	1	0.9660	0.9332	0	0
18	1	1.1794	1.3911	0	1
19	1	-1.8955	3.5929	1	0
20	1	1.1377	1.2943	0	0

Site 1 logit psi = 1*(1) + 1*(-1.9936) + 0*(3.9743)+ 2*(1) +0.1*(0) = 1.0064.

Site 1 psi = exp(1.0064)/(1+exp(1.0064)) = 0.7323.

Site 2 logit psi = 1*(1) + 1*(0.9660) + 0*(.9332)+ 2*(0) +0.1*(0) = 1.9660

Site 2 psi = exp(1.9660)/(1+exp(1.9660)) = 0.8772.

Site 3 logit psi = 1*(1) + 1*(1.1794) + 0*(1.3911)+ 2*(0) +0.1*(1) = 2.2794.

Site 3 psi = exp(2.2794)/(1+exp(2.2794)) = 0.9072.

Site 4 logit psi = 1*(1) + 1*(-1.8955) + 0*(3.5929)+ 2*(1) +0.1*(0) = 1.1045.

Site 4 psi = exp(1.1045)/(1+exp(1.1045)) = 0.7511.

Site 5 logit psi = 1*(1) + 1*(1.1377) + 0*(1.2943)+ 2*(0) +0.1*(0) = 1.1045.

Site 5 psi = exp(1.1045)/(1+exp(1.1045)) = 0.8949.

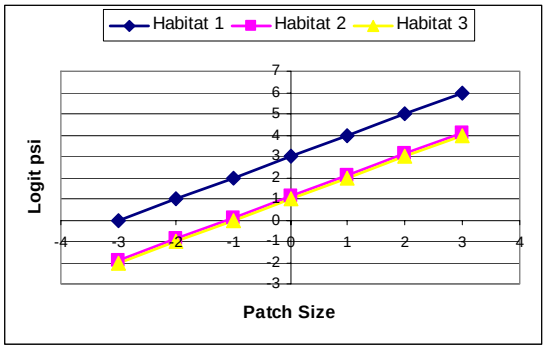
Thus, each site has a unique probability of occupancy, ψ, associated with it, depending on what the site's covariate values are. It's very instructive to study these results. Remember that we specified that the intercept $B_{00} = 1$, $B_1 = 1$, $B_2 = 0$, $B_3 = 2$, and $B_4 = 0.1$.

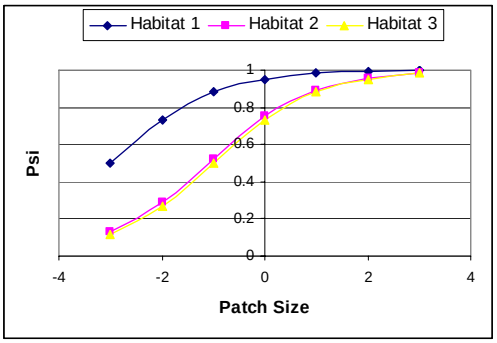
	G	H	I	J	K	L	M	N	O	P	Q
14	Occupancy Covariates							Detection		Occupancy	
15	Psi (Int)	Patch Size	Patch Size ^{2}	Habitat 1	Habitat 2	H1*PS	H2*PS	Logit p	p link	Logit psi	psi link
16	1	-1.9936	3.9743	1	0	-1.994	0.000	1.00000	0.73106	1.0064	0.7323
17	1	0.9660	0.9332	0	0	0.000	0.000	1.00000	0.73106	1.9660	0.8772
18	1	1.1794	1.3911	0	1	0.000	1.179	1.00000	0.73106	2.2794	0.9072
19	1	-1.8955	3.5929	1	0	-1.895	0.000	1.00000	0.73106	1.1045	0.7511
20	1	1.1377	1.2943	0	0	0.000	0.000	1.00000	0.73106	2.1377	0.8945

For site 1, we start with the intercept value of 1, which, when multiplied by B_{00} is 1. This value in turn corresponds to a baseline occupancy probability of exp(1.0)/(1+exp(1.0)) = 0.7311. Now we move to the next beta – patch size. The beta for patch size was positive (+1.0), indicating that as Z scores increased, the

probability of occupancy increased. Site 1 was much smaller than the average patch (Z score of -1.9936) which decreased the probability of occupancy below the baseline value: exp(1 + -1.9936)/(1+exp(1 + -1.9936)) = 0.2702. Now we move to the habitat betas. If site 1 was located in habitat 3, the final probability of occupancy would remain at 0.2702. However, the fact site 1 was located in habitat 1 effectively reversed the negative effect of patch size (the beta for habitat 1 was +2.0, which is a very strong effect: exp(1 + -1.9936 + 2.0)/(1+exp(1 + -1.9936 + 2.0)) = 0.7323. Essentially, by being in habitat 1, the intercept is bumped up by 2 logit notches, with the final probability of occupancy for site 1 = 0.7323. Site 2 had a higher probability of occupancy than site 1: it was located in habitat 3 (the reference habitat), but because it had a larger than average patch size (Z = 0.9960), the probability of occupancy increased. Site 3 had a very high probability of occupancy (0.9072) because it had a large patch size (Z = 1.1794), and the probability was also slightly increased because site 3 was located in habitat 2. It's worthwhile to take the time to understand exactly what the betas mean, and assess the magnitude of their effects.

We could depict our results graphically as follows, where the logit equations are graphed on the left, and the back-transformed probability of site occupancy is depicted on the right:





It's critical that you understand a fundamental point for additive models: the effect of the continuous variable (i.e., the slope of the effect) is the same for all categories (habitats); the effect of the categories (habitat) themselves simply shift the intercept up or down relative to the reference category (in this case, habitat 3). Thus, graphs of the logit equations show that the effect of patch size on occupancy is the same for all habitat types (occupancy probability increases as patch size increases), but the habitats themselves may have different baseline occupancy levels. If you've had a statistics course in linear modeling, you might recognize this model as an Analysis of Covariance (equal slopes model).

LINEAR MODELS WITH INTERACTIVE EFFECTS

The last type of linear model that we will examine is the model that includes interactions between patch size and habitat. This model is a bit of a mind-bender, but it's not too bad if you spend a bit of time on it. In our example, an interaction between patch size and habitat can be interpreted as follows: the effect of patch size on occupancy depends on what habitat you are considering. So, you need to talk about the effect of patch size for each habitat separately. The interaction between patch size and habitat requires two additional parameters be estimated: B_5 and B_6. Additionally, you need to "create" the 2 new pieces of interaction data for each site. First, multiply patch size by habitat 1. Second, multiply patch size by habitat 2. The results are shown in columns L and M. Thus, column L contains a non-zero patch size value for sites located in habitat 1, while column M contains a non-zero patch size value for sites located in habitat 2. (I don't know if standardization affects the interaction model outcomes or not.) The full linear model becomes:

Logit psi = B_0 + B_1 * (patch size) + B_2 * (patch size^2) + B_3 * (habitat1) + B_4 * (habitat2) + B_5 * (patch size*habitat1) + B_6 * (patch size*habitat2).

If B_5 is significantly different from 0, there is evidence that the effect of patch size for habitat 1 differs from the other two habitats; if B_6 is significantly different from 0, there is evidence that the effect (slope) of patch size for habitat 2 differs from the other two habitats. If B_5 and B_6 are both 0, then there is no evidence of a patch size by habitat interaction, and you're back to the additive model (patch size + habitat), in which the effect of patch size is the same for all habitat types.

MODELING LOGIT EQUATIONS IN THE SPREADSHEET

Now, let's see how all of this is applied within the spreadsheet environment. First, let's get oriented to the section of the spreadsheet where you specify the model. Cells F9:M9 list the parameters. There is 1 p parameter (the intercept, given in cell F11 and labeled B_0), and there are 7 ψ parameters (the intercept, labeled B_{00}, plus 6 potential covariates: patch size, patch size^2, habitat1, habitat2, PS*habitat1 interaction, and PS*habitat2 interaction).

	E	F	G	H	I	J	K	L	M
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
10	Estimate?	1	1	0	0	0	0	0	0
11	Beta			0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

In cells F10:M10, you enter a 1 under the corresponding parameter to indicate that you want to estimate that parameter for a given model, and a 0 to indicate that you won't be estimating that parameter for a given model. Cells F10 and G10 MUST both be equal to 1 because you must estimate the intercept. Notice that when a 0 is entered, the cell takes on a pink shade (the cells are conditionally formatted).

Underneath the "Estimate?" row is the Beta row. Solver will work on finding the values in these cells, so you should make sure they are blank before running Solver, with the exception that you must enter a 0 in those cells for any parameter that is not being estimated. (Thus, if a certain parameter is not being estimated, the "Estimate?" cell will be pink, and you must enter a 0 in its corresponding beta beneath it). This is necessary to count the number of parameters estimated correctly, and to ensure that logits for p and ψ are computed correctly. So, the model shown above is set up to estimate the intercept for p and the intercept for psi. In other words, the model depicted is p(.)psi(.). By forcing the betas for all covariate effects to be 0, the covariates do not enter the linear equation for estimating the logit of p or the logit of psi. In this example, Solver will find values for cells F11 and G11 that maximize the multinomial log likelihood.

Now let's look at how the spreadsheet computes logit p, p, logit ψ, and ψ for each site. First, clear out the beta cells (F11:M11).

	N	O	P	Q
14	Detection		Occupancy	
15	Logit p	p link	Logit ψ	ψ link
16	0.00000	0.50000	0.0000	0.5000

Cell N16 (logit p for site 1) has the equation =F16*$F$11, which computes the logit for p. It's basically taking the beta values for p, and multiplying them by the intercept value for site 1 (which is 1 for all sites). The logit link is computed in cell O16 with the equation =EXP(N16)/(1+EXP(N16)). In other words,
P = $Exp(B_0)/(1+exp(B_0))$. These two equations are copied down columns to generate the logit p and p for the remaining sites. Note that since this is a p(.) model, $p_1 = p_2$, and p is the same for all 200 sites.

Logit ψ for site 1 is computed in cell P16 with the equation =SUMPRODUCT($G$11:$M$11,G16:M16). In other words, Logit psi = B_{00}*(1) + B_1*(patch size) + B_2*(patch size2) + B_3*(Habitat1) + B_4*(Habitat2) + B_5*(patch size*habitat1) + B_6*(patch size *habitat2). The SUMPRODUCT function in Excel is really useful…it multiplies the betas by the corresponding site-specific covariates in one simple equation that can be copied down for all sites. The logit link is computed in cell Q16 with the equation =EXP(P16)/(1+EXP(P16)). In other words, Psi = Exp(logit ψ)/(1+exp(logit ψ)). These two equations are copied down columns to generate the logit psi and psi for the remaining sites.

Make sense? If it doesn't, spend a bit of time thinking about the linear equations and their back transformations to probabilities.

MISSING COVARIATE VALUES

OK, time for a quick reality check. In the general occupancy model, we showed you how you code for a site when a particular survey is missed. But what if you surveyed a site but forgot to record some covariate data? You've probably noticed that the spreadsheet dataset is complete, but in reality there will be times where certain data are not collected even when the site was surveyed. Let's suppose that you were unable to gather patch size statistics for one site. What should you do? Gary White provides some potential solutions in the MARK helpfiles: "Probably the best option is to code missing individual covariate values with the mean of the variable for the population measured. Replacing the missing value with the average means that the mean of the observed values will not change, although the variance

will be slightly smaller because all missing values will be exactly equal to the mean and hence not variable. If you have lots of missing values, another option is to code the sites into 2 groups, where all the missing values are in one group. Then, you can use both groups to estimate a common parameter, and only apply the individual covariate to one group. This approach can be tricky, so think through what you are doing before you try this approach." Alternatively, if relatively few sites have missing data, it might be best just to omit them from analysis.

MODEL P(.)PSI(.)

OK, now let's return to the input cells and set up your spreadsheet as follows:

	E	F	G	H	I	J	K	L	M
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
10	Estimate?	1	1	0	0	0	0	0	0
11	Beta	2.000000	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Again, this is model p(.)psi(.) because neither p or psi is constrained by any covariates. Now enter a 2 for the beta in cell F11 and enter a 1 in cell G11 as shown above, and study the logits and parameter values (p and ψ) for the first 5 sites.

	N	O	P	Q
15	Logit p	p link	Logit ψ	ψ link
16	2.00000	0.88080	1.0000	0.7311
17	2.00000	0.88080	1.0000	0.7311
18	2.00000	0.88080	1.0000	0.7311
19	2.00000	0.88080	1.0000	0.7311
20	2.00000	0.88080	1.0000	0.7311

You should notice that all 5 sites have the same logit p (2.000), p (0.8808), logit ψ (1.000), and ψ (0.7311) values. This is what you'd expect from a dot model. It says that all sites have the same p and psi values, irrespective of their site covariates.

If you change the values in cells F11 or J11, you'll see different p and psi estimates, but the new estimates will be the same for all sites. Try it!

Now, in the basic occupancy model we mentioned model over-parameterization. Recall that the saturated model for the basic model (with no covariates) estimates a probability of each history based on the raw data. In this particular example, there are 4 kinds of histories (11, 10, 01, and 00), and the sum of the history probabilities must be 1. Therefore, for this particular data set where no covariates are estimated, you can estimate 4 – 1 = 3 parameters at most, otherwise the model will be overparamaterized (there are 3 "free" parameters in the multinomial equation – the fourth is not free because you can derive it). What about the covariate model? Well, with the covariate model, each site has a unique probability of detection and occupancy (depending on the model), so the saturated model does not really apply because the analysis is done on a site by site basis. Does this mean you can run a model with 100 covariates? No! The general rule of thumb is that you need <u>at least</u> 10 observations (sites) per parameter estimated (more is much better, otherwise you are asking too much from a small number of sample points). In this spreadsheet example, we have 200 sites, so we could run a model with up to 20 parameters, though that wouldn't be advisable. Suppose that you conducted a study where only 50 sites were evaluated. This means that you can run models with $\leq$ 5 parameters. It sounds like a lot of sites, but remember that you must estimate the intercept for p and the intercept for ψ, so you have only three covariates to "play" with in any one model.

MODEL P(.)PSI(Patch Size)

OK, back to the spreadsheet. Now alter your spreadsheet as shown below so that there is covariate effect on psi (patch size).

	E	F	G	H	I	J	K	L	M
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
10	Estimate?	1	1	1	0	0	0	0	0
11	Beta				0.000000	0.000000	0.000000	0.000000	0.000000

This would be called model p(.)psi(patch size). Notice now that we need to enter a 1 in cells F10:H10 to indicate that now we are interested in estimating the beta for the p and ψ intercepts, plus the beta associated with Cov 1 (patch size), so K = 3. Also enter a beta value for these parameters: F11:G11 = 1.000, H11 = -2.00. The -2.000 beta value indicates that as standardized patch size increases, psi decreases.

	E	F	G	H	I	J	K	L	M
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
10	Estimate?	1	1	1	0	0	0	0	0
11	Beta	1.000000	1.000000	-2.000000	0.000000	0.000000	0.000000	0.000000	0.000000

These aren't the maximum likelihood estimates for this dataset (Solver would be used to find those beta values that maximize the log likelihood), but we wanted to show you an example to demonstrate a few quick points.

Now, with these parameter estimates, take a look at the first 5 sites:

	F	G	H	I	J	K	L	M	N	O	P	Q
14	Detection	Occupancy Covariates							Detection		Occupancy	
15	P (Int)	Psi (Int)	Patch Size	Patch Size^2	Habitat 1	Habitat 2	H1*PS	H2*PS	Logit p	p link	Logit psi	psi link
16	1	1	-1.9936	3.9743	1	0	-1.994	0.000	1.00000	0.73106	4.9871	0.9932
17	1	1	0.9660	0.9332	0	0	0.000	0.000	1.00000	0.73106	-0.9320	0.2825
18	1	1	1.1794	1.3911	0	1	0.000	1.179	1.00000	0.73106	-1.3589	0.2044
19	1	1	-1.8955	3.5929	1	0	-1.895	0.000	1.00000	0.73106	4.7910	0.9918
20	1	1	1.1377	1.2943	0	0	0.000	0.000	1.00000	0.73106	-1.2753	0.2183

Notice that all sites have the same p's (cells O16:O20) and different ψ's (cells Q16:Q20), because the sites have different Z values for patch size. If you change site 2's Z scores for patch size to match site 1's Z scores, you'd see that the logit

psi and ψ would be identical for both sites because the two sites have exactly the same covariate values.

PROBABILITY OF GETTING A PARTICULAR ENCOUNTER HISTORY

Keep the beta entries of $B_0 = 1$, $B_1 = 1$, and $B_2 = -2.0$. Now that we know what p and ψ are for each site, we can determine the probability of getting a particular history. Each potential history is written out in cells R15:U15.

	R	S	T	U
14	Probability of History			
15	11	10	01	00

Now, given a site's p and ψ, we can write an equation that reveals the probability of getting a 11 history, a 10 history, a 01 history, and a 00 history.

Probability 11 = ψ*p*p

Probability 10 = ψ*p*(1-p)

Probability 01 = ψ*(1-p)*p

Probability 00 = ψ*(1-p)*(1-p) + (1-ψ)

These equations should be a snap to you by now. Remember that our underlying model for this exercise is p(.)ψ(.), so we don't need to differentiate between p_1 and p_2. The trick is to apply these equations on a site by site basis, using each site's unique p and ψ estimate. The equations in cells R16:U16 do exactly this for site 1:

Probability of 11 = cell R16 =Q16*O16*O16 = ψ*p*p

Probability 10 = cell S16 =Q16*O16*(1-O16) = ψ*p*(1-p)

Probability 01 = cell T16 =Q16*(1-O16)*O16 = ψ*(1-p)*p

Probability 00 = cell U16 =Q16*(1-O16)*(1-O16)+(1-Q16) =ψ*(1-p)*(1-p)+(1-ψ)

Note that the sum of the probabilities must be 1 (sum R16:U16 = 1). These equations are copied down for the remaining sites; and hence the probabilities of getting a particular history depend on the site's unique p and ψ values.

	R	S	T	U	V	W
14	Probability of History					
15	11	10	01	00	Prob. Observed History	Ln L
16	0.530824	0.195279	0.195279	0.078618	0.078618	-2.54315
17	0.150990	0.055546	0.055546	0.737918	0.737918	-0.30392
18	0.109255	0.040193	0.040193	0.810360	0.810360	-0.21028
19	0.530045	0.194993	0.194993	0.079970	0.079970	-2.52611
20	0.116694	0.042929	0.042929	0.797448	0.042929	-3.14820

Column V uses a HLOOKUP function to find the site's actual history, and returns the probability of getting that particular history. The formula is =HLOOKUP(E16,$R$15:$U$515,B16+1,FALSE). Column W simply takes the natural log of the probability of observing the history that was observed.

So, the end result of the process is a probability of observing the history that was observed on a site by site basis. Let's walk through the process step by step. Site 1 had a history of 00. Given the beta estimates entered in the spreadsheet (B_0 = 1; B_{00} = 1, Beta Patch Size = -2 for occupancy), the probability of getting a 11 history is 0.530824, the probability of getting a 10 history is 0.195279, the probability of getting a 01 history is 0.195279, and the probability of getting a 00 is 0.078618. Cell V16 returns the actual probability associated with a 00 history, which is site 1's actual history. Thus, site 1 had a history of 00, and the probability of realizing this history given the model beta estimates is 0.078618. The natural log of 0.078618 = -2.54315 (cell W16).

	R	S	T	U	V	W
14	Probability of History					
15	11	10	01	00	Prob. Observed History	Ln L
16	0.530824	0.195279	0.195279	0.078618	0.078618	-2.54315

THE MULTINOMIAL LOG LIKELIHOOD FOR INDIVIDUAL COVARIATE MODELS

Now, if you recall from the previous worksheet, we need to compute the multinomial log likelihood for the entire data set, which is the frequency of history11*ln(probability of history11) + frequency of history10*ln(probability of history10) + frequency of history01*ln(probability of history01) + frequency of history00*ln(probability of history00). In this case, however, each individual has a frequency of 1. So all we need to do is take the natural log (ln) of each individual's history (column W), multiply each by 1 (the frequency), and add them all up (cell J5) to get the multinomial log likelihood, given the betas entered. Cell J5 reports the model's Log_eL, and has the equation =SUM(W16:W215).

MAXIMIZING THE LN LIKELIHOOD

Now, you can probably guess where we're headed. Remember, our model is currently set up to run p(.)psi(patch size). We want to maximize the multinomial log likelihood function in cell J5 by changing cells F11:H11, which are the betas associated with the two intercepts, plus the covariate for patch size. First, clear out cells F11:H11. Open Solver, and complete the information in the dialogue box. Set cell J5 to a maximum by changing cells F11:H11.

Press Solve. Solver will run through various combinations of betas until it finds a solution that maximizes cell J5. In this way, you are finding the maximum likelihood estimates of B_0 (the intercept for p), B_{00} (the intercept for ψ), and B_1 (the covariate effect of patch size on ψ). Once Solver has found a solution, press the option to keep the solution, and then study the output.

INTERPRETING THE MODEL OUTPUT

With the MLE's maximized, you can now inspect the model output. Here are the results we got:

	J	K	L	M	N
3	**Outputs**				
4	Log_eL	$-2Log_eL$	K	AIC	AICc
5	-216.839	433.679	3	439.679	439.801
6	Model DF	C hat	P (MLE)	ψ (MLE)	
7	197	2.201	0.713798986	0.506352058	

This model had a Log_eL = -216.839 (cell J5), a $-2Log_eL$ = 433.679 (cell K5). K is computed in cell L5 with the formula =SUM(F10:M10). Hopefully now you see why you have to enter a 1 for parameters that are being estimated for a model, and 0's for parameters that are not being estimated. AIC is computed as $-2Log_eL + 2K$ in cell M5, and AICc is computed in cell N5. We will revisit the AICc scores in a few minutes. The model DF is computed in cell J7 as the total sample size minus K. C-

hat is computed as -2Log$_e$L/model DF in cell K7. C-hat is normally computed as the deviance of a model divided by model degrees of freedom. However, for models with individual covariates, deviance is the same thing as -2Log$_e$L. Later in the exercise, we'll estimate c-hat in a different way when we conduct a goodness of fit test. Cell L7 provides the "real" estimate of p, and cell M7 provides the "real" estimate of ψ. MARK and PRESENCE report these, but what exactly are they? They are the logit transformed intercept values. Take a look at the equations:

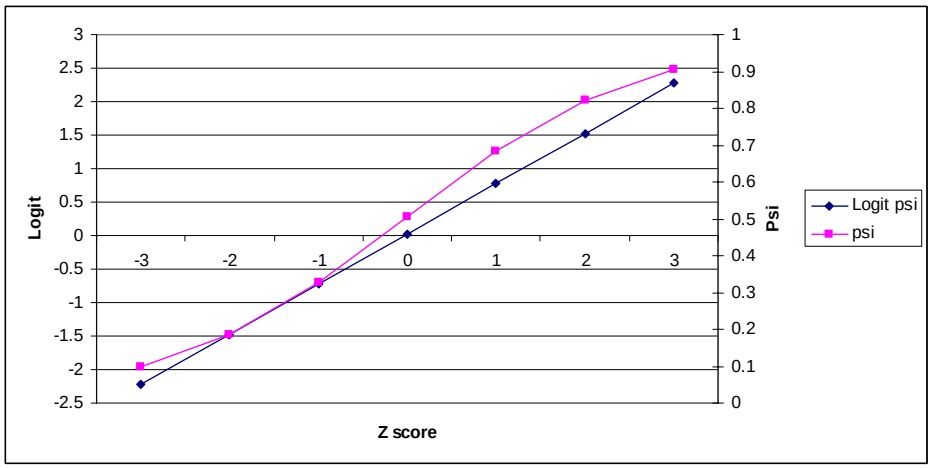
Cell L7 = EXP(F11)/(1+EXP(F11))
Cell M7 =EXP(G11)/(1+EXP(G11)).

In other words, the p and psi estimates apply to intercept values only, and how you interpret them depends on your covariates and coding system. For this particular model, we modeled p as a dot model. The MLE for p (cell L7) therefore corresponds to p for each and every site. For occupancy, we modeled ψ as a function of patch size. The MLE for ψ (cell M7) therefore corresponds to the sites with an average patch size (Z = 0). If we included habitat in the model, the intercept would correspond to sites with an average patch size (Z = 0) and habitat = 3 (the reference habitat, which is the habitat treatment that is coded by 00, in our case habitat 3). The interpretation of these "real" estimates can be tricky, especially because reporting them ignores the covariate effect, which was the purpose of the model to begin with!

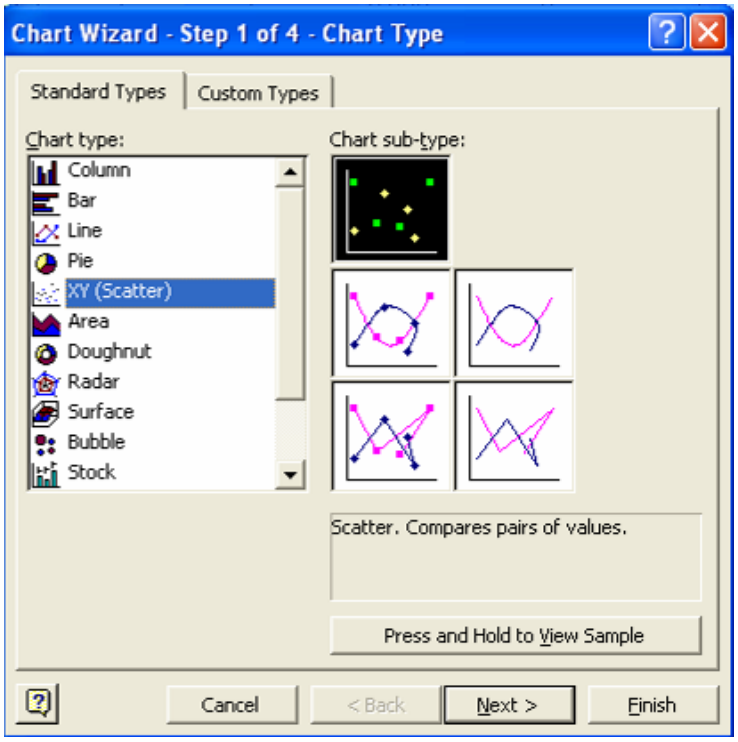
The betas are much more meaningful and should be the focus of interpretation. Here are the betas from this model:

	E	F	G	H	I	J	K	L	M
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
10	Estimate?	1	1	1	0	0	0	0	0
11	Beta	0.913907	0.025410	0.751644	0.000000	0.000000	0.000000	0.000000	0.000000

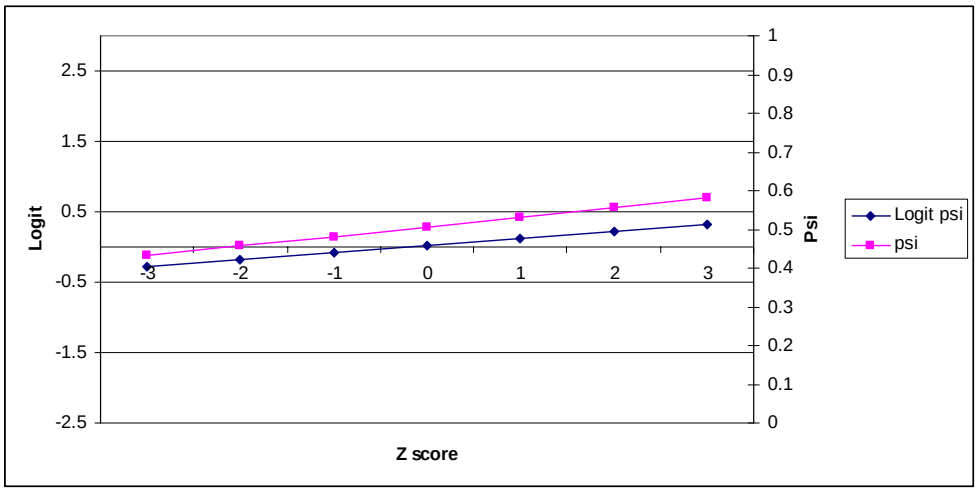
Now, how should we go about interpreting these results? The MLE for B_0 = 0.913907, which corresponds to a p of 0.71380. So our detection rates were on the high side. B_{00} = 0.025410, and B_1 = 0.751644, indicating that as Z patch size increases, occupancy probability increases. Exactly how big is this effect? We can interpret this in a few ways. First, we can make a graph showing the logits of psi (diamonds) and back-transformed psi estimates (squares) across a range of Z scores:



The graph is helpful because it shows you the magnitude of the effect of patch size on occupancy probability. This is a fairly dramatic effect: a site with a Z score of -3 has an occupancy probability of only 0.0971, whereas a site with a Z score of 2 has an occupancy probability of 0.8218. You can make a similar graph by selecting cells H16:H215, and also cells P16:Q215 and, then selecting the scatter graph option.



If B_1 = 0.1 instead of 0.751644, the effect would be much less dramatic.



With B_1 = 0.1, a site with a Z score -3 for patch size has a 0.4318 probability of detection, while a site with a Z score of +2 for patch size has a 0.5561 probability of detection…not nearly as dramatic.

Second, we can discuss this result in terms of odds, which is computed as probability/(1-probability). Given that B_{00} = 0.025410 and B_1 = 0.7516440, the

probability of occupancy at a site where Z = +2.0 is exp(0.025410 + 0.7516440*2)/(1+exp(0.025410 +0.7516440*2)) = 0.8218. The odds of a species being detected on at this site are then computed as:

0.8218/(1-0.8218) = 4.611672.

Therefore, the odds are 4.61:1. Remember, odds is interpreted as wins:losses. That is, if a site is truly occupied we would expect that for every 4.61 sites surveyed resulting in an occurrence, 1 site would result in non-occurrence.

TRACKING RESULTS

OK! Now you have found the MLE's for an occupancy model with site-level covariate effects, specifically model p(.)psi(patch size). How would this model compare to a model where different combinations of covariates are estimated? Well, we need to first record our results someplace on the spreadsheet, then we need to run other models and record their results, and then we can use model selection procedures to compare them. In this exercise, we'll run just five models – just enough to give you the hang of setting up models and running them, and we'll record the AICc value for each model to fill in the table in cells V3:AA10.

	V	W	X	Y	Z	AA
3	Model Selection Results Table					
4	Model	AICc	Rank	Delta	Exp(-0.5*Delta)	Weight
5	p(.)psi(patch size)		#N/A	0.000	1.0000	0.2000
6	p(.)psi(patch size + patch size^2)		#N/A	0.000	1.0000	0.2000
7	p(.)psi(habitat)		#N/A	0.000	1.0000	0.2000
8	p(.)psi(patch size + habitat)		#N/A	0.000	1.0000	0.2000
9	p(.)psi(habitat*patch size)		#N/A	0.000	1.0000	0.2000
10	Minimum AIC =	0.000		Sum =	5.0000	

Remember that you should have a clearly identified model set before you start your analyses, with well-defined rationale for running each model that is grounded

on the biology of the species (Burnham and Anderson 2002). Here are the models we'll run for this exercise:

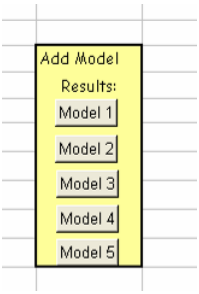
Model 1 (a continuous model): p(.)psi(patch size)

Model 2 (a polynomial model): p(.)psi(patch size + patch size^2)

Model 3 (a categorical model): p(.)psi(habitat)

Model 4 (an additive model): p(.)psi(patch size + habitat)

Model 5 (an interaction model): p(.)psi(patch size*habitat)



We've already run the first model, p(.)psi(patch size), so now just click on the button labeled Model 1 (around cell P5) to add the AICc value from this model to the results table. (If you've disabled the macros, you'll need to copy and paste the AIC result to the table manually).

	V	W	X	Y	Z	AA
3	Model Selection Results Table					
4	Model	AICc	Rank	Delta	Exp(-0.5*Delta)	Weight
5	p(.)psi(patch size)	439.801	1	0.000	1.0000	0.0000
6	p(.)psi(patch size + patch size^2)		#N/A	-439.801	##########	0.2500
7	p(.)psi(habitat)		#N/A	-439.801	##########	0.2500
8	p(.)psi(patch size + habitat)		#N/A	-439.801	##########	0.2500
9	p(.)psi(habitat*patch size)		#N/A	-439.801	##########	0.2500
10	Minimum AIC =	439.801		Sum =	##########	

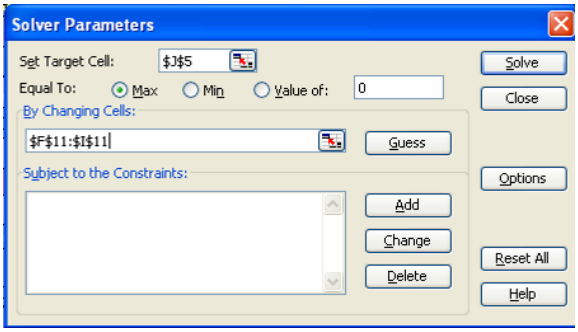
MODEL P(.)PSI(Patch Size + Patch Size^2)

Ok, the next model (Model 2) is the second order polynomial model, p(.)psi(patch size + patch size^2). How should we set up this model? Give it some thought, then plug away and run it (we'll give you the answers next, but try it on your own first).

To set up this model, you'd indicate that the model will estimate the intercept for p, and the intercept for psi, as well as Cov 1 (patch size), Cov 2 (patch size2). So K = 4 for this model. Remember that you MUST enter a 0 for any betas that are not being estimated in this model (cells J11:M11):

	E	F	G	H	I	J	K	L	M
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
10	Estimate?	1	1	1	1	0	0	0	0
11	Beta					0.000000	0.000000	0.000000	0.000000

Next, run Solver. Set cell J5 to a maximum, by changing the values in cells $F$11:$I$11:



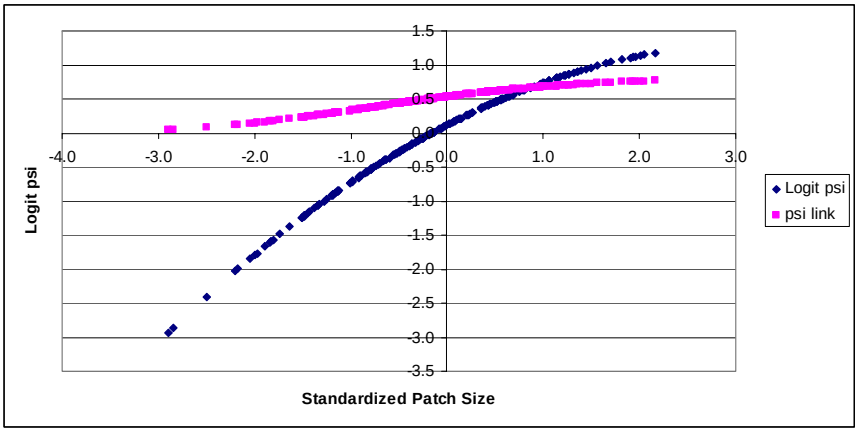
Press Solve, and keep your Solver results. Then press the button labeled Model 2 to add your results to the Results Table.

	V	W	X	Y	Z	AA
3	Model Selection Results Table					
4	Model	AICc	Rank	Delta	Exp(-0.5*Delta)	Weight
5	p(.)psi(patch size)	439.801	1	0.000	1.0000	0.0000
6	p(.)psi(patch size + patch size^2)	441.317	2	1.516	0.4686	0.0000
7	p(.)psi(habitat)		#N/A	-439.801	##########	0.3333
8	p(.)psi(patch size + habitat)		#N/A	-439.801	##########	0.3333
9	p(.)psi(habitat*patch size)		#N/A	-439.801	##########	0.3333
10	Minimum AIC =	439.801		Sum =	##########	

We'll study these results a bit later, but for now just note that the AICc score is higher for this model by ~2 units (strictly due to the addition of one parameter compared to the last model). The betas from this model indicate that the second order effect is small but negative (-0.111534).

	E	F	G	H	I	J	K	L	M
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
10	Estimate?	1	1	1	1	0	0	0	0
11	Beta	0.914812	0.115535	0.731197	-0.111534	0.000000	0.000000	0.000000	0.000000

A graph of the resulting psi's would look like this:



MODEL P(.)PSI(habitat)

OK, model 3 is the "categorical" model, where we estimate psi as a function of habitat. The set up for this model would be as shown:

	E	F	G	H	I	J	K	L	M
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
10	Estimate?	1	1	0	0	1	1	0	0
11	Beta			0.000000	0.000000			0.000000	0.000000

Run this model by setting cell J5 to a maximum, by changing cells F11:G11, J11:K11. Keep your Solver results, and then press the button labeled Model 3 to add your results to the Results Table.

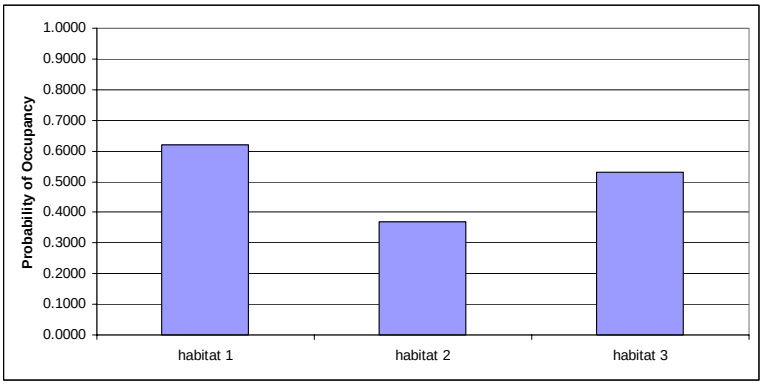
	V	W	X	Y	Z	AA
3	Model Selection Results Table					
4	Model	AICc	Rank	Delta	Exp(-0.5*Delta)	Weight
5	p(.)psi(patch size)	439.801	1	0.000	1.0000	0.0000
6	p(.)psi(patch size + patch size^2)	441.317	2	1.516	0.4686	0.0000
7	p(.)psi(habitat)	455.28361	3	15.482	0.0004	0.0000
8	p(.)psi(patch size + habitat)		#N/A	-439.801	##########	0.5000
9	p(.)psi(habitat*patch size)		#N/A	-439.801	##########	0.5000
10	Minimum AIC =	439.801		Sum =	##########	

This model seems to have a lot less support than the previous two models (as evidenced by its much higher AICc score). When we look at the parameter estimates, we can see that there are large differences among the three habitat types.

	E	F	G	H	I	J	K	L	M
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
10	Estimate?	1	1	0	0	0	0	0	0
11	Beta	0.916290	0.117419	0.000000	0.000000	0.368617	-0.659616	0.000000	0.000000

Remember that the intercept for this categorical model represents habitat 3, so psi for habitat 3 is exp(0.117419)/(1+exp(0.117419)) =0.5293. Habitat 1 has a beta value of 0.368617, so probability of occupancy is computed as exp(0.117419 + 0.368617)/(1+ exp(0.117419 + 0.368617)) = 0.6192. Habitat 2 has a covariate

estimate of -0.659616, so probability of occupancy is computed as exp(0.117419 - 0.659616)/(1+ exp(0.117419 - 0.659616)) = 0.3677.

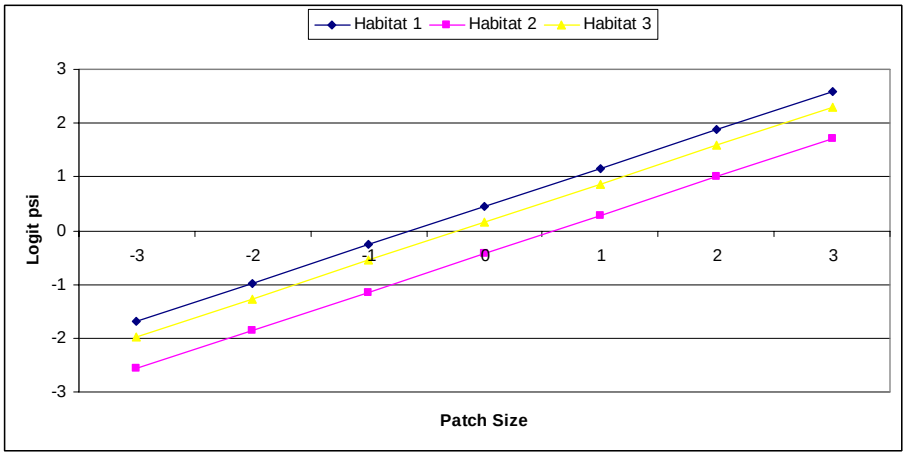


Model P(.)PSI(Patch Size + Habitat)

Model 4 is the additive linear model, p(.)psi(patch size + habitat). Set up this model and run it, and add your results to the results table:

	V	W	X	Y	Z	AA
3	Model Selection Results Table					
4	Model	AICc	Rank	Delta	Exp(-0.5*Delta)	Weight
5	p(.)psi(patch size)	439.801	2	0.607	0.7382	0.0000
6	p(.)psi(patch size + patch size^2)	441.317	3	2.123	0.3459	0.0000
7	p(.)psi(habitat)	455.28361	4	16.089	0.0003	0.0000
8	p(.)psi(patch size + habitat)	439.194	1	0.000	1.0000	0.0000
9	p(.)psi(habitat*patch size)		#N/A	-439.194	##########	1.0000
10	Minimum AIC =	439.194		Sum =	##########	

Take time now to study the betas, and see if you can interpret their meaning. Remember, in an additive model, the effect of patch size is the same across the three habitat types, meaning that the slope is the same regardless of habitat type. But, the habitats themselves may differ. A graph of our logit results looks like this:



As you can see, the habitat effect is not as strong as the patch size effect, but there are still differences… for any given patch size, habitat 1 has the highest occupancy rates, and habitat 2 has the lowest occupancy rates.

Model P(.)PSI(Patch Size * Habitat)

OK, one more model, and then we'll study the output. This is interaction model p(.)psi(patch size*habitat). Remember that to run this model, you must estimate the effects of patch size and habitat, as well as the interaction covariates. Go ahead and set it up and run it, and add your results to the Results Table.

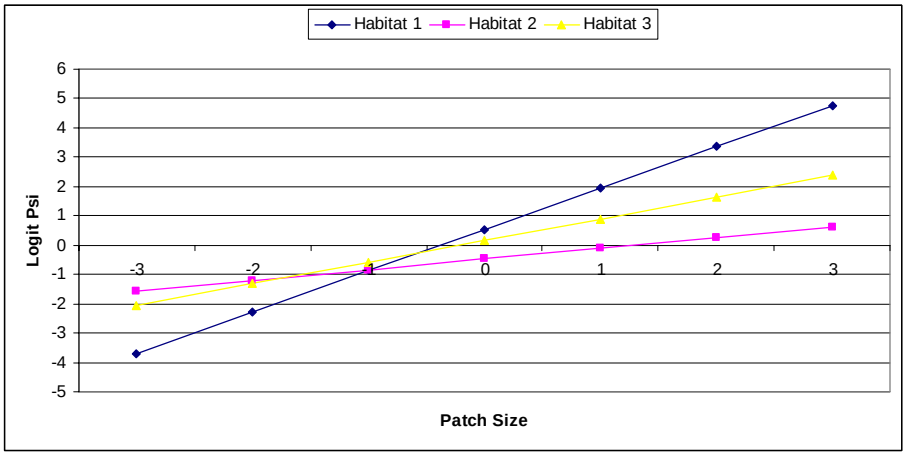
	E	F	G	H	I	J	K	L	M
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
10	Estimate?	1	1	1	0	1	1	1	1
11	Beta				0.000000				

You should get the following beta results:

	E	F	G	H	I	J	K	L	M
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
10	Estimate?	1	1	1	0	1	1	1	1
11	Beta	0.937816	0.159759	0.738235	0.000000	0.381478	-0.637870	0.671764	-0.372036

We've indicated that the interaction model can be a mind-bender, so let's walk through these results carefully. First, the ψ intercept is 0.159759, and as we've

indicated previously this is the baseline logit value for habitat 3. The patch size estimate is 0.738235, and in the interaction model, this is the slope of the effect of patch size for the reference habitat, or habitat 3. To get the intercepts for habitats 1 and 2, add the intercept to the corresponding parameter estimates for hab1 and hab2 as we did previously. Thus, the intercept for habitat 1 is 0.159759 + 0.381478 = 0.5412, the intercept for habitat 2 is 0.159759 + -0.637870 = -0.478111. To get the slopes for habitats 1 and 2, add the patch size parameter for the reference habitat to the estimates for PS*hab1 and PS*hab2. Thus, the slope for habitat 1 is 0.738235 + 0.671764 = 1.41, and the slope for habitat 2 is 0.738235 + -0.372036 = 0.3662. You can certainly graph these results to help you get a better idea of what's going on:



This graph shows what interactions are all about…..the effect of patch size (its slope) varies depending on what habitat you are talking about. In this particular example, in all three habitats logit psi increases with increasing patch size, but the effect is more dramatic for habitat 1 than habitats 2 and 3.

COMPARING MODELS

OK! We've been running model after model and have been studying the results, but how do you know which model best describes the field data you collected? First, let's take a look at the results for the 5 different models:

	V	W	X	Y	Z	AA
3	Model Selection Results Table					
4	Model	AICc	Rank	Delta	Exp(-0.5*Delta)	Weight
5	p(.)psi(patch size)	439.801	3	0.607	0.7382	0.2483
6	p(.)psi(patch size + patch size^2)	441.317	4	2.123	0.3459	0.1163
7	p(.)psi(habitat)	455.28361	5	16.089	0.0003	0.0001
8	p(.)psi(patch size + habitat)	439.194	1	0.000	1.0000	0.3363
9	p(.)psi(habitat*patch size)	439.430	2	0.235	0.8889	0.2990
10	Minimum AIC =	439.194		Sum =	2.9734	

Now let's compare the results from the different models. Our goal is to determine which model best "fits" our observed data so that we can infer something about detection probability and probability of site occupancy – the purpose of occupancy modeling. You probably know by now that in many cases you can fit models better by estimating more parameters. The model selection paradigm (presented by Ken Burnham and David Anderson in their book, Model Selection and Multimodel Inference) uses AIC as a measure of parsimony – and this consists of a measure of fit of the data ($-2\text{Log}_e L$) and the number of parameters (K): $AIC = -2\text{Log}_e L + 2K$. (AICc is a second order correction of AIC for small sample sizes). In the words of Cooch and White, "AIC is a good, well-justified criterion for selecting the most parsimonious model, i.e., the model which best explains the variation in the data while using the fewest parameters." For two models, the one with the lower AICc value is considered a more parsimonious model. As you add parameters to a certain model, the $-2\text{Log}_e L$ may get smaller (the model fit may be better), but the number of parameters is increased. As a result, as you add parameters to a model, its bias

is reduced but the variance in each parameter is increased, such that precision is lost. So there is a trade-off in how well the model fits the data and the number of parameters that need to be estimated from the data. Also, from a practical perspective, in many cases estimating the value of an additional parameter is a costly enterprise, so we want a model that explains the data well while at the same time keeps the number of parameters that need to be estimated at a minimum. The model selection paradigm provides a method for comparing model AICc scores as a means for weighing the evidence of competing models. Let's compare the models now:

	V	W	X	Y	Z	AA
3	Model Selection Results Table					
4	Model	AICc	Rank	Delta	Exp(-0.5*Delta)	Weight
5	p(.)psi(patch size)	439.801	3	0.607	0.7382	0.2483
6	p(.)psi(patch size + patch size^2)	441.317	4	2.123	0.3459	0.1163
7	p(.)psi(habitat)	455.28361	5	16.089	0.0003	0.0001
8	p(.)psi(patch size + habitat)	439.194	1	0.000	1.0000	0.3363
9	p(.)psi(habitat*patch size)	439.430	2	0.235	0.8889	0.2990
10	Minimum AIC =	439.194		Sum =	2.9734	

First, we find the model with the lowest AICc score – this is the most parsimonious model. In this case, it happens to be model 4, p(.)psi(patch size + habitat). The lowest AICc score of the four is calculated in cell W10 with a MIN function. Cells X5:X9 rank the models from best to worst with a RANK function. You can see that model 4 was ranked first, the interaction model (model 5) was ranked 2nd, and the habitat model p(.)psi(habitat) was ranked last. The AICc values are very close for models 4 and 5; model 5 must have had a lower -2Log$_e$L (it fit the data better than the additive model), but suffered an increased 2K penalty because it estimated 2 additional parameters than model 4.

The delta column (cells Y5:Y9) computes the difference in AICc scores between the best model (rank = 1) and the other models. So the best ranked model will always have Delta = 0. If the delta values are within 2 AICc units of the best ranking model, there is strong evidence of support for both that model and for the best ranked model. If the delta values are between 2 and 7, then there is considerable support for that model as well as the top ranked model. See Burnham and Anderson for a much more thorough and better-explained discussion of this topic. So, for this dataset, four of the five models appear to be supported by the data. While it's nice to have a single model "blow the other models away" to keep interpretation nice and clean, it's often the case in ecological studies where there is support for multiple models, as is the case here. However, you can see that model 3 has essentially no support, such that we don't need to consider it further. Its delta score (16.089) indicates that it is much less supported by the data than the top ranked model.

The weight of evidence (cells AA5:AA9) for each model is computed with two steps. First, we take the exponent of -1/2 times the delta value for each model (cells Z5:Z9). Why? Because a while ago we multiplied the log likelihood by -2 (Akaike did this for historical reasons), and to get back to the basic likelihood we need to take the exponent (which negates the log) and multiply by -1/2 (which negates the multiplication by -2). Then these scores are added (cell Z10). Then the Akaike weights are computed in cells AA5:AA9 as the model's EXP (-1/2 * delta) score divided by the sum (cell Z10). These weights are interpreted as probability of being the best K-L model in the model set. From these weights, you can see if one model has most of the support, or if several models explain the data equally well. So, for our example, model p(.)psi(patch size + habitat) has an AIC

weight of 0.3363, indicating that this model has a 33.63% chance of being the best K-L model in the model set. The next best-supported model is the full model p(.)psi(patch size * habitat), with an AIC weight of 0.2990, indicating that it has a 29.90% chance of being the best K-L model in the model set. In this example, you probably wouldn't bet your lunch on any one model as being the best model, so which model is the best model for drawing inferences isn't as clear-cut as one would hope.

MODEL AVERAGING AND MULTI-MODEL INFERENCE

So, as a quick summary, we used model selection procedures to compare the AICc scores among the five models we ran, and we found substantial support for four of them, and virtually no support for the fifth. Each model provided us with some parameter estimates for drawing inferences about detection probability and probability of occupancy for our 200 study sites. We know that none of the models gives the "correct" estimates, but which ones should we report? Is there a way to use the information from the model selection process and not disregard the information from any model? The answer is yes, and the process is called model averaging. Model averaging is covered in detail in other spreadsheet exercises, so we'll just mention the basic concept now. Basically, there are 200 sites, and each of the 5 models estimated a p and psi for <u>each</u> site. How could you get a model averaged estimate for each site? Simple! You run model 1, get the estimates for p and psi for each site, and then multiply those estimates by the model's AICc weight. Then you run model 2, get the estimates of p and psi for each site, and then multiply those estimates by model 2's AICc weight. You do the same thing for models 3, 4 and 5. The result is that you now have five weighted estimates of p and psi for each site. To get the model averaged estimate, just add the weighted

estimates up…it's that easy. However, estimating the standard errors of these estimates is a bit more involved; see Cooch and White for more details.

TIME FOR A BREAK….

Ahhh! Much better!

ASSESSING MODEL FIT

OK. Hopefully you have some idea of how to run an occupancy model with covariates, and how to compare among different models in a candidate set. We've run five models, and were able to rank them and obtain their model weights. However, we've missed a CRITICAL step – we haven't assessed goodness of fit yet. What if all 5 of the model's we've run don't really "fit" the data? Well, the model selection procedures will still rank them from best to worst. If none of the models fit the data, model selection procedures will simply rank a bunch of lousy models, from the least lousy to the lousiest.

Until recently, a method of assessing goodness of fit for occupancy models was lacking. Fortunately, Darryl MacKenzie and Larissa Bailey figured out a method for assessing fit that is fairly straight-forward. The citation is: MacKenzie, D., and L.

Bailey. 2004. Assessing fit of site occupancy models. Journal of Agricultural, Biological and Ecological Statistics 9:300-318.

The idea is to make sure at least ONE model in your candidate set "fits" the occupancy framework. Usually this model is the full or global model, or the model with the most parameters. What might make the data not fit the occupancy framework? Let's return to the model assumptions: 1) The system is closed to changes in the occupancy status of site during the sampling period. This means that the species cannot go extinct from a site, or cannot colonize a vacant site over the course of sampling. Individuals within the population can be born, die, or move, but these processes cannot influence the occupancy status of a site between sampling periods. 2) Species are not falsely detected. 3) Detection at a site is independent of detection at other sites. Larissa Bailey notes that lack of fit generally comes in two "flavors" of problems (1) violations of the assumptions listed above, or (2) an inappropriate model structure, in which the data contain covariates that weren't considered or were improperly modeled. What tends to happen when a model doesn't fit is that the precision of parameter estimates is underestimated (the standard errors look better than they actually are).

THE MacKENZIE BAILEY GOODNESS OF FIT TEST

So, how do you test fit? Well, first select the model (1, 2, 3, 4, or 5) in which you will assess the goodness of fit. In this exercise, we'll assess the fit of model p(.)psi(patch size), which is model 1, only because GOF testing can be a time-intensive experience in the spreadsheet environment, and this model converges on a solution more quickly than the models 4 or 5. Keep in mind, however, that what's typically done is to assess fit on the most parameterized model, in our case, the

full model p(.)psi(patch size*habitat). The rationale is that as the number of parameters increases, the -2Log$_e$L decreases, indicating a closer fit to the saturated model's -2Log$_e$L. Hence, the model with the most parameters is assumed a priori to have the lowest -2Log$_e$L. But, as we've said before, for logistical reasons, we will be evaluating the fit of model p(.)psi(patch size). So, start by re-running model 1 again. Here are the parameter estimates from this model:

	E	F	G	H	I	J	K	L	M
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
10	Estimate?	1	1	1	0	0	0	0	0
11	Beta	0.913907	0.025410	0.751644	0.000000	0.000000	0.000000	0.000000	0.000000

Now scroll over to columns R:U and examine the "Goodness of Fit" statistics (cells R3:U10).

	R	S	T	U
3	MacKenzie-Bailey Goodness of Fit			
4		Observed	Expected	$(O-E)^2/E$
5	11	50	50.0	0.000
6	10	18	20.0	0.207
7	01	22	20.0	0.193
8	00	110	110.0	0.000
9		200	200	Chi-Square
10				0.3995

The method is deceptively simple. In cells S5:S8, you count up the number of sites that had a 11 history, a 10 history, a 01 history, and a 00 history. This is done with a COUNTIF function. For example, cell S11 has the equation =COUNTIF($E$16:$E$215,R5), which counts up the number of "11" histories in cells E16:E215. The expected frequencies are simply the sum of the history probabilities across all sites. For instance, cell T5 computes the expected number of sites that should have a 11 history under model p(.)psi(patch size). The equation in that cell is =SUM(R16:R215). Similarly, cell T6 computes the expected number

of sites that should have a 10 history under model p(.)psi(patch size) with the equation =SUM(S16:S515), cell T7 computes the expected number of sites that should have a 01 history under model p(.)psi(patch size) with the equation =SUM(T16:T515), and so on.

Now, we use the good old Pearson's Chi-Square approach to assess fit, which is $(O-E)^2/E$ for each history, added together. For example, cell U5 has the equation =(S5-T5)^2/T5, which is the observed number of sites with a 11 history, minus the expected number of sites with a 11 history. The result is squared, and then divided by the expected value. This is done for each of the histories. Remember, each cell is chi-square distributed with 1 degree of freedom, so individual cell values greater than about 3.8 indicate a lack of fit. The sum is provided in cell U10 and is the model's Chi-Square value.

Wow! This model has a Chi-Square value of around 0.3995. Does it fit or not?

	R	S	T	U
3	MacKenzie-Bailey Goodness of Fit			
4		Observed	Expected	$(O-E)^2/E$
5	11	50	50.0	0.000
6	10	18	20.0	0.207
7	01	22	20.0	0.193
8	00	110	110.0	0.000
9		200	200	Chi-Square
10				0.3995

The model appears to be a good fit because none of the values in cells U5:U8 exceed 3.8 (and they should fit because we simulated data in a way that none of

the model assumptions were violated). And if this model fits (which is ranked third out of five models), the top two models in the model set should fit as well.

THE BOOTSTRAP

But let's suppose that the chi square value was a lot higher. How would you know whether to conclude the data fit or not? Well, currently the best way to test "fit" is to simulate a new dataset (called a bootstrap dataset) based on the parameter estimates (betas) from the model you are assessing (model 1: p(.)psi(patch size)). Here are the steps we'll be using to assess fit with a bootstrap, as specified by MacKenzie and Bailey:

1. Run model p(.)psi(patch size) and obtain beta estimates and the Pearson Chi-Square test statistic.
2. Use those beta estimates to simulate a bootstrap dataset (i.e., generate new encounter histories for each site – the covariates for each site are the same as the original data).
3. Once you have a simulated bootstrap dataset, then run the analysis again, and compute the Pearson Chi Square test statistic and store it. Because you simulated the data based on the results from an actual model, you KNOW the simulated data fit will fit the model.
4. Repeat the process about 1000 times.
5. Take all 1000 bootstrap Chi-Square results, order them, and find out where your model's Chi-Square (calculated from the real data) value falls within this bootstrap distribution. It sounds like a lot of work, but it's not really if you use a macro in the spreadsheet to run your trials for you. Plus, MARK

and PRESENCE both include methods for simulating bootstrap data, and they complete the analysis in a flash.

OK, let's try it. Start by running the model p(.)psi(patch size) again. Once again, here are the beta results from this model:

	E	F	G	H	I	J	K	L	M
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
10	Estimate?	1	1	1	0	0	0	0	0
11	Beta	0.913907	0.025410	0.751644	0.000000	0.000000	0.000000	0.000000	0.000000

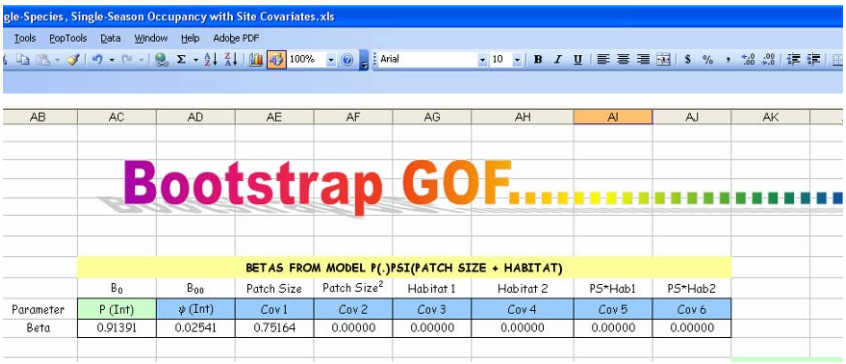
And here are the MacKenzie-Bailey Pearson Chi Square results:

	R	S	T	U
3	MacKenzie-Bailey Goodness of Fit			
4		Observed	Expected	$(O-E)^2/E$
5	11	50	50.0	0.000
6	10	18	20.0	0.207
7	01	22	20.0	0.193
8	00	110	110.0	0.000
9		200	200	Chi-Square
10				0.3995

Upon inspection, this model looks to be a good fit…..and it should be because the original data were simulated and did not violate any of the occupancy model assumptions. But we can almost guarantee that "real" data will be quite a bit messier, so let's continue. Copy the Chi-Square value from this model (cell U10) and paste its value into cell AN8. (Go to Edit | Paste Special | Paste Values….don't paste the formula or you'll screw things up). This cell is labeled Chi-Square$_R$ (where the R is for real data, as opposed to bootstrap data).

	AM	AN
8	Chi-Square$_R$	0.399469401
9	Percentile	#N/A
10	Chi-Square$_B$	#DIV/0!
11	c hat	#DIV/0!

Now, select cells F11:M11 (the maximized beta values from model p(.)psi(patch size) and paste the values into cells AC11:AJ11 as shown below. This portion of the spreadsheet is labeled BOOTSTRAP GOF, and it is here where you'll be simulating data based on the beta estimates from model p(.)psi(patch size).



So far, so good. Now, let's study columns AC:AD. Given the betas from model p(.)psi(patch size), the predicted p for each site is given in column AC, and the predicted ψ for each site is given in column AD. Remember, these values are estimated based on the maximized Log$_e$L for model p(.)psi(patch size). The first five sites are shown, and they are linked the original p and ψ estimates provided in columns O and Q.

	AB	AC	AD	AE	AF	AG	AH	AI	AJ
8		BETAS FROM MODEL P(.)PSI(PATCH SIZE + HABITAT)							
9		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
10	Parameter	P (Int)	ψ (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
11	Beta	0.91391	0.02541	0.75164	0.00000	0.00000	0.00000	0.00000	0.00000
12									
13									
14		PREDICTED VALUES		RANDOM NUMBERS					
15	Site	Predicted p	Predicted ψ	p1	p2	ψ	Bootstrap History		
16	1	0.71380	0.18648	0.07352	0.23457	0.79512	00		
17	2	0.71380	0.67951	0.46121	0.86373	0.93078	00		
18	3	0.71380	0.71339	0.13123	0.32454	0.48481	11		
19	4	0.71380	0.19792	0.77144	0.47305	0.98876	00		
20	5	0.71380	0.70693	0.88030	0.33814	0.41534	01		

Now we come to part where we simulate new histories for each site (bootstrap histories), based on p and ψ values for each site from model p(.)psi(patch size), and based on random numbers associated with p_1, p_2, and ψ. Cells AE16:AG515 are simply random numbers between 0 and 1 (each of these cells has the equation =RAND(), which is Excel's random number function). Remember that there are two surveys per site, so we need a random p for each survey.

A simulated, bootstrap history is generated for <u>each</u> site based on the site's predicted p and ψ values, and on the random numbers provided for that site. Let's walk through a specific example for site 1: Cell AH16 has the equation = =IF(AND(AG16<AD16,AE16<AC16),1,0)&IF(AND(AG16<AD16,AF16<AC16),1,0), and consists of two IF functions that are joined together. The first IF function (shown in red here) results in a 0 or 1 for the first survey, and the second IF function (shown in blue here) results in a 0 or 1 for the second survey. For survey 1, if cell AG16 (the random ψ) is less than cell AD16 (site 1's ψ estimate), AND if cell AE16 (the random p_1) is less than cell AC16 (site 1's p estimate), then the species was present and detected in the first survey and a 1 is recorded; otherwise the species was not detected. For survey 2, if cell AG16 (the random ψ) is less

than cell AD16 (site 1's ψ estimate), AND if cell AF16 (the random p_2) is less than cell AC16 (site 1's p estimate), then the species was present and detected in the first survey and a 1 is recorded; otherwise the species was either not present, and thus not detected, or was simply not detected even though the site was occupied.

This formula is copied down for all 200 sites, and provides us with a "bootstrapped histories" for each site. What's the big deal? Well, because we used random numbers to simulate histories, we know that we have not violated any of the occupancy model assumptions, namely: 1) The system is demographically closed to changes in the occupancy status of site during the sampling period – we used the same ψ value for both surveys so there was no change in occupancy pattern. 2) Species are not falsely detected – we either detected the species or not based on the random number for each survey, and there were no mistakes in data recording that would change a 0 → 1 or 1 → 0. 3) Detection at a site is independent of detection at other sites – each site has its own random numbers and in no way does a random number for one site influence the random number associated with another site. We also know that there are no structural problems with the data because we simulated data based on a model with specified covariates, and we analyzed the data with the exact same model structure. In short, the simulated data are known to fit.

Notice when you press F9, the calculate key, Excel generates new random numbers, and hence new bootstrapped histories. Let's take a look at the first three sites shown below and determine how they ended up with 00, 11, and 10 histories, respectively.

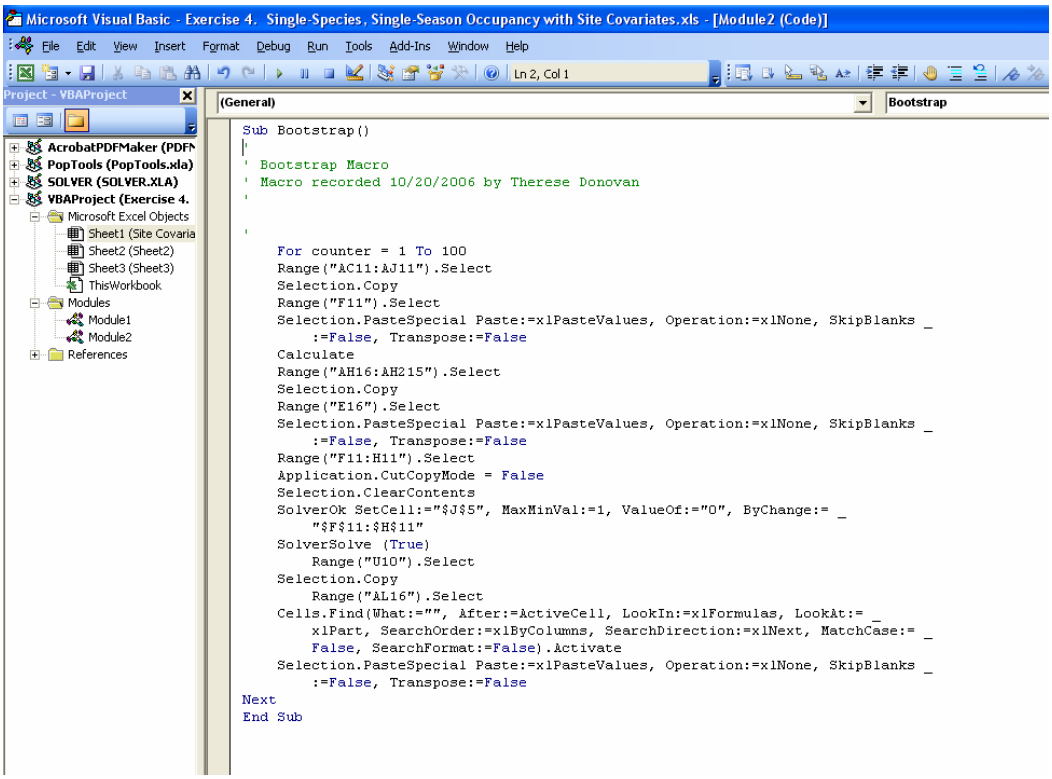
	AB	AC	AD	AE	AF	AG	AH
14		PREDICTED VALUES		RANDOM NUMBERS			
15	Site	Predicted p	Predicted ψ	p1	p2	ψ	Bootstrap History
16	1	0.71380	0.18648	0.67384	0.51682	0.84414	00
17	2	0.71380	0.67951	0.38257	0.48904	0.14045	11
18	3	0.71380	0.71339	0.61317	0.96827	0.69246	10

Site 1's random ψ was 0.84414, and its real ψ (according to model p(.)psi(patch size)) was 0.18648. Because the random ψ was greater than the real ψ, the site was considered to be unoccupied. So, regardless of the detection parameters, this site's history is 00. How about site 2? Site 2's random ψ was 0.14045, which is less than site 2's predicted ψ (0.67951). So site 2 is occupied, but was the species detected? The random number for p_1 was 0.38257, and the real p is 0.71380 for site 1 (according to model p(.)psi(patch size)). Because the random p_1 was less than the real p, the species was detected and a 1 was returned for the first visit for site 1. The random p_2 for site 1 was 0.48904, and this number is less than the real p (0.71380), so the species was detected in the second survey, resulting in a 11 history for site 2. How about site 3? Site 3's random ψ was 0.69246, which is less than site 3's predicted ψ (0.71339). So site 3 is occupied, but was the species detected? The random number for p_1 was 0.61317, and the real p is 0.71380 for site 1 (according to model p(.)psi(patch size)). Because the random p_1 was less than the real p, the species was detected and a 1 was returned for the first visit for site 1. The random p_2 for site 1 was 0.96827, and this number is greater than the real p (0.71380), so the species was not detected in the second survey, resulting in a 10 history for site 3.

	AB	AC	AD	AE	AF	AG	AH
14		PREDICTED VALUES		RANDOM NUMBERS			
15	Site	Predicted p	Predicted ψ	p1	p2	ψ	Bootstrap History
16	1	0.71380	0.18648	0.67384	0.51682	0.84414	00
17	2	0.71380	0.67951	0.38257	0.48904	0.14045	11
18	3	0.71380	0.71339	0.61317	0.96827	0.69246	10

Now, when you press F9, you simulate an entirely new bootstrapped encounter history for each site. Press it 23 times, and you've generated 23 simulated datasets which are all known to fit the data. Press it 345 times, and you've generated 345 simulated datasets which are all known to fit the data.

The next step is to copy cells AH16:AH215 into cells E16:E215 (replacing the original encounter histories), and run Solver on the bootstrap data, and record the new MacKenzie-Bailey Chi-Square statistic. Then press F9 again to simulate a new bootstrap encounter history, and repeat the process. We've written a macro to automate these steps for you. Here is the VBA code:



Just press the little smiley face and you'll run this code 100 times, representing 100 bootstrap trials. You can run more or fewer trials if you'd like, just go to Macros | Boostrap | Edit, and change the code

For counter = 1 to 100 to **For counter = 1 to X** (and replace the X with whatever number you'd like). Then, close out of the visual basic editor. Ideally, you'd run 1,000 or even 10,000 trials, but that's beyond our purpose here.

Important note: If you can't get the bootstrap to run, here's what you should try first: Go to Tools | Macro | Macros | Bootstrap | Edit. Then, with the Visual Basic module active, click References on the Tools menu,
and then select the Solver.xla check box under Available References. If Solver.xla doesn't appear under Available References, click Browse and open Solver.xla in the \Office\Library\Solver\ subfolder.
****

Go ahead and press the smiley face – this might be a good time to go grab some lunch – the spreadsheet is very slow compared to MARK or PRESENCE. After the 100 trails are completed, take a good look the bootstrap results in columns AL:AN.

	AK	AL	AM	AN
13	Bootstrap Results			
14				
15				
16	Trial	Chi-Square	Ordered Chi-Sq	Percentile
17	1	0.214801547	1.11883E-08	0
18	2	5.962476377	0.001172144	0.01
19	3	3.668143228	0.00226856	0.02
20	4	2.185105527	0.004905685	0.03
21	5	0.896189276	0.014029521	0.04
22	6	1.001054034	0.022549021	0.05
23	7	0.112190275	0.025494112	0.06
24	8	0.787374889	0.02733818	0.07
25	9	1.477754987	0.027869608	0.08
26	10	0.724009531	0.030668584	0.09

Our first 10 bootstrap Chi-Square results are provided in cells AL17:AL26…your results will be different because you'll undoubtedly simulate different histories than we did. Given these results, the 100 Chi-Square values are ordered according with the PERCENTILE function in column AM. For instance, a Chi-Square result of 0.001172144 is in the 0.01 percentile, indicating that 1% of the bootstrap trials had a lower Chi-Square result, and 99% of the trials had a larger Chi-Square result. A Chi-Square value of 0.02786 is in the 0.08 percentile, indicating that 8% of the trials had a lower Chi-Square result, and 92% of the trials had a larger Chi-Square result. The goal now is to find where the original model p(.)psi(patch size)'s Chi-Square values falls within this bootstrap distribution:

	AM	AN
8	Chi-Square$_R$	0.399469401
9	Percentile	0.49
10	Chi-Square$_B$	1.343354815
11	c hat	0.297367008

In cell AN8, you entered the chi-square value from the original data for model p(.)psi(patch size). Cell AN9 finds where this value "falls" in the bootstrap distribution. The formula is =VLOOKUP(AN8,AM17:AN117,2); it looks up the realized chi-square value, and reports the percentile. Our realized chi-square observed fell within in the 49th percentile of the bootstrapped chi-square values. Because we know the bootstrapped chi-squares represent values where the data are known to fit, we can interpret this as "given the data fit the model, a chi-square value of 0.39947 is not all that unlikely. There is no evidence that the observed data do not fit." Typically, if the observed chi-square value is in the 95th percentile or higher, you'd conclude there is evidence for lack of fit.

The average of the Chi-Square test statistics is computed in cell AN10 with the equation =AVERAGE(AL17:AL116). And, finally, c hat is computed as the observed Chi-Square / Average bootstrap Chi-Square in cell AN11.

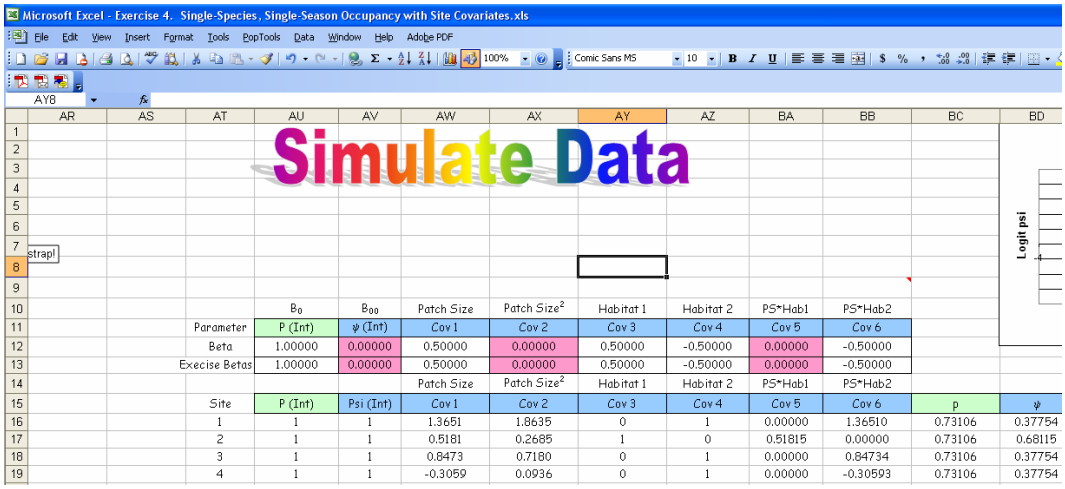
Now what? Well, c-hat is the statistic you use for adjusting the standard errors for the model parameters. Hopefully, c hat is 1 or less than 1, indicating no adjustment is needed. If c hat is larger than 1, (e.g., 2), it indicates a lack of fit and you should adjust your standard errors by multiplying the estimated standard errors by the square root of c hat. We haven't added standard errors to the spreadsheet, but MARK and PRESENCE allow you to make these adjustments easily.

For our dataset, model p(.)psi(patch size) appears to fit the data, and c hat is well below 1, indicating no need to inflate our standard error estimates. Thus, we can now safely draw inferences from the results of our models because we now know

that at least one model in the model set fits the observed field data, and a couple of models rank higher than the model in which we assessed fit.

SIMULATING COVARIATE DATA

OK! We're finally closing in the end of the site-level covariate exercise. We've covered a LOT of ground, but we still need to learn how to simulate data for analysis. Scroll to the right of the sheet, and you'll find a section labeled Simulate Data:



		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2		
	Parameter	P (Int)	ψ (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6		
	Beta	1.00000	0.00000	0.50000	0.00000	0.50000	-0.50000	0.00000	-0.50000		
	Execise Betas	1.00000	0.00000	0.50000	0.00000	0.50000	-0.50000	0.00000	-0.50000		
				Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2		
	Site	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6	p	ψ
	1	1	1	1.3651	1.8635	0	1	0.00000	1.36510	0.73106	0.37754
	2	1	1	0.5181	0.2685	1	0	0.51815	0.00000	0.73106	0.68115
	3	1	1	0.8473	0.7180	0	1	0.00000	0.84734	0.73106	0.37754
	4	1	1	-0.3059	0.0936	0	1	0.00000	-0.30593	0.73106	0.37754

The parameters for this model are listed in cells AU10:BB10. All you need to do is enter some beta values in cells AU12:BB12, and then press F9 to simulate new data. The beta values shown below are the actual values used to simulate the data we've been analyzing. Thus, we simulated data where there is a patch size and a habitat effect on ψ, and there is an interaction between patch size and habitat for habitat 2. (No wonder this model was the top ranked model!). If you change the values in row 12 for your own purposes, that's fine…we pasted in the beta values used in this exercise in row 13 for future reference.

	AT	AU	AV	AW	AX	AY	AZ	BA	BB
10		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
11	Parameter	P (Int)	ψ (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
12	Beta	1.00000	0.00000	0.50000	0.00000	0.50000	-0.50000	0.00000	-0.50000
13	xecise Betas	1.00000	0.00000	0.50000	0.00000	0.50000	-0.50000	0.00000	-0.50000

Now, the next step is to assign some covariate values to each site:

	AT	AU	AV	AW	AX	AY	AZ	BA	BB
14				Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
15	Site	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
16	1	1	1	1.3651	1.8635	0	1	0.00000	1.36510
17	2	1	1	0.5181	0.2685	1	0	0.51815	0.00000
18	3	1	1	0.8473	0.7180	0	1	0.00000	0.84734
19	4	1	1	-0.3059	0.0936	0	1	0.00000	-0.30593
20	5	1	1	-0.4005	0.1604	0	0	0.00000	0.00000

The sites are listed in column AT, and in AU we assign a 1 for the intercept of p and in AV we assign a 1 for the intercept of ψ. Cov 1 (patch size) is a continuous variable, so we need to enter a Z score for each site. The formulae in cell AW16 generates Z scores for site 1 with the equation =NORMSINV(RAND()), which generates a random Z score from a distribution whose mean is 0 and whose standard deviation is 1. Cov 2 (patch size2) is computed in cell AX16 as =AW16^2. Cov 3 and 4 are categorical (habitat 1 and habitat 2), and together determine each site's habitat. In cell AY16, we entered the equation =IF(RAND()<0.33,1,0) to generate values for Cov 5. This function, draws a random number, and if the random number is less than 0.33, the site is coded for habitat 1 (Cov 3 = 1); otherwise the site is coded 0. In cell AZ16, the formula determines the value for Cov4 as =IF(AY16=1,0,IF(RAND()<0.5,1,0)). If the site is coded 1 for habitat 1 (AY16 = 1), then Cov 4 must be 0. Otherwise, if a random number is less 0.5, Excel returns a 1 and the site is located in habitat 2; otherwise Excel returns a 0 and the site is located in habitat 3. In this way, we roughly assign habitats to the 200 sites in roughly equal numbers. The last step is to create data for the interaction terms. In column BA, we multiply the site's patch size by habitat 1, and in column BB we multiply the site's patch size by habitat 2.

An important thing to notice here is that the assignments for the various covariates values for site 1 are completely independent of each other. That is, the Z score for Cov 1 does not depend on any of the other covariate values for site 1 (We haven't simulated data where one covariate value depends on the value of another covariate. That is, we haven't simulated data that are dependent in any way).

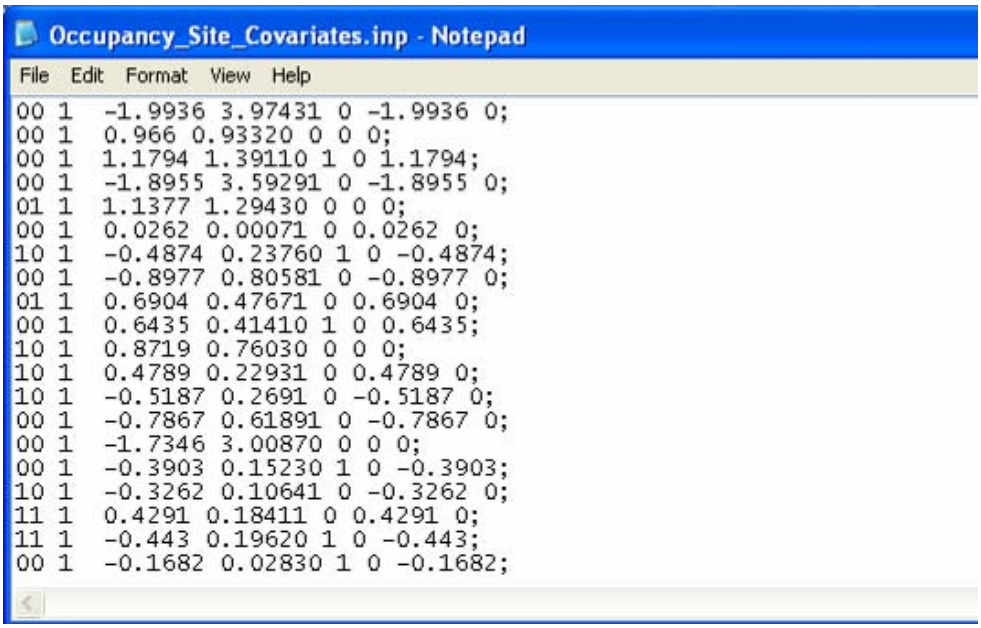
Now that each site has covariate values for covariates one through six, and beta values specified in cells AU12:BB12, we can compute each site's ψ and p parameters as we've done previously, and we can simulate data in the same way we simulated data for the bootstrap. For site 1, p is computed in cell BC16 with the equation =EXP($AU$12)/(1+EXP($AU$12)), which is the back-transformed logit equation. Site 1's ψ is computed in cell BD16 with the equation =EXP(SUMPRODUCT($AV$12:$BB$12,AV16:BB16))/(1+EXP(SUMPRODUCT($AV$12:$BB$12,AV16:BB16))). Remember, the results from these equations are completely dependent on the site's covariate values and the beta values you enter. Columns BE:BG simply create histories based on the site's p and ψ values (as determined by the betas entered and covariate values), and use random numbers to create the history.

	BE	BF	BG	BH	BI	BJ
14	Random Numbers					
15	p1	p2	ψ	Survey 1	Survey 2	History
16	0.47836265	0.55366131	0.63279057	0	0	00
17	0.47126102	0.60457341	0.35881049	1	1	11
18	0.52119428	0.17612179	0.48430619	0	0	00
19	0.06685597	0.909578	0.28887726	1	0	10
20	0.08385558	0.9480222	0.92669552	0	0	00

Take some time to look at the spreadsheet equations before moving on.

CREATING INPUT FILES FOR MARK AND PRESENCE

The final step is to compare your spreadsheet result with results generated in MARK and PRESENCE. The very first thing to do, however, is to replace the bootstrap histories with the original, raw data. The original data are provided in cells A16:A215. Copy these cells and paste them into cells E16:E215. The PRESENCE input file can be created within PRESENCE itself, so you don't need to do anything in that regard. To develop a MARK input file, select cells Y16:Y215, and copy them into Notepad.



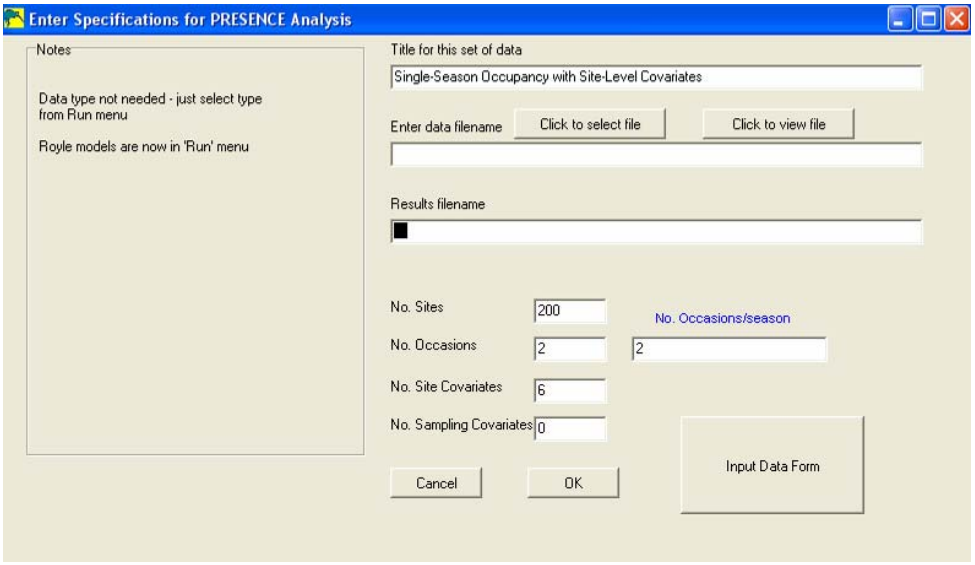
Save this file with an INP extension (e.g., "Occupancy_Site_Covariates.inp"), and we'll import this file into MARK later on.

SINGLE-SPECIES, SINGLE-SEASON OCCUPANCY WITH SITE COVARIATES IN PROGRAM PRESENCE

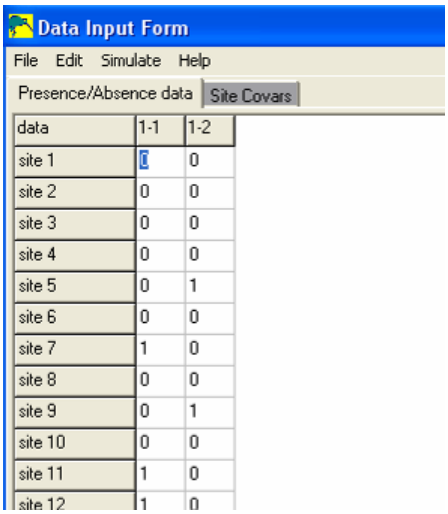
INPUT DATA

In this exercise, we will analyze the data in the spreadsheet Site Covariates in program PRESENCE. Recall that there are no detection covariates, and that there are several site-level covariates (patch size, patch size^2, two habitat covariates that code for three habitat types, as well as the two covariates for the patch size by habitat interactions), making a total of 6 covariates. Remember that this worksheet has only two sampling sessions. Before you begin, make sure that the data you are working with are the original data. That is, make sure the histories in column E match those in column A. If they don't match, copy the histories in column A into column E, using the Paste Special | Paste Values option.

Open PRESENCE and go to File | New. In the Enter Specifications Form, we'll create the PRESENCE input file. Enter a title for the analysis (e.g., Single-Season Occupancy with Site Covariates). Enter 200 for the number of sites, and 2 for the number of occasions. Again, in this particular dataset, all of the covariates (n = 6) are considered site level covariates.



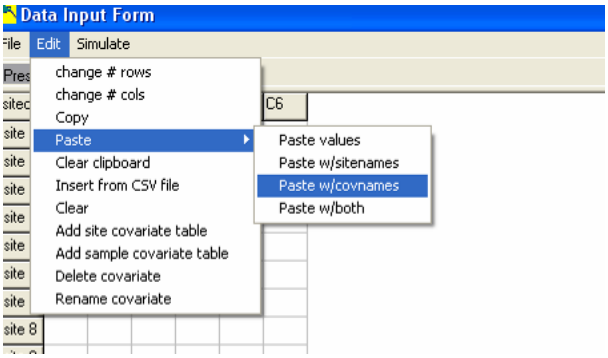
When you are finished, click on the Input Data Form. PRESENCE then presents you with the Data Input Form, where you can paste in your data. Notice that there are two tabs on the form: Presence/Absence Data and Site Covars. First, we'll paste in the encounter histories in the tab labeled Presence/Absence Data. On your spreadsheet, select C16:D215 and go to Edit | Copy. Then go to the Data Input Form, select the first box and select Edit | Paste, and select the Paste Values option.



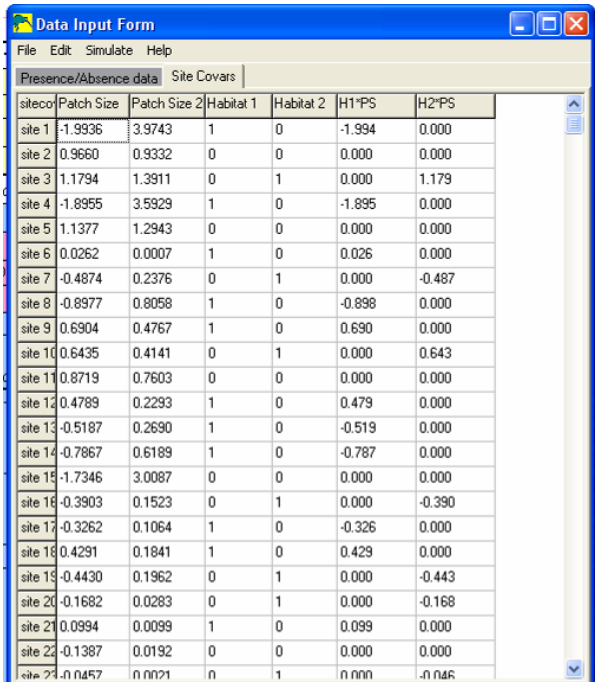
data	1-1	1-2
site 1	1	0
site 2	0	0
site 3	0	0
site 4	0	0
site 5	0	1
site 6	0	0
site 7	1	0
site 8	0	0
site 9	0	1
site 10	0	0
site 11	1	0
site 12	1	0

Note: a fast way to select cells C16:D215 is to select cells C16:D16, and then press the Control + Shift + down arrow key….then press Control+C to copy the selected cells and paste them in to PRESENCE.

Now, click on the tab labeled Site Covars. We'll first paste in the covariates associated with detection. On the spreadsheet, select cells H15:M215, and copy them. (Yes, copy the column heading). Then select the Data Input Form again, and select the first box of data on the left-hand side, and go to Edit | Paste | Paste w/covnames.

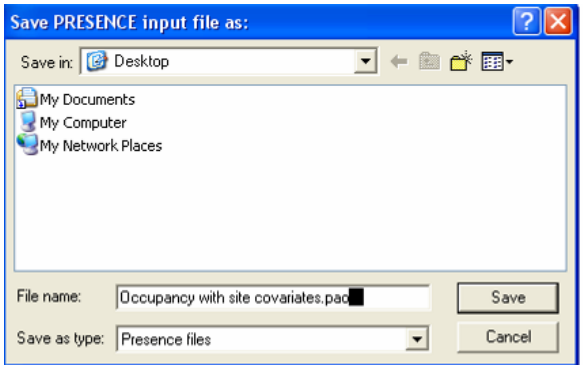


Your Data Input Form should look like this:



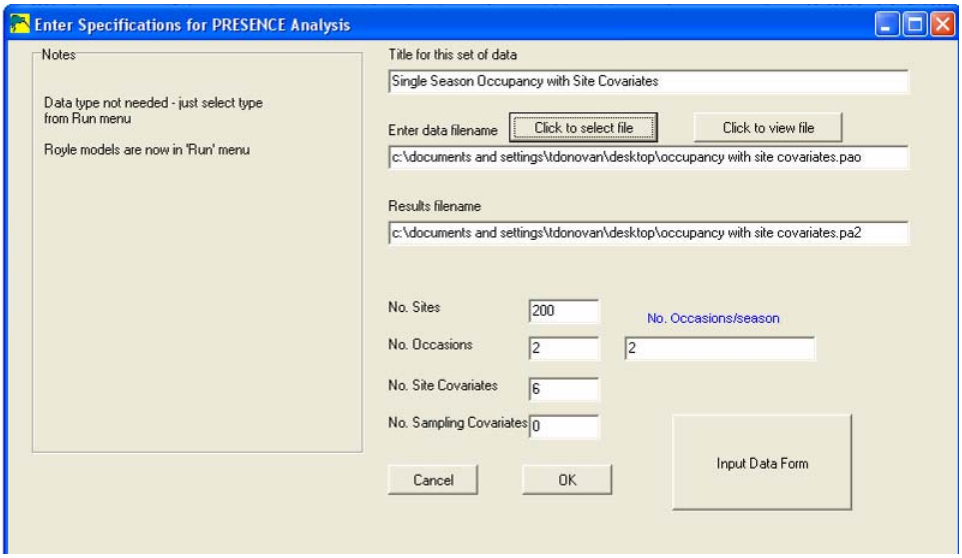
siteco	Patch Size	Patch Size 2	Habitat 1	Habitat 2	H1*PS	H2*PS
site 1	-1.9936	3.9743	1	0	-1.994	0.000
site 2	0.9660	0.9332	0	0	0.000	0.000
site 3	1.1794	1.3911	0	1	0.000	1.179
site 4	-1.8955	3.5929	1	0	-1.895	0.000
site 5	1.1377	1.2943	0	0	0.000	0.000
site 6	0.0262	0.0007	1	0	0.026	0.000
site 7	-0.4874	0.2376	0	1	0.000	-0.487
site 8	-0.8977	0.8058	1	0	-0.898	0.000
site 9	0.6904	0.4767	1	0	0.690	0.000
site 10	0.6435	0.4141	0	1	0.000	0.643
site 11	0.8719	0.7603	0	0	0.000	0.000
site 12	0.4789	0.2293	1	0	0.479	0.000
site 13	-0.5187	0.2690	1	0	-0.519	0.000
site 14	-0.7867	0.6189	1	0	-0.787	0.000
site 15	-1.7346	3.0087	0	0	0.000	0.000
site 16	-0.3903	0.1523	0	1	0.000	-0.390
site 17	-0.3262	0.1064	1	0	-0.326	0.000
site 18	0.4291	0.1841	1	0	0.429	0.000
site 19	-0.4430	0.1962	0	1	0.000	-0.443
site 20	-0.1682	0.0283	0	1	0.000	-0.168
site 21	0.0994	0.0099	1	0	0.099	0.000
site 22	-0.1387	0.0192	0	0	0.000	0.000
site 23	-0.0457	0.0021	0	1	0.000	-0.046

Note that you can re-size the column widths by selecting the line that delineates two columns, and then dragging your mouse to the left or right. Now, you need to save this file as the PRESENCE input file. Go to File | Save As, and save the file as "Occupancy with Site Covariates.pao" and put it somewhere where you can retrieve it.

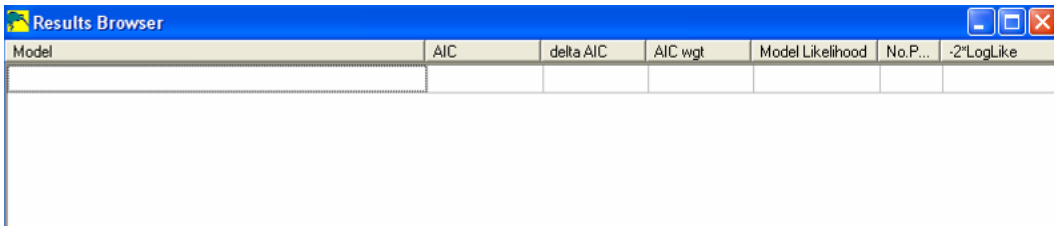


Click Save. Now, return to the Enter Specifications form and tell PRESENCE where the data is located. Click on the button labeled "Click to select file", and then navigate to the location where you stored your .pao file.

Note: To save the data for input to MARK, repeat the save process, except change the data-type box from '.pao' to '.inp'.



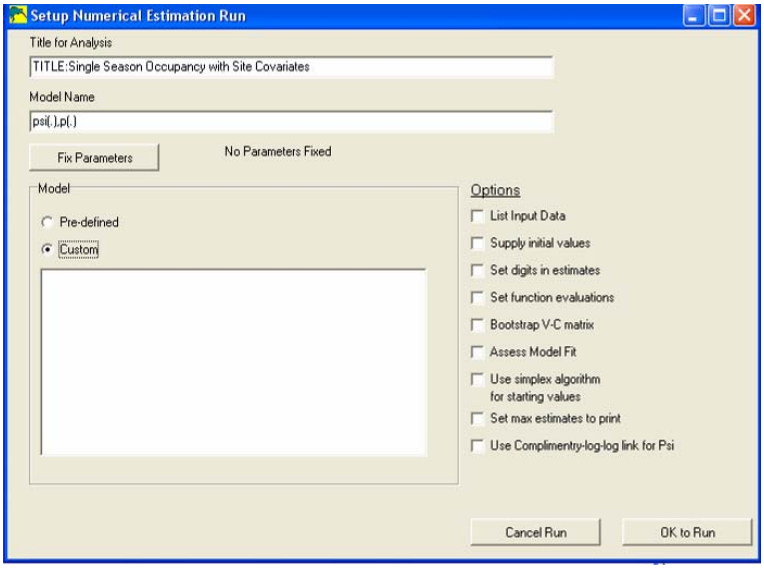
Click OK, and PRESENCE then shows the Results Browser and your data is now ready for analysis.



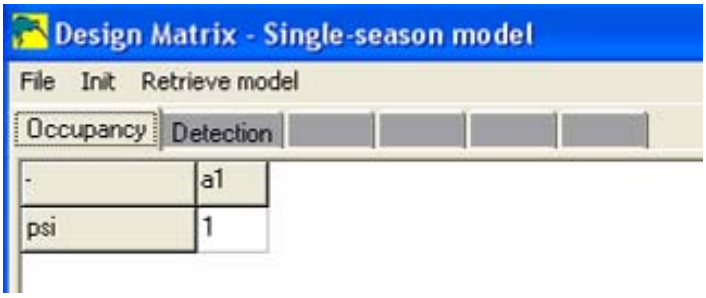
MODEL P(.)PSI(Patch Size)

Let's run the first covariate model: p(.)psi(patch size). In this model, we'll still estimate only three parameters, p_1, p_2, and ψ for each site, but $p_1 = p_2$ and will be constant for all sites, and we'll force ψ to be functions of covariates. To run this model in PRESENCE, go to the main form and select Run | Analysis: Single-season.

Enter a model name (e.g., p(.)psi(patch size)), and then select the Custom option for the model type.



By selecting this option, PRESENCE will immediately produce a new window, which is affectionately known as the Design Matrix. Here is where you define your linear models for each parameter.



Note that the Design Matrix (DM) in PRESENCE consists of two tabs, one for occupancy (ψ) and one for detection (p_1 and p_2). You'll see that the tabs will change when we get to other kinds of models later in the book. Now, there are many ways to specify a model in PRESENCE and you'll undoubtedly find your own style once you

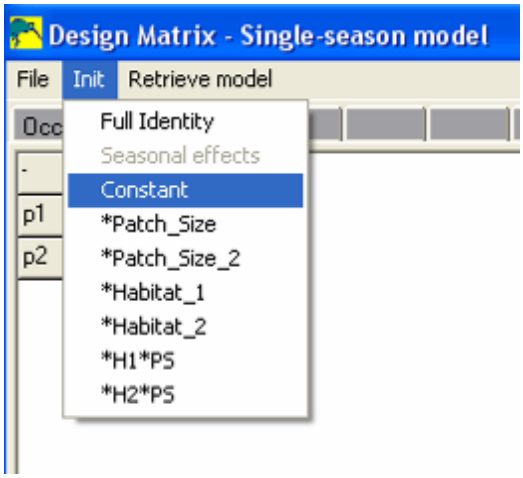
get some experience. Here we'll describe our approach, but keep in mind that there are several ways to run a model in the Design Matrix.

The tab labeled Occupancy is shown first, and here is where you specify a model constraining ψ. The rows in the DM indicate the parameter name; in this case, there is only one parameter, ψ, and is labeled "psi." The columns of DM specify the linear model associated with a given parameter. That is, the columns represent the betas. By default, we begin with the intercept model, and you can see that PRESENCE calls the intercept a1. (In our spreadsheet we called it B_{00}…no matter). So the current model shown indicates the following linear model:

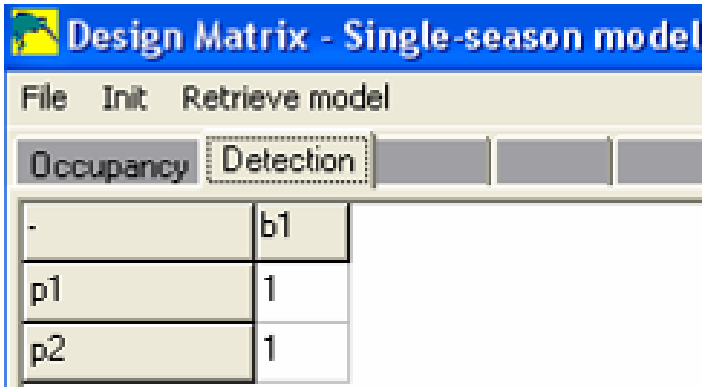
Logit ψ = 1 * a1, where a1 is the intercept.

Very soon we will add covariates to this model.

If you click on the tab labeled Detection, you can specify a model constraining p_1 and p_2. To begin, choose the Init | Constant option, which indicates to PRESENCE that your detection parameters are constant across the two surveys.



Now, when you click on the detection tab in the DM, you should see the following screen.



The tab labeled Detection is where you specify a model constraining the two detection parameters, p1 and p2. If your data consisted of three survey occasions, there would be three rows of parameters, labeled p1, p2, and p3. The rows in the DM indicate the parameter name; in our current exercise, there are two parameters, p1 and p2. The columns of DM specify the linear model associated with a given parameter. That is, the columns represent the betas. Because we specified that p was constant, you can see that there is only one column, and it is labeled b1. This is the beta for the intercept for p_1 and p_2. Note that the 1 entered for b1 is stacked, which is PRESENCE's way of forcing p1 = p2. So the current model shown indicates the following linear model:

Logit p1 = 1 * b1, where b1 is the intercept,

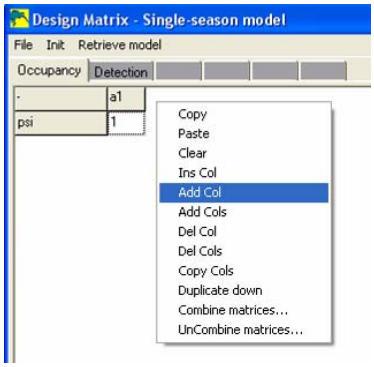
Logit p2 = 1 * b1, where b1 is the intercept.

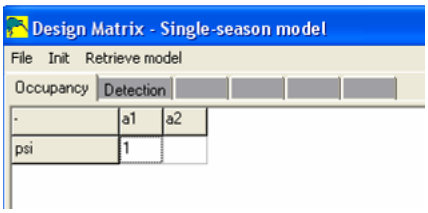
Thus, Logit p = 1 * p1.

Make sense?

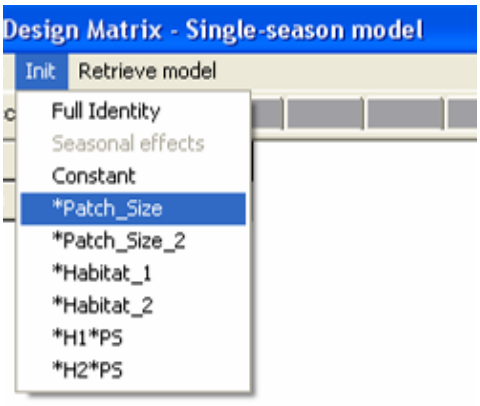
OK, so let's switch back to the occupancy tab and force occupancy to be a function of patch size. We need to specify the following linear model:

Logit ψ = a1 + a2*patch size, where a1 is the intercept and a2 is effect size (slope) of patch size. This should look very familiar to you because it is exactly what we covered in the spreadsheet exercise. So, we need to add a column to the DM's occupancy tab to specify the model. Select the number 1 under the column labeled a1, and then right-click to bring up another menu. Then select the "Add Col" option to add a column.





Now, click in the blank square under a2, and then go to Init | *Patch Size as shown below:



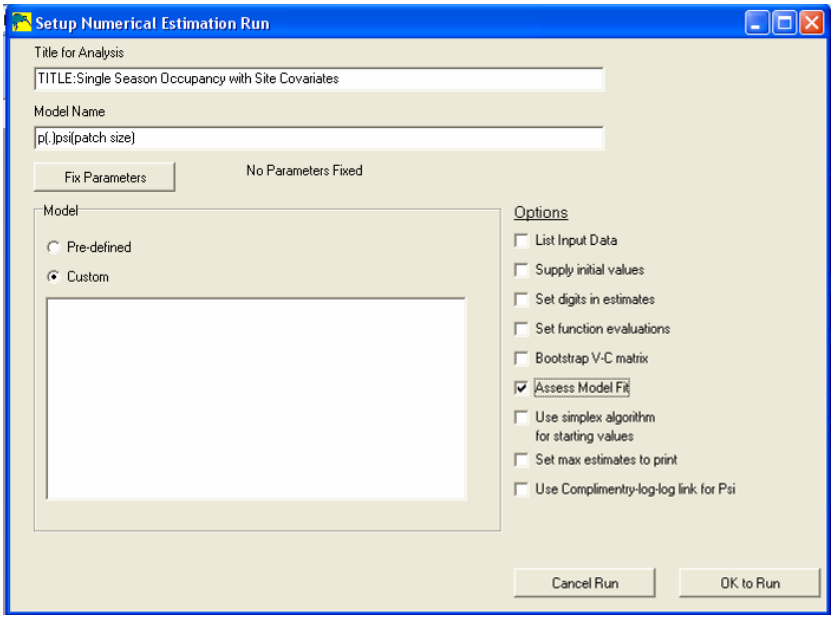
Now your DM for Occupancy should look like this:

OK! You have now specified a linear model for occupancy within PRESENCE, and the model is:

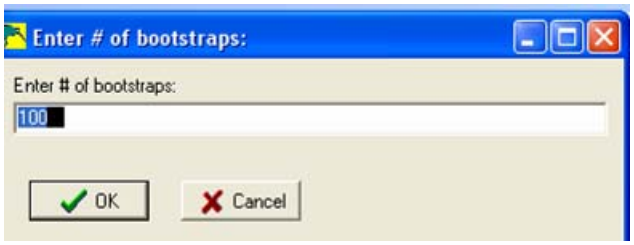
$$\text{Logit } \psi = a1*1 + a2*\text{Patch Size}.$$

Just read this equation across the row. Just like in the spreadsheet, Presence will find the values for a1 and a2 that maximize the multinomial log likelihood.

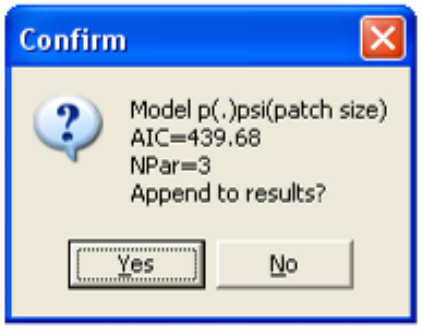
Now we are ready to run model psi(patch size)p(date). Click back on the Setup Numerical Estimation Form. If you haven't already entered a name for this model, do that now: **p(.) psi(patch size)**. Because we ran the MacKenzie and Bailey GOF test on this model in the spreadsheet, we'll also run it in PRESENCE now by selecting the Option labeled Assess Model Fit.



Click OK to Run. A dialogue box will appear asking you how many bootstrap analyses you'd like to run. Remember, we ran 20 in the spreadsheet. PRESENCE is way, way faster and you can run many more bootstrap trials. Typically 1,000 – 10,000 trials are run, but for now, 100 is fine – click OK.



After PRESENCE is finished, it will bring up a dialogue box asking if you'd like to append the results.



Click Yes, and your results will be added to the Results Browser:

Results Browser

Model	AIC	deltaAIC	AIC wgt	Model Likelihood	no.Par.	-2*LogLike
p(.)psi(patch size)	439.68	0.00	1.0000	1.0000	3	433.68

Now, let's study the output from this model in PRESENCE. Right-click on the model name, and select view model output:

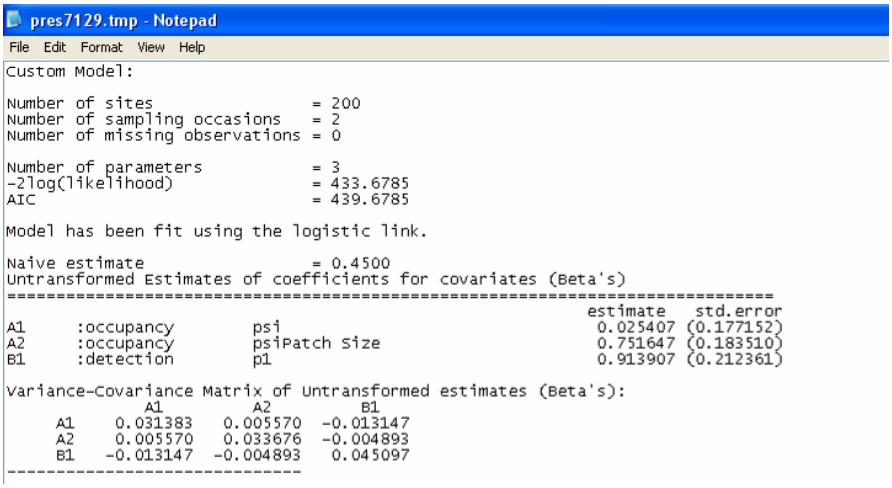
pres7129.tmp - Notepad

File Edit Format View Help

```
=======(p(.)psi(patch size))======


PRESENCE - Presence/Absence-Site Occupancy data analysis
Fri Jun 08 21:32:22 2007,       Version 2.070605
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
==>i=C:\Documents and Settings\tdonovan\Desktop\occupancy with site covariates.pao
==>l=C:\Documents and Settings\tdonovan\Desktop\occupancy with site covariates.pa2.out
==>name=p(.)psi(patch size)
==>model=100
==>j=C:\Documents and Settings\tdonovan\Desktop\occupancy with site covariates.dm
==>boot2=100
model=100 N,T-->200,2
modtype-->1 Single-Season data Model selected

Data checksum = 30212
NSi-->6
site_covname[0]=Patch Size
site_covname[1]=Patch Size 2
site_covname[2]=Habitat 1
site_covname[3]=Habitat 2
site_covname[4]=H1*PS
site_covname[5]=H2*PS
NSa-->0
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
 Single Season Occupancy with Site Covariates
- - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
modtype=1 N=200 T=2 Groups=1 bootstraps=0

-->0

Matrix 1: rows=2, cols=3
-,a1,a2,
psi  1 Patch Size
========================

Matrix 2: rows=3, cols=2
-,b1,
p1  1
p2  1
========================
```

Take some time to get familiar with the PRESENCE output. PRESENCE lists information about the model type, sample size, etc. Under the section labeled Matrix 1, PRESENCE indicates that this is a design matrix with two rows and three columns, and that a1 and a2 are the coefficients to be estimated. In Matrix 2, b1 is the coefficient to be estimated. Thus, the total number of parameters to be estimated in this model is 3.

Now let's look at the rest of the output:



pres7129.tmp - Notepad

File Edit Format View Help

```
Custom Model:

Number of sites                 = 200
Number of sampling occasions    = 2
Number of missing observations  = 0

Number of parameters            = 3
-2log(likelihood)               = 433.6785
AIC                             = 439.6785

Model has been fit using the logistic link.

Naive estimate                  = 0.4500
Untransformed Estimates of coefficients for covariates (Beta's)
------------------------------------------------------------------------------
                                                                  estimate   std.error
A1      :occupancy        psi                                    0.025407 (0.177152)
A2      :occupancy        psiPatch Size                          0.751647 (0.183510)
B1      :detection        p1                                     0.913907 (0.212361)

Variance-Covariance Matrix of Untransformed estimates (Beta's):
                A1          A2          B1
     A1     0.031383    0.005570   -0.013147
     A2     0.005570    0.033676   -0.004893
     B1    -0.013147   -0.004893    0.045097
------------------------------
```

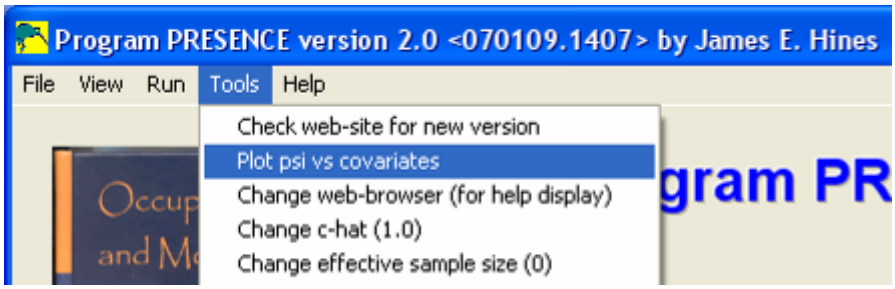
The -2Log$_e$L for this model is 443.6785, which corresponds to cell K5 in the spreadsheet. PRESENCE indicates that the logistic link was used in the modeling process, and that naïve estimate is 0.6180 (as we saw for our first model). After the double-dashed line, PRESENCE reports the beta estimates and associated standard errors. The beta estimates are provided in cells F11:H11, and match fairly closely.

	E	F	G	H	I	J	K	L	M	N
3	**Summarized Inputs**					**Outputs**				
4	11	10	01	00	Total	Log$_e$L	-2Log$_e$L	K	AIC	AICc
5	50	18	22	110	200	-216.839	433.679	3	439.679	439.801
6						Model DF	C hat	P (MLE)	ψ (MLE)	
7						197	2.201	0.713798986	0.506352058	
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2	
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6	
10	Estimate?	1	1	1	0	0	0	0	0	
11	Beta	0.913907	0.025410	0.751644	0.000000	0.000000	0.000000	0.000000	0.000000	

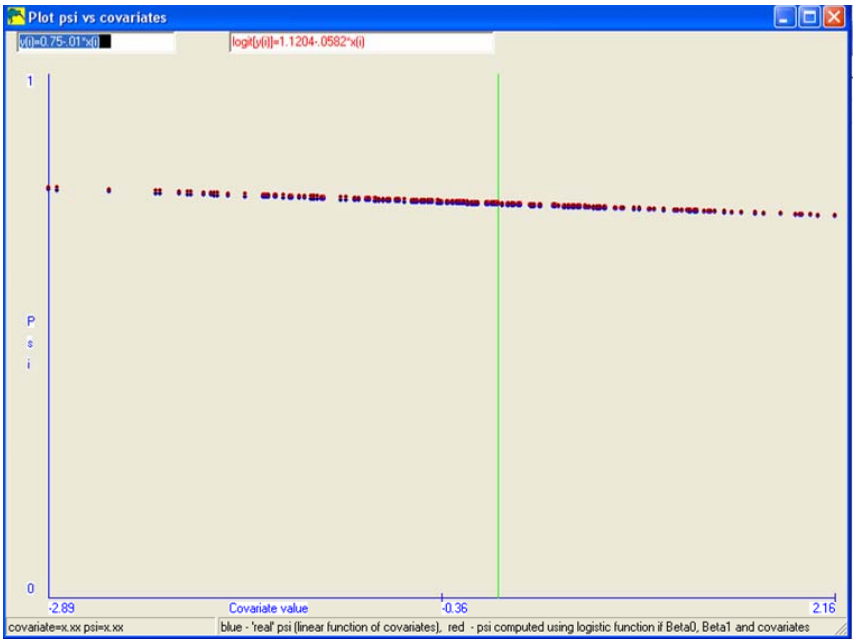
The beta values and their confidence limits should be studied, because the confidence limits will help you interpret your results. Was there an effect of standardized patch size on psi? The PRESENCE output indicates that the beta for date was positive 0.751647, with a standard error of 0.183510. From this information, you can compute the 95% confidence intervals: the upper confidence interval is 0.751647 + 1.96*0.183510 = 1.111; the lower confidence interval is

0.751647 - 1.96*0.183510 = 0.3919. These confidence intervals don't overlap 0 at all, indicating that this is a "significant" result. The question "how significant?" is best answered by plotting the standardized patch against psi to understand the biological implications of the result, as we did in the spreadsheet exercise (and not by reporting a statistical p value such as p = 0.0034).

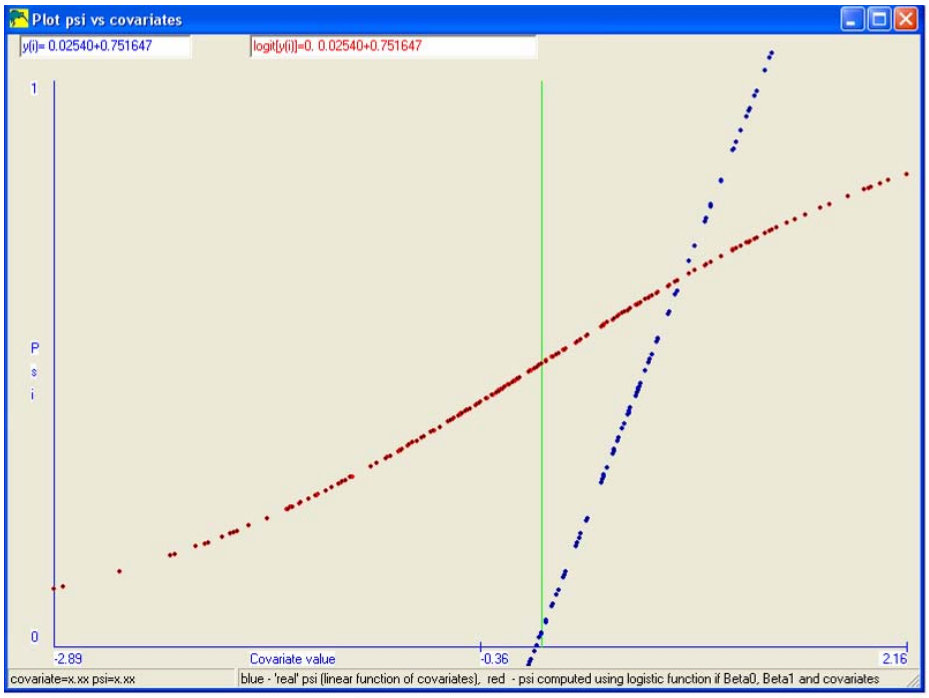
In PRESENCE, you can quickly view the relationship between ψ and a covariate (this option is not set up to visualize the relationship between p and a covariate). Just go to Tools | Plot psi vs covariates.



PRESENCE then presents a graph that looks something like this:

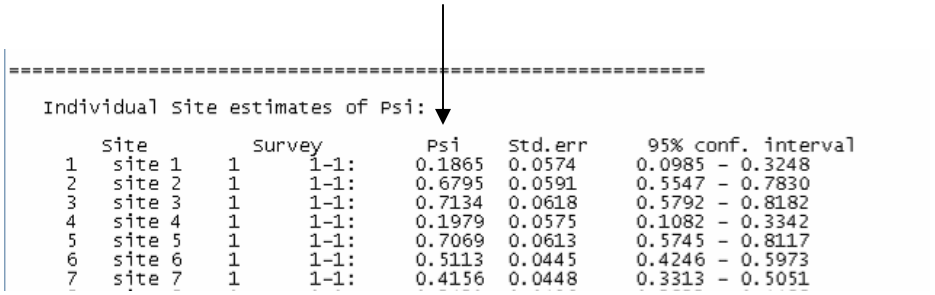


Note that this graph is not correct because you need to enter the correct linear equation in the upper portion of the graph (copying the parameter estimates from the output):



The series in blue shows the logit equation, and the series in red shows the back-transformed logit equation which shows the probability (ψ) as a function of standardized patch size. Although you wouldn't use this graph in a manuscript, it quickly gives you an indication of how strong an effect size is for a particular covariate.

Next, the output provides the p and ψ estimates on a site by site basis, based on the beta estimates from the model. This section of the output is divided into sections; the first section focuses on the psi estimates:

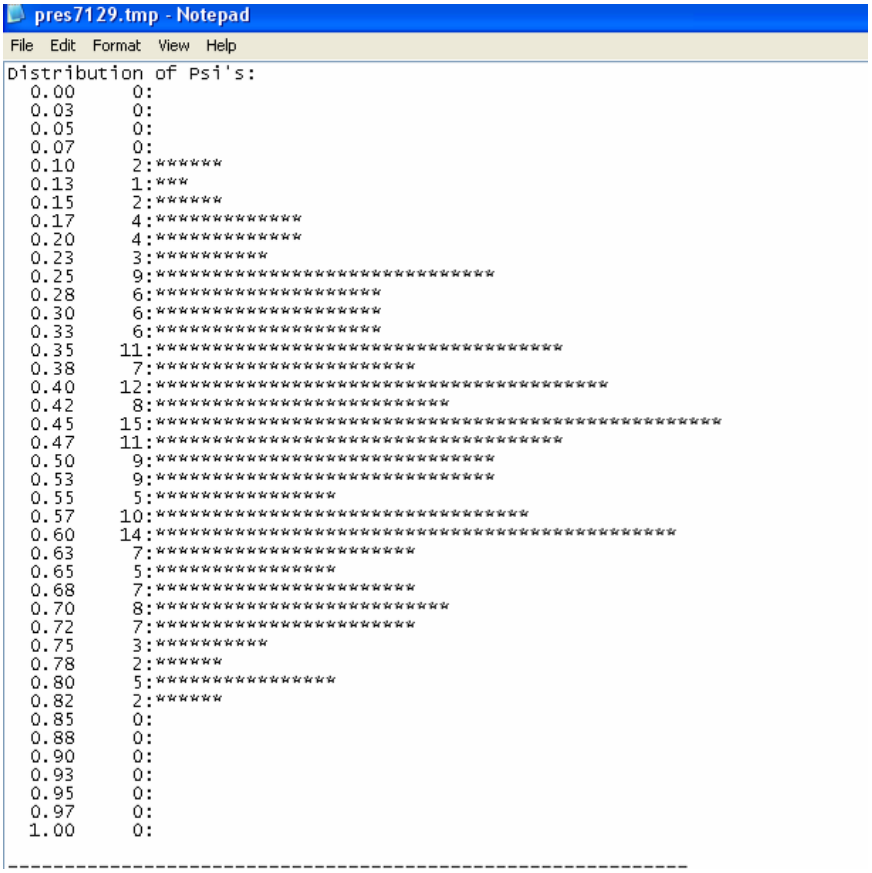


```
============================================================

  Individual Site estimates of Psi:

      Site          Survey         Psi     Std.err     95% conf. interval
  1   site 1    1      1-1:     0.1865   0.0574     0.0985 - 0.3248
  2   site 2    1      1-1:     0.6795   0.0591     0.5547 - 0.7830
  3   site 3    1      1-1:     0.7134   0.0618     0.5792 - 0.8182
  4   site 4    1      1-1:     0.1979   0.0575     0.1082 - 0.3342
  5   site 5    1      1-1:     0.7069   0.0613     0.5745 - 0.8117
  6   site 6    1      1-1:     0.5113   0.0445     0.4246 - 0.5973
  7   site 7    1      1-1:     0.4156   0.0448     0.3313 - 0.5051
```

For brevity, the output for only the first 7 sites is shown, and the spreadsheet comparisons are shown below (column Q). The nice thing about PRESENCE is that the standard errors and confidence are provided for each site. (We'll have to add that to the spreadsheet some day).

	G	H	I	J	K	L	M	N	O	P	Q
14	Occupancy Covariates							Detection		Occupancy	
15	Psi (Int)	Patch Size	Patch Size ^{2}	Habitat 1	Habitat 2	H1*PS	H2*PS	Logit p	p link	Logit psi	psi link
16	1	-1.9936	3.9743	1	0	-1.994	0.000	0.91391	0.71380	-1.4730	0.1865
17	1	0.9660	0.9332	0	0	0.000	0.000	0.91391	0.71380	0.7515	0.6795
18	1	1.1794	1.3911	0	1	0.000	1.179	0.91391	0.71380	0.9119	0.7134
19	1	-1.8955	3.5929	1	0	-1.895	0.000	0.91391	0.71380	-1.3993	0.1979
20	1	1.1377	1.2943	0	0	0.000	0.000	0.91391	0.71380	0.8805	0.7069
21	1	0.0262	0.0007	1	0	0.026	0.000	0.91391	0.71380	0.0451	0.5113
22	1	-0.4874	0.2376	0	1	0.000	-0.487	0.91391	0.71380	-0.3410	0.4156

Scroll down a bit further, and you'll see the distribution of ψ estimates for all 200 sites, based on the model parameter estimates:



This distribution shows the expected probability of site occupancy for each of the 200 sites, given the model's parameter estimates (e.g., 11 sites had an expected ψ of 0.35, and 15 sites had an expected ψ of 0.45).

A bit further down, you'll see the parameter estimates for p:

```
------------------------------------------------------------

   Individual Site estimates of p:

       Site          Survey           p     Std.err      95% conf. interval
    1   site 1    1      1-1:      0.7138  0.0434      0.6219 - 0.7909

============================================================
```

Since we ran the dot model for p, there would be no variation in p among sites…each site has an identical estimate for p.

ASSESSING MODEL FIT

Does the output indicate the model p(.) psi(patch size) fits the observed data?

The last part of the output gives the results of the MacKenzie and Bailey GOF test.

```
============================================================

Assessing Model Fit

History(cohort)    Observed    Expected    Chi-square
00(0)                110         109.9659         0.00
01(0)                 22          20.0341         0.19
10(0)                 18          20.0341         0.21
11(0)                 50          49.9659         0.00
Test Statistic    =           0.3995
------------------------------------------------------------
Test Statistic    =           0.3995
Probability of test statistic >= observed
from 100 parametric bootstraps = 0.9109
Estimate of c-hat = 0.1419
------------------------------------------------------------

CPU time: 1.0 seconds
```

The first portion of the output reveals the observed and expected frequencies for each history, based on the model's parameter estimates. These match the spreadsheet:

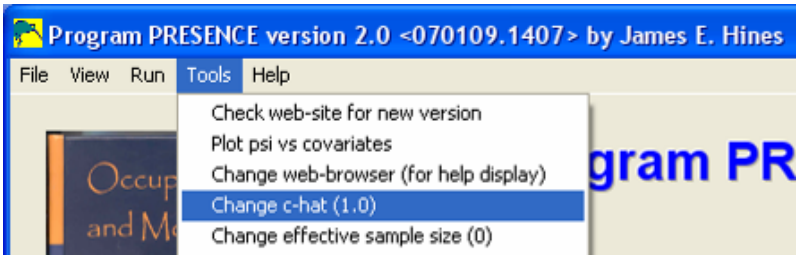
	R	S	T	U
3	MacKenzie-Bailey Goodness of Fit			
4		Observed	Expected	$(O-E)^2/E$
5	11	50	50.0	0.000
6	10	18	20.0	0.207
7	01	22	20.0	0.193
8	00	110	110.0	0.000
9		200	200	Chi-Square
10				0.3995

The second portion of the output focuses on the bootstrap results. Remember, these results are completely dependent on the simulated data, and hence the results will always vary from run to run; i.e., your results will be slightly different than ours. In our analysis, PRESENCE ran 100 bootstrap trials, and the observed Chi-Square test statistic (0.3995) fell within the lower portion of bootstrap

distribution. Specifically, 91.09% of the bootstrap Chi-Square values were larger than 0.399, and (1-91.09%) = 8.91% of the bootstrap Chi-Square values were smaller than the observed value. Based on these results, we can conclude that there is no evidence of lack-of-fit. These results are not very consistent in the 100 spreadsheet runs (I'm not sure why), where the observed test statistic is given in cells AN8, and the percentile is given in cell AN9.

	AM	AN
8	$Chi\text{-}Square_R$	0.399469401
9	Percentile	0.49
10	$Chi\text{-}Square_B$	1.343354815
11	c hat	0.297367008

Now, you have a choice of what to do next. First, because the observed data appear to "fit" the model, you can continue running models with a clear conscience. If c-hat was greater than 1, you want to adjust c-hat because many analysts feel that any c-hat estimate greater than 1 potentially biases the standard error estimates. To do this, return to the main menu and go to Tools | Change c-hat.



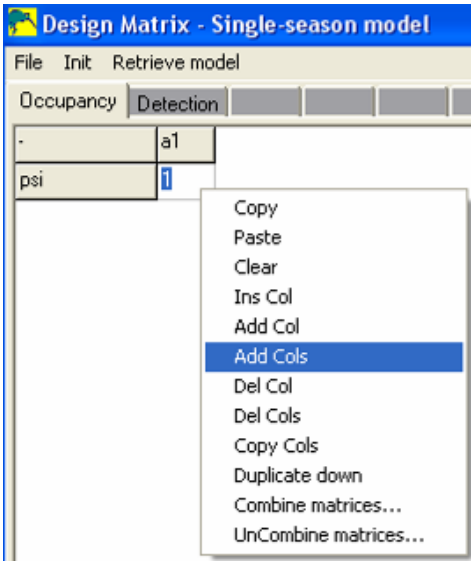
Then enter the new c-hat value, and click OK. You should see that the standard errors for the different parameter estimates have increased as a result.

To retrieve the design matrix for a model, run a new model, then click the 'Retrieve model' menu, then the 'refresh model list'. Next, click 'Retrieve model' again and the list of models run will appear. Choose one of them and the design matrix should be filled with the matrix from the chosen model.

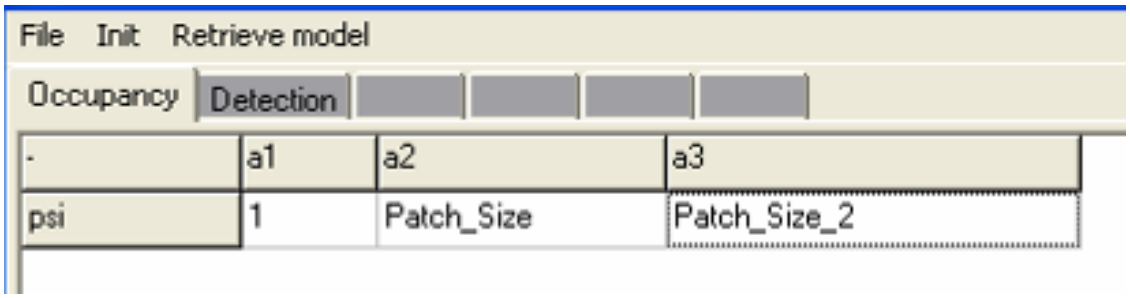
MODEL P(.)PSI(Patch Size + Patch Size^2)

Congratulations! You've run your first linear model in PRESENCE! The next model is model p(.)psi(patch size + patch size^2), which is the polynomial model. Click on the PRESENCE's main form, select Run | Analysis: Single-season again. The Setup Numerical Estimation Form and the DM appear. See if you can set up this model now.

In this model, the linear equation for psi will be 1 * a1 + a2 * patch size + a3 * patch size 2. For the occupancy DM tab, we will need to add two columns to our design matrix. Right click on the number 1 under a1, and then select Add Cols.



Enter 2 in the dialogue box that pops up to add 2 columns. Then initialize the two new columns by going to Init | Patch_Size and Init | Patch_Size_2. The occupancy side of the model should look like this:

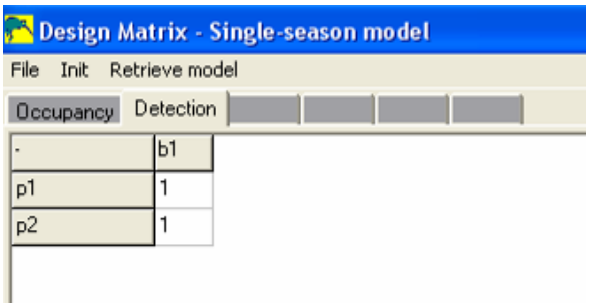


Now, we have specified the following linear model in PRESENCE:

$$\text{Logit psi} = 1 * a1 + a2 * \text{patch size} + a3 * \text{patch size}^2.$$

Let's think about what this DM specifies. In this model, ψ is constrained to be a function of its intercept, patch size, and patch size2. If a3 = 0, we're back to the patch size model we ran a few minutes ago.

OK, now select the detection tab. For the detection DM tab, we only need to make sure that p1 = p2 as we specified in the previous model.



Run this model, and call it p(.)p(patch size + patch size2), and add the results to the Results Browser.

Results Browser

Model	AIC	deltaAIC	AIC wgt	Model Likelihood	no.Par.	-2*LogLike
p(.)psi(patch size)	439.68	0.00	0.6715	1.0000	3	433.68
p(.)psi(patch size + patch size 2)	441.11	1.43	0.3285	0.4892	4	433.11

Now let's look at the key results from this model:

pres4280.tmp - Notepad

File Edit Format View Help

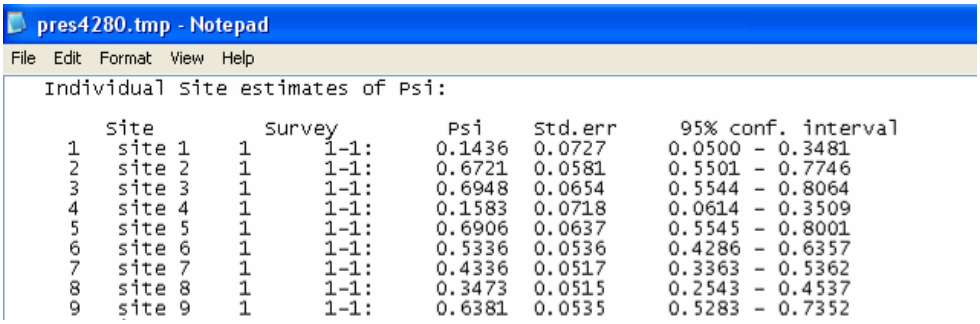
```
Number of sites                 = 200
Number of sampling occasions    = 2
Number of missing observations  = 0

Number of parameters            = 4
-2log(likelihood)               = 433.1117
AIC                             = 441.1117

Model has been fit using the logistic link.

Naive estimate                  = 0.4500
Untransformed Estimates of coefficients for covariates (Beta's)
------------------------------------------------------------------------------
                                                              estimate   std.error
A1      :occupancy        psi                                0.115533 (0.215190)
A2      :occupancy        psiPatch Size                      0.731200 (0.182769)
A3      :occupancy        psiPatch Size 2                   -0.111534 (0.149506)
B1      :detection        p1                                 0.914813 (0.212306)
```

This model's AIC value is 441.1117, which is also reported in cell M5 in the spreadsheet. The beta estimates for the model are then listed in a table in PRESENCE, along with the standard error estimates. Note that the table specifies the name of the parameter (e.g., A1, A2, A3, B1) and also provides the name of the associated covariate. These estimates match those in cells F11:I11 in the spreadsheet to the fourth decimal place…not too bad.

	E	F	G	H	I	J	K	L	M
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
10	Estimate?	1	1	1	1	0	0	0	0
11	Beta	0.914812	0.115535	0.731197	-0.111534	0.000000	0.000000	0.000000	0.000000

If you study the PRESENCE output where the site-by-site ψ and p estimates are listed, you should see that the spreadsheet also matches PRESENCE.

Let's start by examining the occupancy estimates.



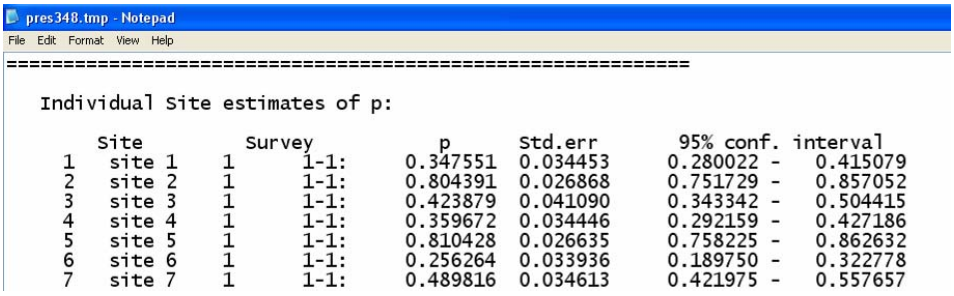
pres4280.tmp - Notepad

File Edit Format View Help

Individual Site estimates of Psi:

	Site		Survey	Psi	Std.err	95% conf. interval
1	site 1	1	1-1:	0.1436	0.0727	0.0500 - 0.3481
2	site 2	1	1-1:	0.6721	0.0581	0.5501 - 0.7746
3	site 3	1	1-1:	0.6948	0.0654	0.5544 - 0.8064
4	site 4	1	1-1:	0.1583	0.0718	0.0614 - 0.3509
5	site 5	1	1-1:	0.6906	0.0637	0.5545 - 0.8001
6	site 6	1	1-1:	0.5336	0.0536	0.4286 - 0.6357
7	site 7	1	1-1:	0.4336	0.0517	0.3363 - 0.5362
8	site 8	1	1-1:	0.3473	0.0515	0.2543 - 0.4537
9	site 9	1	1-1:	0.6381	0.0535	0.5283 - 0.7352

OK, now let's turn to the detection estimates. Given the parameter estimates from the model, estimates for p for the first seven sites are shown on the PRESENCE output:



pres348.tmp - Notepad

File Edit Format View Help

Individual Site estimates of p:

	Site		Survey	p	Std.err	95% conf. interval
1	site 1	1	1-1:	0.347551	0.034453	0.280022 - 0.415079
2	site 2	1	1-1:	0.804391	0.026868	0.751729 - 0.857052
3	site 3	1	1-1:	0.423879	0.041090	0.343342 - 0.504415
4	site 4	1	1-1:	0.359672	0.034446	0.292159 - 0.427186
5	site 5	1	1-1:	0.810428	0.026635	0.758225 - 0.862632
6	site 6	1	1-1:	0.256264	0.033936	0.189750 - 0.322778
7	site 7	1	1-1:	0.489816	0.034613	0.421975 - 0.557657

And these correspond to cells O16:O22.

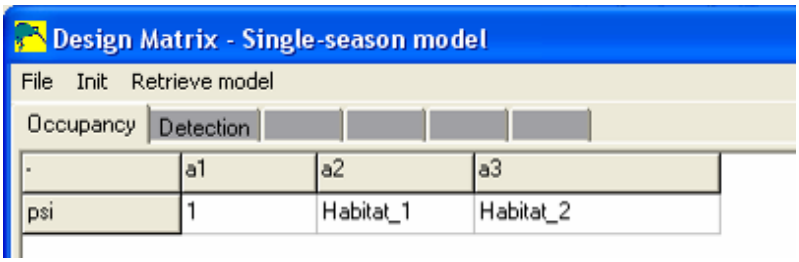
	E	F	G	H	I	J	K	L	M	N	O
13		Int	Cov 1	Cov 2	Cov 3	Int	Cov 4	Cov 5	Cov 6		
14		Detection Covariates				Occupancy Covariates					
15	History	P (Int)	Date	Temp	Rain	Psi (Int)	Patch Size	Habitat 1	Habitat 2	Logit p	p link
16	00	1	-0.703	0.12712234	1	1	-0.888	0	1	-0.62943	0.34764
17	11	1	0.736	0.08616731	0	1	1.166	1	0	1.41355	0.80433
18	00	1	-1.081	-0.5083373	0	1	0.227	0	0	-0.30653	0.42396
19	00	1	-0.647	1.07703412	1	1	0.338	0	0	-0.57712	0.35960
20	11	1	0.777	-1.6276723	0	1	1.425	1	0	1.45265	0.81041
21	00	1	-1.163	-0.1677861	1	1	0.917	0	0	-1.06545	0.25627
22	11	1	-0.081	-0.5704866	1	1	0.368	0	0	-0.04084	0.48979

Study the coefficients and the standard errors, and think about how you would interpret these results.

MODEL P(.)PSI(Habitat)

OK, now let's run p(.)psi(habitat). Remember that there are three habitat types, and we need to estimate two parameters to derive estimates for the three habitats. The covariate "habitat1" pertains to habitat 1, "habitat2" pertains to habitat 2, and habitat 3 is the reference habitat (which is the intercept only). So the occupancy tab in the DM should have 3 covariates, which represents the linear equation: Logit ψ = a1 + a2*habitat1 + a3*habitat2.

Go to Run | Analysis | Single-season, and choose the « custom » radio button. Add two columns to the occupancy design matrix, and initialize the linear equation.

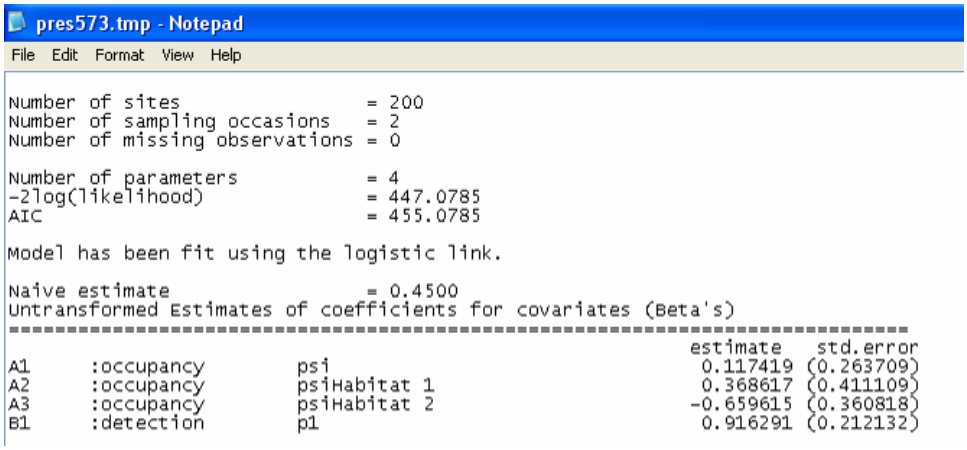


Name the model p(.)psi(habitat), and then run the model and add the results to the results browser.

Results Browser

Model	AIC	deltaAIC	AIC wgt	Model Likelihood	no.Par.	-2*LogLike
p(.)psi(patch size)	439.68	0.00	0.6713	1.0000	3	433.6785
p(.)psi(patch size + patch size 2)	441.11	1.43	0.3284	0.4892	4	433.1117
p(.)psi(habitat)	455.08	15.40	0.0003	0.0005	4	447.08

Select the model name on the results browser, and then right-click to view the model output.



The spreadsheet results are shown below for comparison.

	E	F	G	H	I	J	K	L	M	N
3	**Summarized Inputs**					**Outputs**				
4	11	10	01	00	Total	Log_eL	$-2Log_eL$	K	AIC	AICc
5	50	18	22	110	200	-223.539	447.078	4	455.078	455.284
6						Model DF	C hat	P (MLE)	ψ (MLE)	
7						196	2.281	0.714285626	0.529321065	
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2	
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6	
10	Estimate?	1	1	0	0	1	1	0	0	
11	Beta	0.916290	0.117419	0.000000	0.000000	0.368617	-0.659616	0.000000	0.000000	

Let's quickly review how to interpret the output for a categorical variable (habitat), starting with the reference habitat, habitat 3. We noted that habitat 3 is the reference habitat, coded 0 for the covariate habitat1 (a2) and coded 0 for the covariate habitat 2 (a2). So the linear equation for the reference habitat is:

Logit psi $_{\text{habitat 3}}$ = a1 * 1 + a2 * 0 + a3 * 0 = a1. Filling in the parameter estimates from this equation, we get

Logit psi $_{\text{habitat 3}}$ = 0.117419 * 1 + 0.368617 * 0 + -0.659615 * 0 = 0.1174

Psi $_{\text{habitat 3}}$ = exp(0.117419)/(1+exp(0.117419)) = 0.5293.

Habitat 1 was coded 1 for the covariate habitat1 and 0 for the covariate habitat2. So the linear equation for habitat 1 is:

Logit psi $_{habitat\ 1}$ = a1 * 1 + a2 * 1 + a3 * 0 = a1. Filling in the parameter estimates from this equation, we get

Logit psi $_{habitat\ 1}$ = 0.117419 * 1 + 0.368617 * 1 + -0.659615 * 0 = 0.4860

Psi $_{habitat\ 1}$ = exp(0.4860)/(1+exp(0.4860)) = 0.6912.

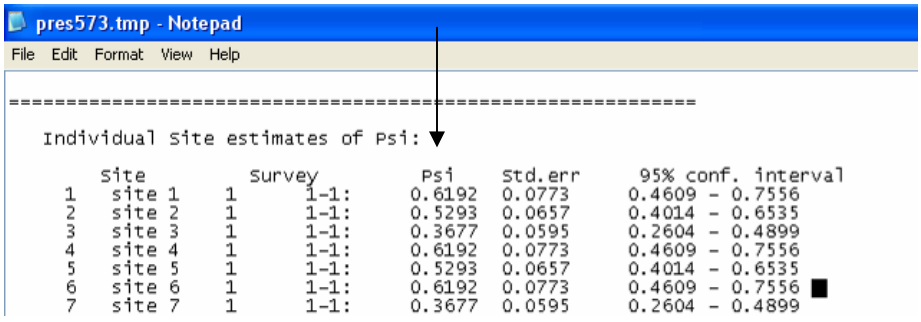
Habitat 2 was coded 0 for the covariate habitat1 and 1 for the covariate habitat2. So the linear equation for habitat 1 is:

Logit psi $_{habitat\ 2}$ = a1 * 1 + a2 * 0 + a3 * 1 = a1. Filling in the parameter estimates from this equation, we get

Logit psi $_{habitat\ 2}$ = 0.117419 * 1 + 0.368617 * 0 + -0.659615 * 1 = -0.5422

Psi $_{habitat\ 2}$ = exp(-0.5422)/(1+exp(-0.5422)) = 0.3677.

These results are confirmed in the Presence and spreadsheet output, where sites have occupancy probabilities that are determined by their habitat type:

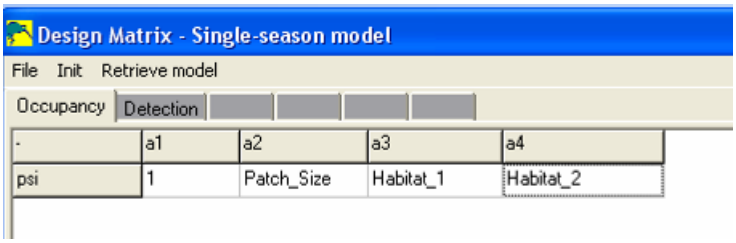


	G	H	I	J	K	L	M	N	O	P	Q
14	Occupancy Covariates							Detection		Occupancy	
15	Psi (Int)	Patch Size	Patch Size ^{2}	Habitat 1	Habitat 2	H1*PS	H2*PS	Logit p	p link	Logit psi	psi link
16	1	-1.9936	3.9743	1	0	-1.994	0.000	0.91629	0.71429	0.4860	0.6192
17	1	0.9660	0.9332	0	0	0.000	0.000	0.91629	0.71429	0.1174	0.5293
18	1	1.1794	1.3911	0	1	0.000	1.179	0.91629	0.71429	-0.5422	0.3677
19	1	-1.8955	3.5929	1	0	-1.895	0.000	0.91629	0.71429	0.4860	0.6192
20	1	1.1377	1.2943	0	0	0.000	0.000	0.91629	0.71429	0.1174	0.5293
21	1	0.0262	0.0007	1	0	0.026	0.000	0.91629	0.71429	0.4860	0.6192
22	1	-0.4874	0.2376	0	1	0.000	-0.487	0.91629	0.71429	-0.5422	0.3677

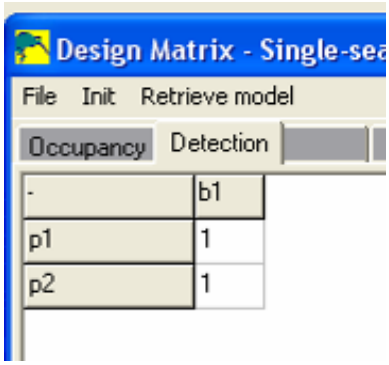
MODEL P(.)PSI(Patch Size + Habitat)

The next model is the additive model p(.)psi(patch size + habitat). So we'll be running the same model we just did (habitat) and will simply add the effect of patch size. So, this model will estimate 4 occupancy parameters and 1 detection parameter (the intercept), or 5 in total.

Go to Run | Analysis | Single-season, choose the custom radio button, and add 3 columns to Design matrix.



The detection tab should also have 1 covariate, b1, in which the number 1 is stacked for p1 and p2, forcing them to be constant:



Name the model p(.)psi(patch size + habitat), run the model, and add the results to the results browser:

Results Browser

Model	AIC	delta AIC	AIC wgt	Model Likelihood	no.Par.	-2*LogLike
p(.)psi(patch size + habitat)	438.88	0.00	0.5004	1.0000	5	428.88
p(.)psi(patch size)	439.68	0.80	0.3354	0.6703	3	433.6785
p(.)psi(patch size + patch size 2)	441.11	2.23	0.1641	0.3279	4	433.1117
p(.)psi(habitat)	455.08	16.20	0.0002	0.0003	4	447.08

Take a look at the model results:

pres5543.tmp - Notepad

File Edit Format View Help

```
========================

Custom Model:

Number of sites                 = 200
Number of sampling occasions    = 2
Number of missing observations  = 0

Number of parameters            = 5
-2log(likelihood)               = 428.8845
AIC                             = 438.8845

Model has been fit using the logistic link.

Naive estimate                  = 0.4500
Untransformed Estimates of coefficients for covariates (Beta's)
------------------------------------------------------------------------------
                                                          estimate   std.error
A1      :occupancy      psi                              0.157533 (0.281225)
A2      :occupancy      psiPatch Size                    0.713230 (0.181997)
A3      :occupancy      psiHabitat 1                     0.292716 (0.428624)
A4      :occupancy      psiHabitat 2                    -0.586246 (0.383614)
B1      :detection      p1                               0.934801 (0.209517)
```

	E	F	G	H	I	J	K	L	M	N
3	**Summarized Inputs**					**Outputs**				
4	11	10	01	00	Total	Log_eL	$-2Log_eL$	K	AIC	AICc
5	50	18	22	110	200	-214.442	428.885	5	438.885	439.194
6						Model DF	C hat	P (MLE)	ψ (MLE)	
7						195	2.199	0.718048196	0.53930243	
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2	
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6	
10	Estimate?	1	1	1	0	1	1	0	0	
11	Beta	0.934801	0.157535	0.713225	0.000000	0.292713	-0.586242	0.000000	0.000000	

We covered how to interpret the results in detail when we ran this model in the spreadsheet, so we'll just very quickly overview it now. The effect of patch size (the slope) is 0.7132 (a positive slope, indicating as patch size increases, probability of occupancy increases). This slope is the same for all three habitat types. If the general linear model is logit psi = intercept + patch size * 0.7132, we

just need to determine what this looks like for each habitat. We know that habitat 3 is the intercept, so the equation is

Logit psi $_{\text{habitat 3}}$ = intercept + patch size * 0.7132

Logit psi $_{\text{habitat 3}}$ = 0.157533 + patch size * 0.7132.

For habitat 1, we just bump the intercept according to the parameter estimate for habitat 1:

Logit psi $_{\text{habitat 1}}$ = (intercept) + patch size * 0.7132

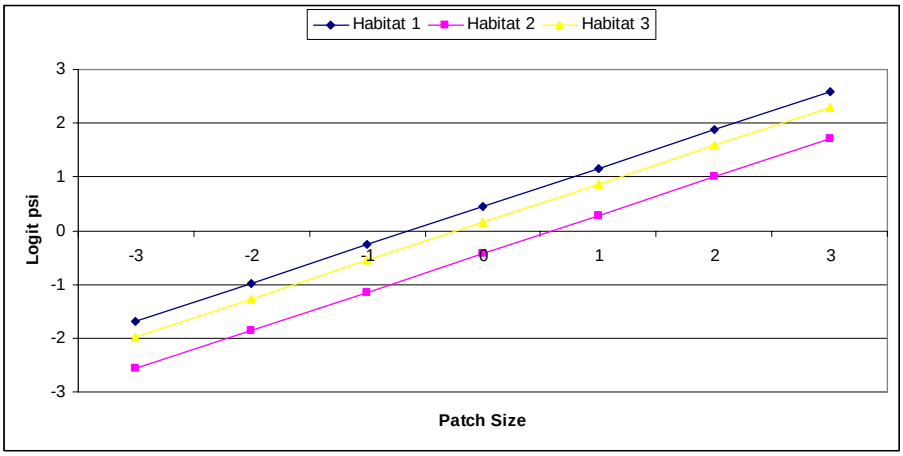
Logit psi $_{\text{habitat 1}}$ = (0.157533 + 0.292716) + patch size * 0.7132.

For habitat 2, we just bump the intercept according to the parameter estimate for habitat 2:

Logit psi $_{\text{habitat 2}}$ = (intercept) + patch size * 0.7132

Logit psi $_{\text{habitat 2}}$ = (0.157533 + -0.586246) + patch size * 0.7132.

A graph of the results looks like this:



MODEL P(.)PSI(Habitat * Patch Size)

OK! We have one model to run, and then we'll study the results browser. The last model is model p(.)psi(habitat * patch size), which is the interaction model where we estimate a unique intercept and a unique patch size slope for each habitat. Remember, to run this model we basically run the same model we just did (patch size + habitat), but we add the two covariates that alter the slope for habitats 1 and 2. So we will be estimating a total of 7 parameters. See if you can set up and run this model now.

Here is our design matrix for occupancy:

Design Matrix - Single-season model

File Init Retrieve model

Occupancy | Detection

-	a1	a2	a3	a4	a5	a6
psi	1	Patch_Size	Habitat_1	Habitat_2	H1*PS	H2*PS

Go ahead and run this model, and add the results to the results browser:

Results Browser

Model	AIC	deltaAIC	AIC wgt	Model Likelihood	no.Par.	-2*LogLike
p(.)psi(patch size * habitat)	438.85	0.00	0.3368	1.0000	7	424.85
p(.)psi(patch size + habitat)	438.88	0.03	0.3318	0.9851	5	428.88
p(.)psi(patch size)	439.68	0.83	0.2224	0.6603	3	433.6785
p(.)psi(patch size + patch size 2)	441.11	2.26	0.1088	0.3230	4	433.1117
p(.)psi(habitat)	455.08	16.23	0.0001	0.0003	4	447.08

Take a look at the parameter estimates for this model, as well as the spreadsheet.

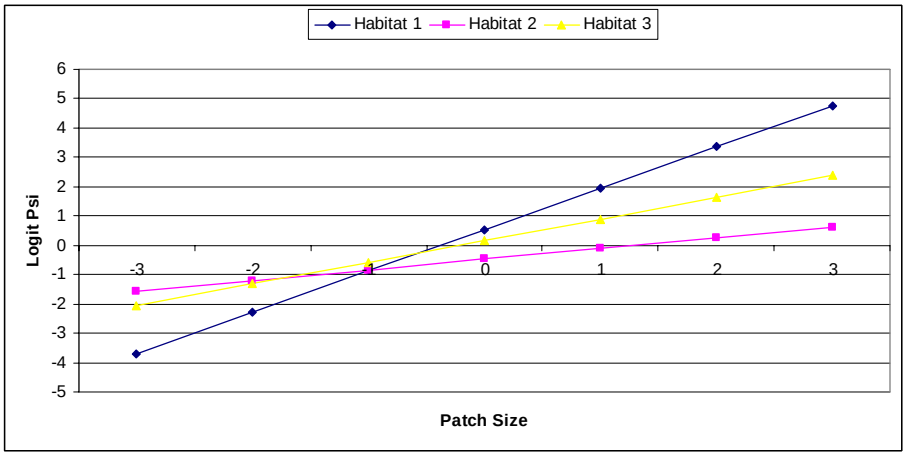
pres4841.tmp - Notepad

File Edit Format View Help

```
Custom Model:

Number of sites                 = 200
Number of sampling occasions    = 2
Number of missing observations  = 0

Number of parameters            = 7
-2log(likelihood)               = 424.8466
AIC                             = 438.8466

Model has been fit using the logistic link.

Naive estimate                  = 0.4500
Untransformed Estimates of coefficients for covariates (Beta's)
------------------------------------------------------------------------------
                                                          estimate   std.error
A1      :occupancy      psi                              0.159767 (0.283868)
A2      :occupancy      psiPatch Size                    0.738310 (0.288369)
A3      :occupancy      psiHabitat 1                     0.381497 (0.476545)
A4      :occupancy      psiHabitat 2                    -0.637876 (0.380352)
A5      :occupancy      psiH1*PS                         0.671564 (0.581300)
A6      :occupancy      psiH2*PS                        -0.372152 (0.392795)
B1      :detection      p1                               0.937813 (0.208857)
```

	E	F	G	H	I	J	K	L	M	N
3	**Summarized Inputs**					**Outputs**				
4	11	10	01	00	Total	Log_eL	$-2Log_eL$	K	AIC	AICc
5	50	18	22	110	200	-212.423	424.846	7	438.846	439.430
6						Model DF	C hat	P (MLE)	ψ (MLE)	
7						193	2.201	0.71865833	0.539855063	
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2	
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6	
10	Estimate?	1	1	1	0	1	1	1	1	
11	Beta	0.937816	0.159759	0.738235	0.000000	0.381478	-0.637870	0.671764	-0.372036	

We spent quite a bit of time explaining how to interpret the parameter estimates in the spreadsheet portion of the exercise, so we'll simply just provide the graph of the results here:



THE RESULTS BROWSER: MODEL SELECTION METHODS

Now that you've run all 5 models, let's look at the Results Browser more carefully.

Results Browser

Model	AIC	deltaAIC	AIC wgt	Model Likelihood	no.Par.	-2*LogLike
p(.)psi(patch size * habitat)	438.85	0.00	0.3368	1.0000	7	424.85
p(.)psi(patch size + habitat)	438.88	0.03	0.3318	0.9851	5	428.88
p(.)psi(patch size)	439.68	0.83	0.2224	0.6603	3	433.6785
p(.)psi(patch size + patch size 2)	441.11	2.26	0.1088	0.3230	4	433.1117
p(.)psi(habitat)	455.08	16.23	0.0001	0.0003	4	447.08

The results show the AIC, delta AIC, AIC weights, Model Likelihood, No. Parameters, and -2Log$_e$L for each model, and match the spreadsheet results shown in cells V3:AA10. The results are slightly different because Presence shows AIC

results, whereas the spreadsheet used AICc in the results table.

	V	W	X	Y	Z	AA
3	Model Selection Results Table					
4	Model	AICc	Rank	Delta	Exp(-0.5*Delta)	Weight
5	p(.)psi(patch size)	439.801	3	0.607	0.7382	0.2483
6	p(.)psi(patch size + patch size^2)	441.317	4	2.123	0.3459	0.1163
7	p(.)psi(habitat)	455.28361	5	16.089	0.0003	0.0001
8	p(.)psi(patch size + habitat)	439.194	1	0.000	1.0000	0.3363
9	p(.)psi(habitat*patch size)	439.430	2	0.235	0.8889	0.2990
10	Minimum AIC =	439.194		Sum =	2.9734	

PRESENCE conveniently sorts the model by the AIC score, showing the best fit model on top. These are the results you would show in a paper (in addition to the parameter estimates for each model). To export these results, right click somewhere in the Results Browser, and select Copy Results to Clipboard. Then you can paste them into a spreadsheet or word processing document.

As you can see, there is support for multiple models. At the moment, model averaging is not available in PRESENCE.

That wraps up the PRESENCE portion of the exercise.

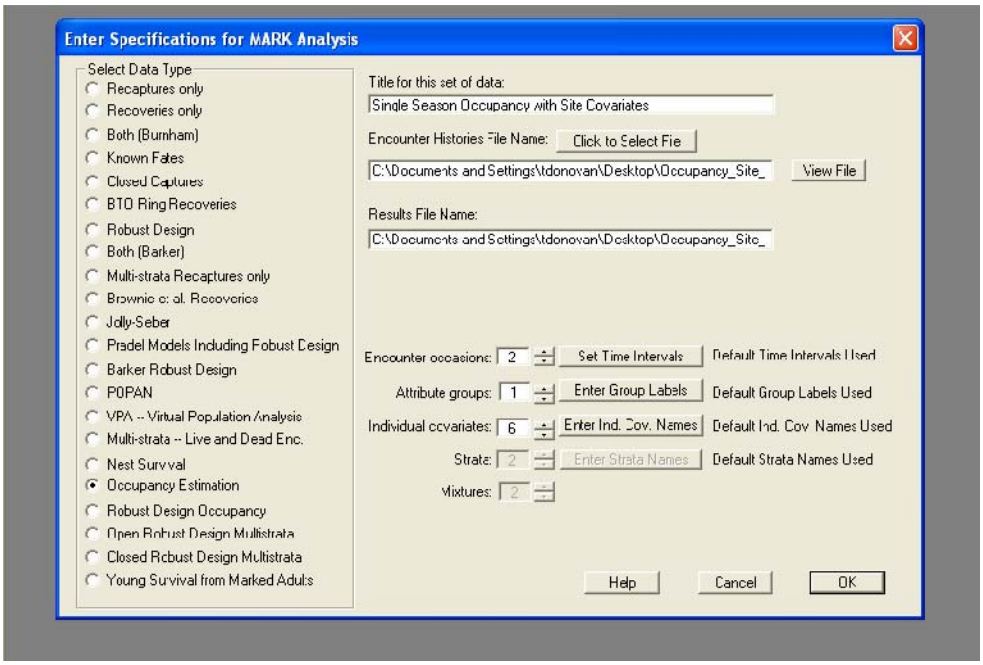
SINGLE-SPECIES, SINGLE SEASON OCCUPANCY WITH SITE COVARIATES IN PROGRAM MARK

MARK INPUT DATA

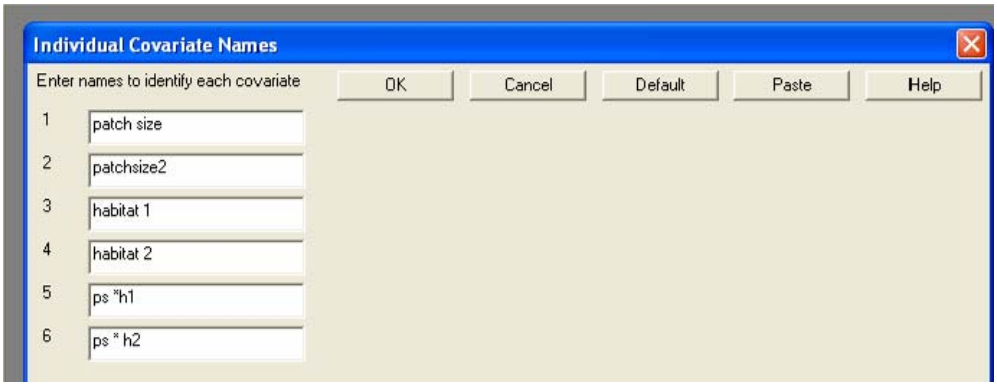
In this exercise, we will analyze the data in the spreadsheet Site Covariates in program MARK. Recall that the occupancy covariates included patch size, patch size^2, habitat (2 covariates), and two habitat*patch size covariates. Remember that this worksheet has only two sampling sessions. The input data are given in cells Y16:Y215. If you haven't already done so, copy these cells and paste them into NotePad. Then save the file (e.g., "Occupancy_Site_Covariate.inp") and include the quotes. This creates a file called Occupancy Covariate.inp, which you'll import into MARK. (This step is unnecessary if you saved a copy of the data as 'MARK' input in PRESENCE.)

GETTING STARTED

Open MARK and select File | New. Select the "Occupancy" radio button. Then upload your .inp file and name the analysis. In this case, there are 2 encounter occasions, and 6 individual covariates.



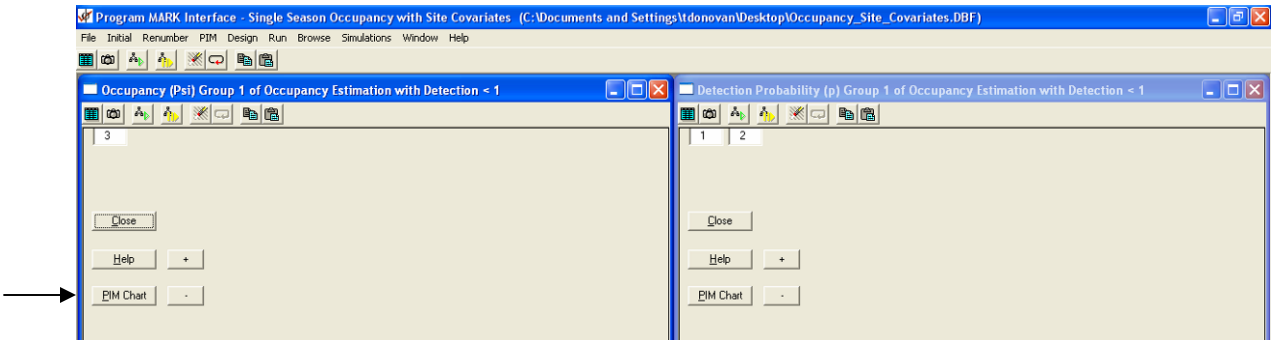
Click on the button labeled Enter Ind. Cov. Name, and enter the names of each covariate as they appear in order in the input file. (Note: MARK will remove the spaces in these names).



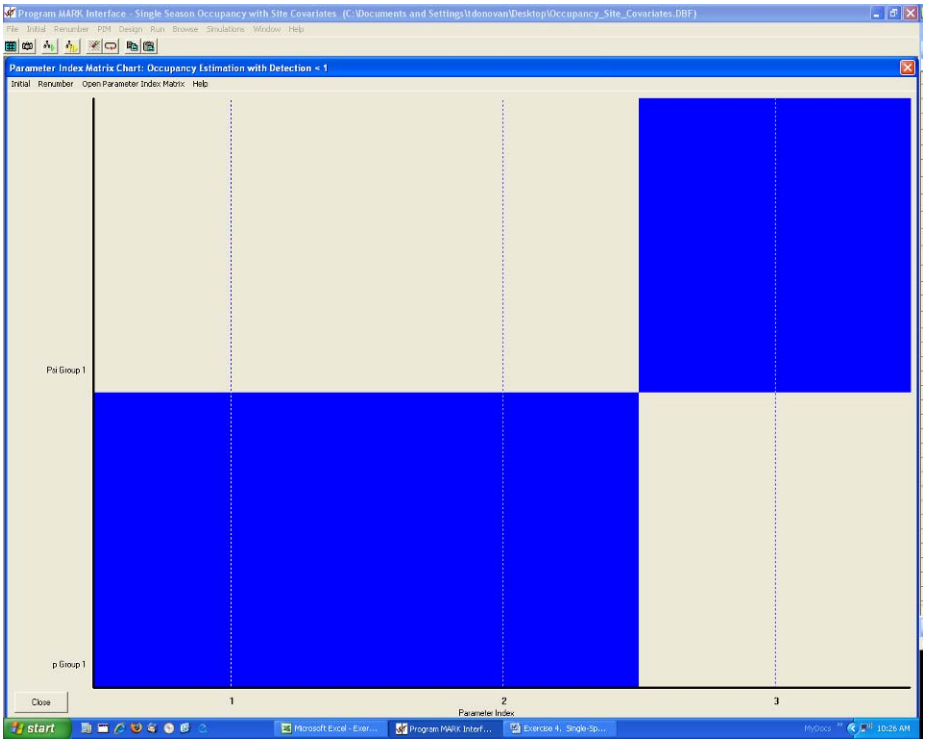
We'll use these names in the design matrix. If you have an analysis with a lot of covariates, it's useful to enter the covariate names in a spreadsheet, then copy the cells and select the Paste button shown in the figure above. This can save you a lot of time in the long run. Click OK. MARK will tell you that it is creating a DBF file to hold the results. Click OK.

MARK PIMS

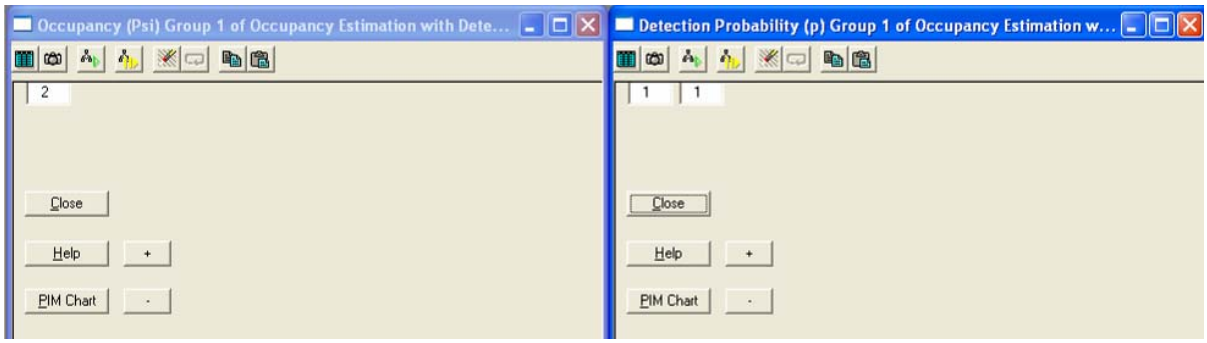
The detection probability PIM will appear. Open the occupancy PIM as well (go to PIM | Open Parameter Index Matrix, then select the Occupancy PIM. Then go to Window | Tile so that you can see both PIMs at once).

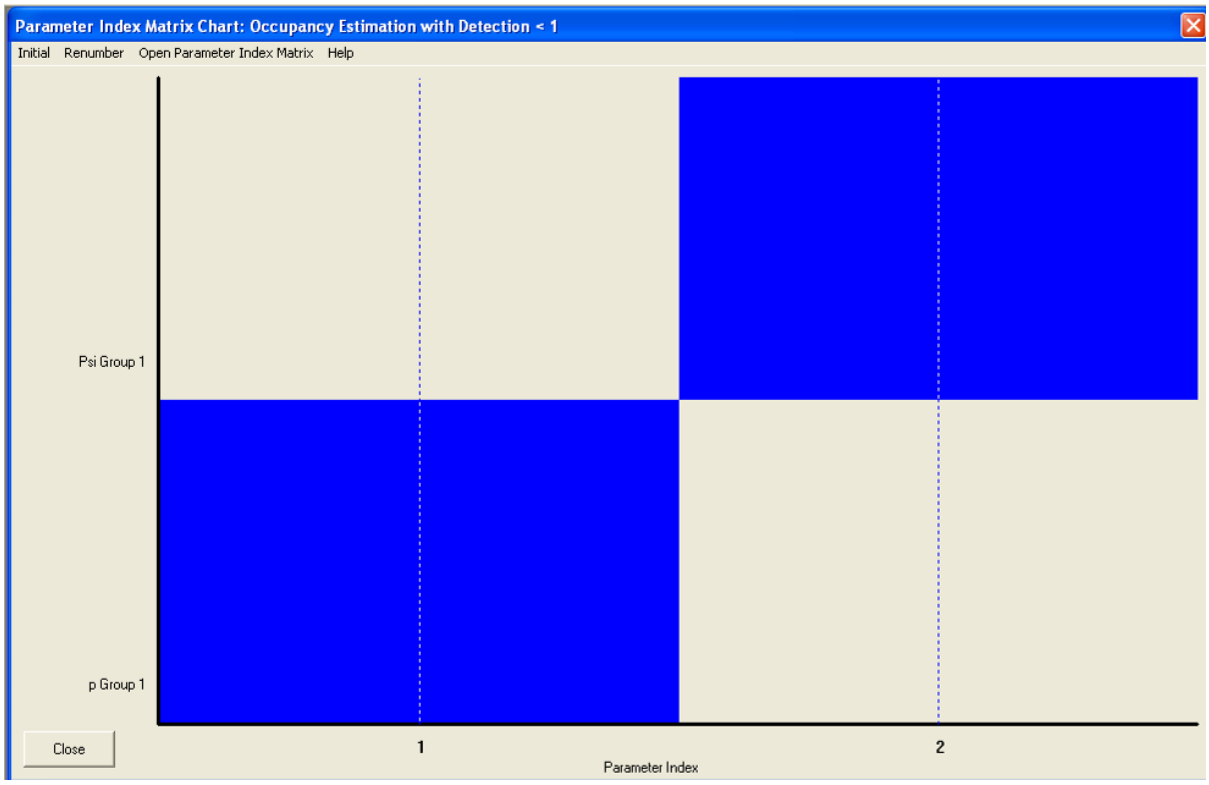


Notice that for this model, we are estimating 2 detection probabilities (p_1 and p_2, which are labeled 1 and 2), and one psi parameter (labeled 3). You can also look at this as a PIM chart by simply clicking on the button labeled PIM Chart.



So far, this looks pretty much like the analysis we did earlier. Note we haven't done anything with covariates yet, and also note that the PIMs and PIM chart indicate that this model will estimate 3 real parameters. In the spreadsheet exercise, however, we estimated two parameters only (p and ψ) by forcing $p_1 = p_2$. So, we need to first modify this model so that the PIMs reflect a model with only 2 parameters. Go ahead with these modifications, either by modifying the PIM or the PIM chart:



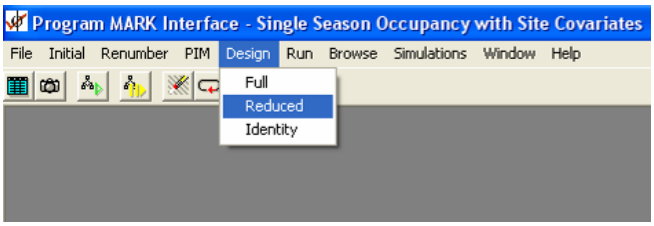


If we were to run this model right now, it would be called p(.)psi(.). Parameter 1 is the estimate the intercept for p, and parameter 2 is the estimate for the intercept for psi. But since this isn't part of our model set, we won't run it. However, it will be the basis for the models that we'll run.

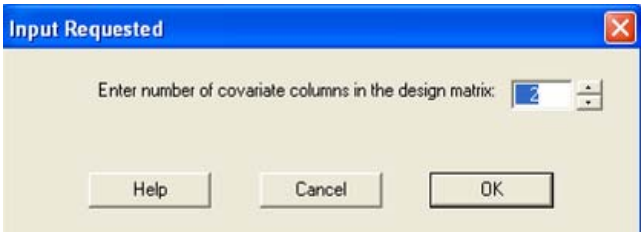
MODEL P(.)PSI(Patch Size)

The first model is model p(.)psi(patch size), where we'll force psi to be a function of patch size. Remember, we model linear equations (logits), and then will back-transform the logits to probabilities (ψ and p) with the logit link. We'll use the Design Matrix to constrain occupancy to be a function of the covariate patch size. Now, there are several ways to develop a Design Matrix…..we'll describe one approach, but you'll undoubtedly form your own preferences as you become familiar with MARK, and will pick up some short cuts along the way.

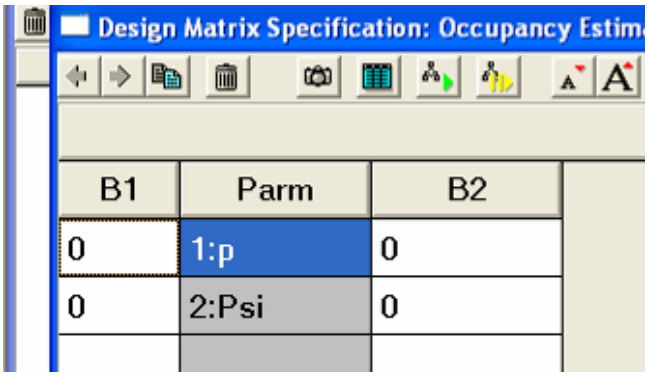
First, go to Design | Reduced.



Then enter 2 in the dialogue box, and press OK.

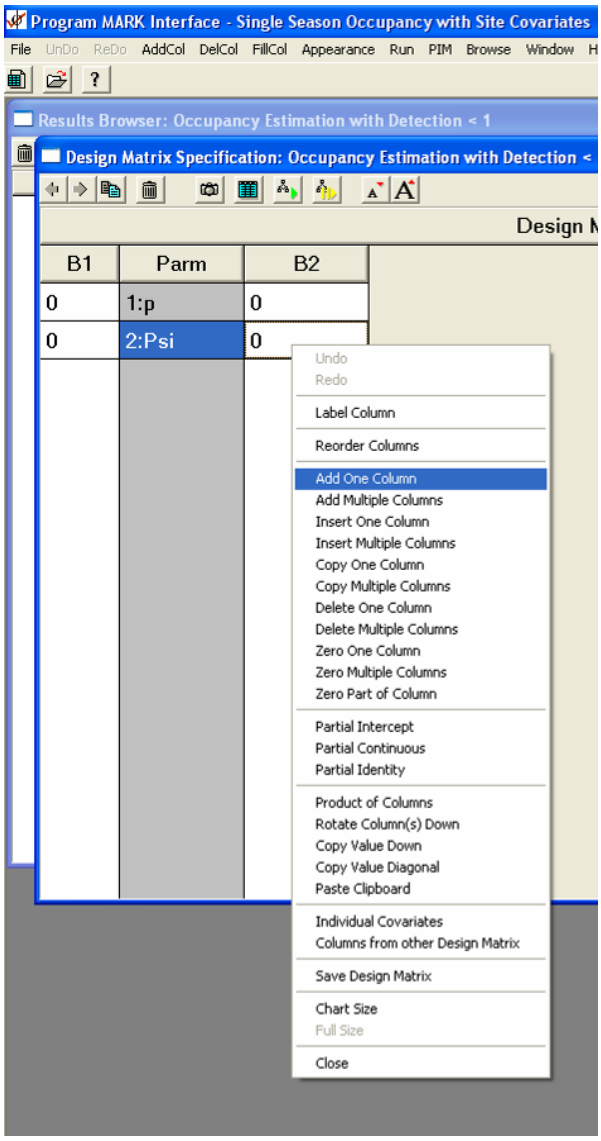


You'll then be presented with the following design matrix:

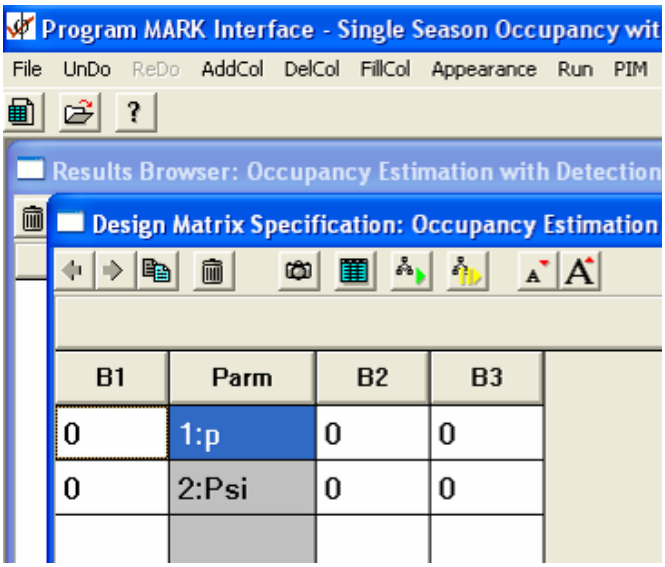


In the PIM, we established that p1 = p2, so this design matrix contains only two rows (row 1 is labeled 1:p and row 2 is labeled 2:Psi). We indicated that we wanted a design matrix with two columns, and these are labeled B1 and B2. Note the grey column labeled "Parm" identifies the parameters. So Parameter 1 refers to p, and is indicated by the first row, Parameter 2 refers to psi is indicated by the second row. Thus, the number of rows is established by the PIM, and is the number of real parameters you'll be estimating. Now, let's think for a moment about how many

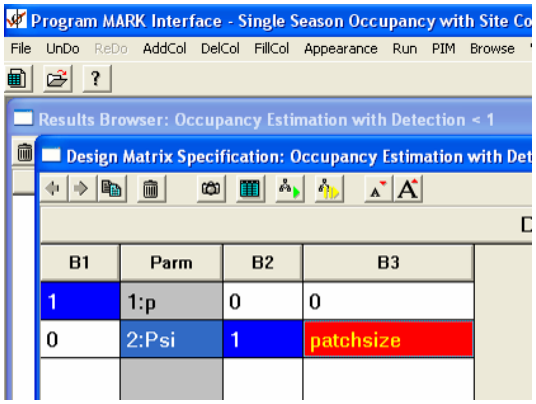
total parameters we need to estimate for model p(.)psi(patch size). The answer is three: the beta intercept for p, the beta intercept for ψ, and the beta for the effect of patch size. You can see that there are two columns in the Design Matrix (DM), so we need to add one more. To add more columns, click on any cell in the DM, and then right click to view a list of options.



Notice that this is an impressive list of options, but we'll use just a couple of these options in this exercise. Select the option labeled "Add One Column." You should now see a new DM:

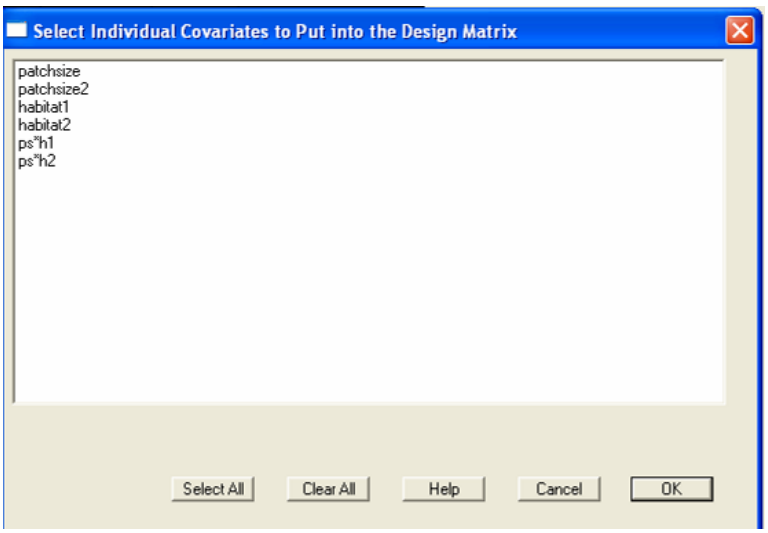


What now? Well, we need to simply specify the linear model for each parameter. That is, we'll need to specify the value for the intercept for p, and we'll need to specify the value for the intercept and value for patch size to modify psi. Here's where we're headed:



In this DM, we indicate in row 1 that p (or rather, logit p) is a function of its intercept (B1), and we indicate in row 2 that psi (or rather, logit psi) is a function of its intercept (B2) plus patch size (B3 * patch size). So, the number of rows in the DM specifies the number of real parameters, and the number of columns sets the "constraints" on how those parameters are estimated. The column headings indicate the beta number and the name of the effect. Don't let the assigned beta number throw you for a loop – MARK has to identify the betas in some way, and the number assigned is not important as long as you know what it refers to. So, how did we build this DM?

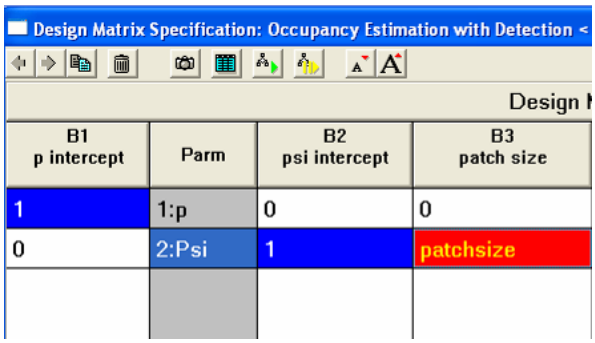
Well, to start, indicate to MARK that we need to estimate the intercept for p by entering a 1 in the top left cell. This corresponds to column F in the spreadsheet. Then, indicate to MARK that we need to estimate the intercept for psi by entering a 1 under B2 for the psi parameter. This corresponds to column G in the spreadsheet. Then, sticking with the row for psi, right-click on the cell at the intersection of the column labeled B3 and the row labeled psi, select the option labeled "Individual Covariates" and you should be presented with the following dialogue box:



Select the word "patchsize" and press OK. You could also just type in the name of the covariate you entered when you started the project, but using this option will ensure that you don't make typos.

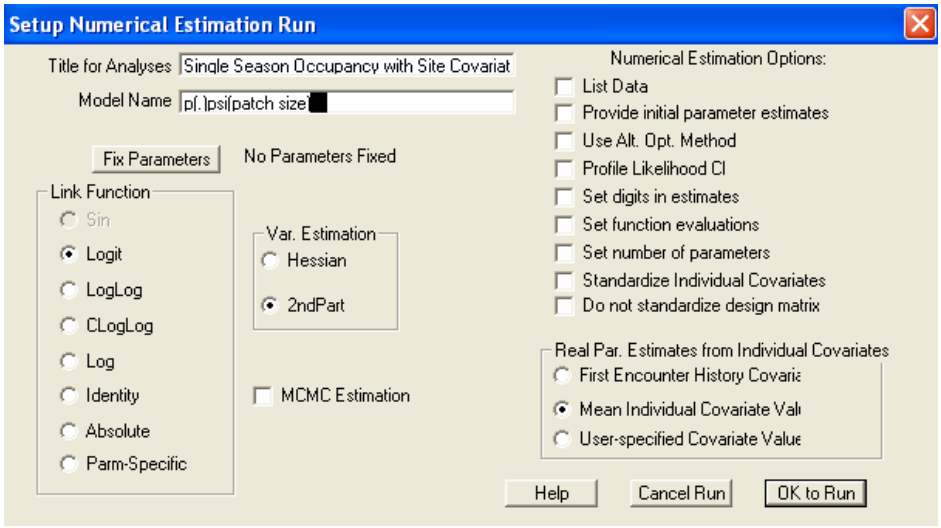
Now, it's VERY important that you label your columns. Go to Appearance | Label Columns, and enter the labels for each column:

There are several reasons for labeling your columns, the most important is that you know what B1, B2, and B3 refer to in the MARK output. As was the case for entering covariate names, you can enter the column headings in Excel and copy and paste the column headings in with the Paste button.

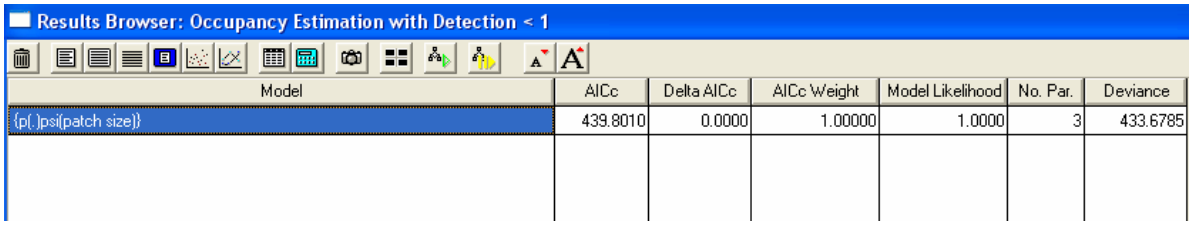


Go ahead and run this model, and call it p(.)psi(patch size). Note that the logit link is the default link now. In the lower right hand corner under Numerical Estimation

Options, select the option labeled "Mean Individual Covariate Value" for now. We'll try another option in a minute or two.



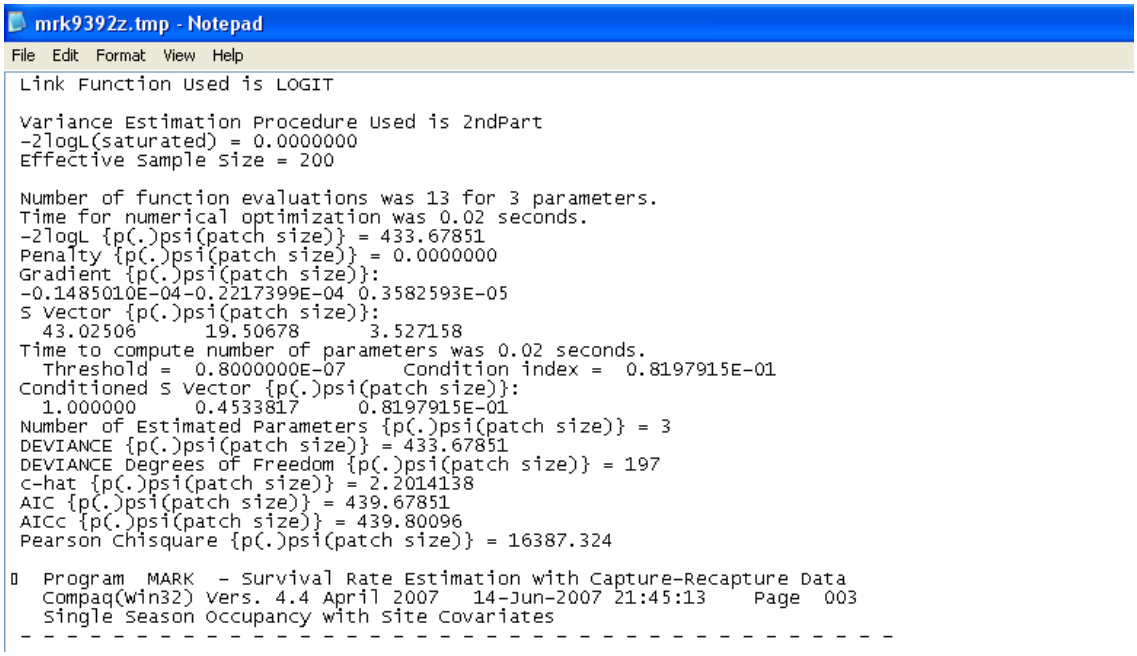
Add the results to the Results Browser.



Note that the model with the covariates has an AICc weight of 1 and that No. Par (or K) is 3. If your spreadsheet is open, you should see that the AICc from the spreadsheet matches the AICc output from MARK. Let's study this output in detail:

MODEL P(Date)PSI(Patch Size) OUTPUT

First, click on the button to the right of the trash can to view the full output:



mrk9392z.tmp - Notepad

File Edit Format View Help

```
 Link Function Used is LOGIT

 Variance Estimation Procedure Used is 2ndPart
 -2logL(saturated) = 0.0000000
 Effective Sample Size = 200

 Number of function evaluations was 13 for 3 parameters.
 Time for numerical optimization was 0.02 seconds.
 -2logL {p(.)psi(patch size)} = 433.67851
 Penalty {p(.)psi(patch size)} = 0.0000000
 Gradient {p(.)psi(patch size)}:
 -0.1485010E-04-0.2217399E-04 0.3582593E-05
 S Vector {p(.)psi(patch size)}:
   43.02506      19.50678      3.527158
 Time to compute number of parameters was 0.02 seconds.
   Threshold =  0.8000000E-07     Condition index =  0.8197915E-01
 Conditioned S Vector {p(.)psi(patch size)}:
   1.000000     0.4533817     0.8197915E-01
 Number of Estimated Parameters {p(.)psi(patch size)} = 3
 DEVIANCE {p(.)psi(patch size)} = 433.67851
 DEVIANCE Degrees of Freedom {p(.)psi(patch size)} = 197
 c-hat {p(.)psi(patch size)} = 2.2014138
 AIC {p(.)psi(patch size)} = 439.67851
 AICc {p(.)psi(patch size)} = 439.80096
 Pearson Chisquare {p(.)psi(patch size)} = 16387.324

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```

Here are the spreadsheet results for comparison:

	J	K	L	M	N
3	**Outputs**				
4	Log_eL	$-2Log_eL$	K	AIC	AICc
5	-216.839	433.679	3	439.679	439.801
6	Model DF	C hat	P (MLE)	ψ (MLE)	
7	197	2.201	0.713798986	0.506352058	

The effective sample size is 200, which is the number of sites (cell I5 in the spreadsheet). The $-2Log_eL$, AIC, and AICc match. In a covariate model, Deviance is the same as the $-2Log_eL$, and so Deviance DF and c-hat match the spreadsheet output. The Pearson Chi-Square was not computed on the spreadsheet because we computed the MacKenzie-Bailey Chi-Square instead.

Now let's study MARK's beta and parameter estimates:

LOGIT Link Function Parameters of {p(.)psi(patch size)}

Parameter	Beta	Standard Error	95% Confidence Interval Lower	Upper
1:p intercept	0.9139073	0.2123614	0.4976790	1.3301356
2:psi intercept	0.0254072	0.1771525	-0.3218117	0.3726261
3:patch size	0.7516468	0.1835111	0.3919651	1.1113285

	F	G	H	I	J	K	L	M
8	B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2
9	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6
10	1	1	1	0	0	0	0	0
11	0.913907	0.025410	0.751644	0.000000	0.000000	0.000000	0.000000	0.000000

You should see that the beta estimates from MARK match the spreadsheet. The beta values and their confidence limits should be studied, because the confidence limits will help you interpret your results. Was there an effect of of patch size on ψ? The MARK output indicates that the beta for patch size was positive (0.752), with confidence intervals from 0.3919651 to 1.1113285. These confidence intervals don't overlap 0 at all, indicating that this is a "significant" result. The question "how significant?" is best answered by plotting the standardized patch size against psi to understand the biological implications of the result, as we did in the spreadsheet exercise, and not by reporting a statistical p value such as (p = 0.0034).

Now let's look at the "real" parameters reported in MARK:

```
                  Real Function Parameters of {p(.)psi(patch size)}

     Following estimates based on unstandardized individual covariate values:
         Variable   Value
         ---------  -------------
         PATCHSIZE  -0.0966970
         PATCHSIZE2 1.0354810
         HABITAT1   0.2550000
         HABITAT24  0.3850000
         PS*H1      0.0131640
         PS*H2      -0.0894735
                                                          95% Confidence Interval
 Parameter                 Estimate       Standard Error      Lower           Upper
 ------------------------  -------------  --------------  --------------  --------------
    1:p                    0.7137990       0.0433833       0.6219137       0.7908631
    2:Psi                  0.4881835       0.0437225       0.4036569       0.5733913

     Time in seconds for last procedure was 0.03

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```
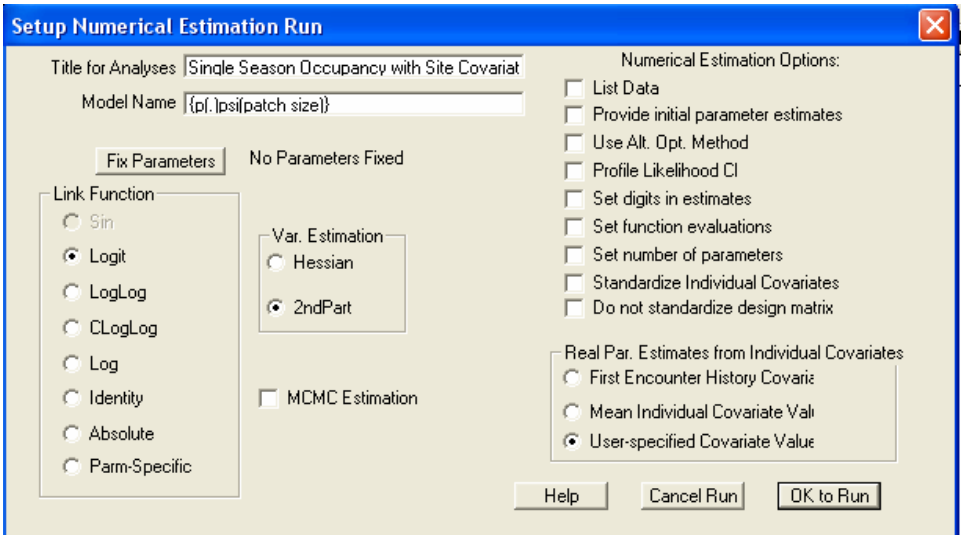
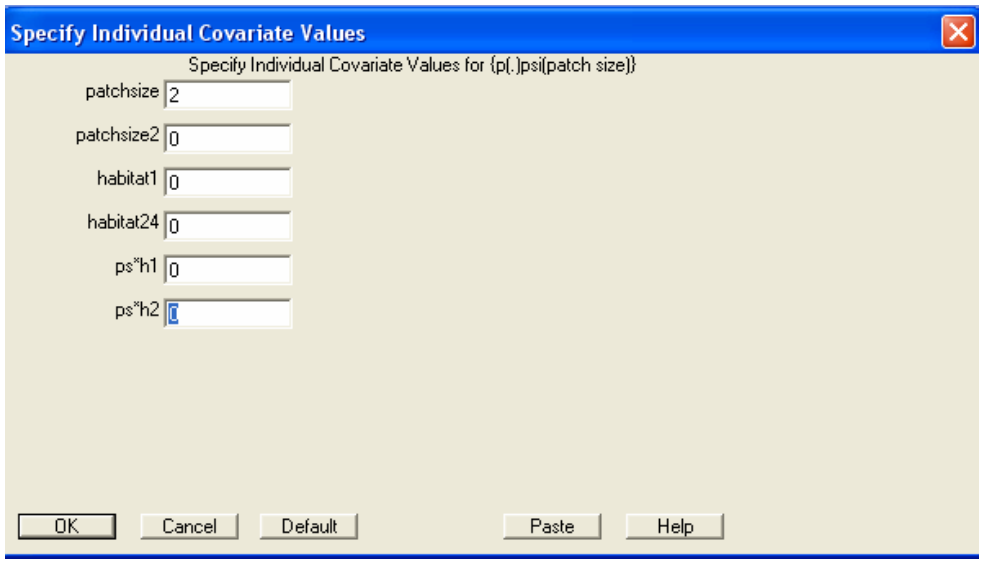

Note that p is estimated as 0.7137990, and psi is estimated as 0.4881835. What do these values come from? Well, because we clicked on the option labeled "Mean Individual Covariate Values" in the model input box in MARK, this p and ψ pertain to a site whose Patch Size Z score was -0.0966970, as shown in the MARK output (this model did not consider covariate values for patch size^2, habitat or the interactions).

If we wanted MARK to output a specific p and ψ for a particular site, we could choose the option labeled "User-specified Covariate Values", and run the model again:



In this case, you need to enter the values for a site of interest. For example, if you wanted the p and ψ estimates for a site with a standardized patch size = 2, you'd enter the following data and run the model:



MARK will output the same results, with the exception that now the output under "real parameters" reflects the conditions at this particular site:

Real Function Parameters of {p(.)psi(patch size)}

Following estimates based on user-specified individual covariate values not standardized:

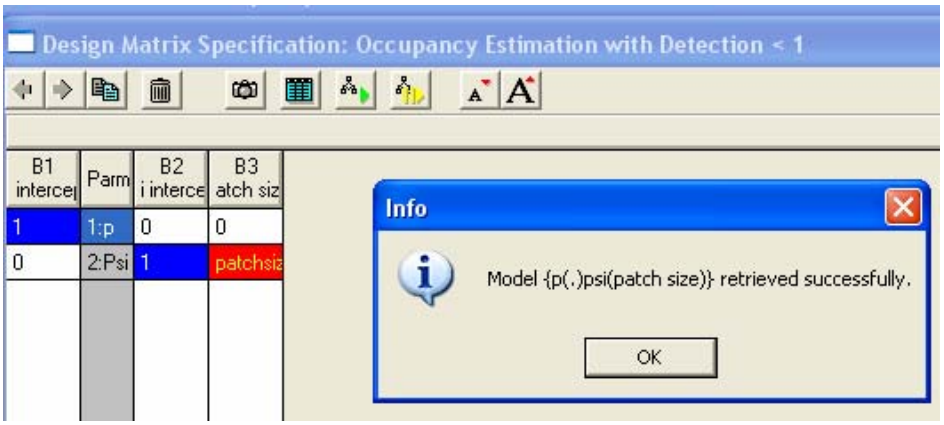
Variable	Value
PATCHSIZE	2.0000000
PATCHSIZE2	0.0000000
HABITAT1	0.0000000
HABITAT24	0.0000000
PS*H1	0.0000000
PS*H2	0.0000000

Parameter	Estimate	Standard Error	95% Confidence Interval Lower	Upper
1:p	0.7137990	0.0433833	0.6219137	0.7908631
2:Psi	0.8218161	0.0635545	0.6632998	0.9152404

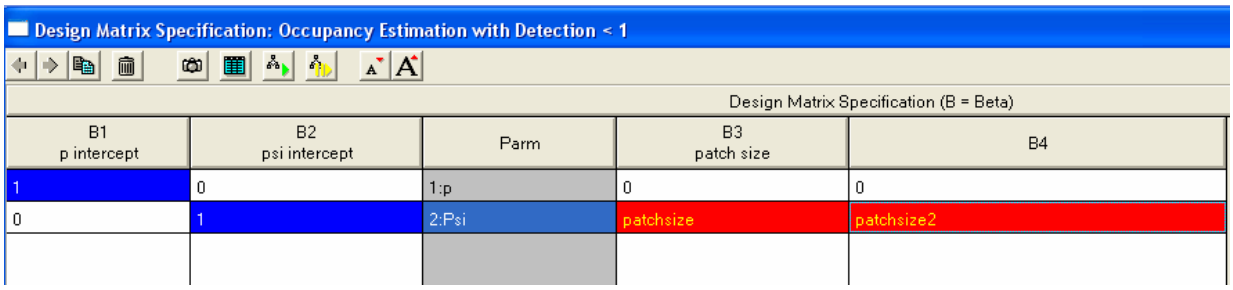
The spreadsheet provides you with these estimates on a site by site basis, and it would be nice if MARK could output the results for all sites as well. (I'm not sure how to do that). So, to recap, the betas are the most critical output in a covariate model because they allow you to interpret the effect size (the beta value itself) and its precision (the standard errors and confidence intervals). The real parameters in the MARK output pertain to conditions at a given site only, or pertain to conditions at the average site, depending on what option you selected.

MODEL P(.) PSI (Patch Size ^{2})

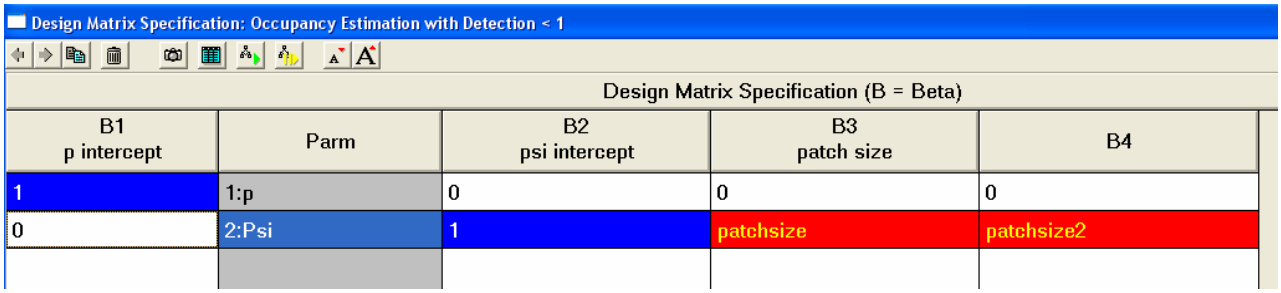
Our next model is model p(.)psi(patch size + patch size^2), or the polynomial model. This model is the same as the last model, but we add the additional parameter for patch size squared. As such, we can build this new model by using the patch size model as the basis. First, click on the model named p(.)psi(patch size) in the results browser in MARK. Then, click on the button with the four black squares on it to retrieve the design matrix for that model, and you should see the following:



Now, select the cell labeled patch size, and then right click to see a list of options, and choose Add One Colume. Then, right click on the new cell in the psi row, select the Individual Covariates option, and then choose the covariate named patchsize2. Your design matrix should now look like this :



If you prefer, you can move the columns around so that the label separates the p equation from the psi equation, so it's easier to see that logit equation for psi is Logit psi = B2 * 1 + B3 * patch size * + B4 * patch size2



That's all there is to it. Run this model by clicking on the button with the small green arrow, and add the results to the results browser. Don't forget to name the model p(.)psi(patch size + patch size2) :

Results Browser: Occupancy Estimation with Detection < 1

Model	AICc	Delta AICc	AICc Weight	Model Likelihood	No. Par.	Deviance
{p(.)psi(patch size)}	439.8010	0.0000	0.68091	1.0000	3	433.6785
{p(.)psi(patch size + patch size2)}	441.3169	1.5159	0.31909	0.4686	4	433.1117

Examine the output of this model by selecting the model name in the results browser, and then select the button to the right of the garbabe can :

mrk7632z.tmp - Notepad

File Edit Format View Help

```
Time for numerical optimization was 0.01 seconds.
-2logL {p(.)psi(patch size + patch size2)} = 433.11174
Penalty {p(.)psi(patch size + patch size2)} = 0.0000000
Gradient {p(.)psi(patch size + patch size2)}:
-0.2968615E-05 0.3057378E-04-0.3650606E-05 0.1176625E-04
S Vector {p(.)psi(patch size + patch size2)}:
  43.28597      19.41515      3.583626      0.6332733
Time to compute number of parameters was 0.02 seconds.
  Threshold =  0.1000000E-06     Condition Index =  0.1462999E-01
Conditioned S Vector {p(.)psi(patch size + patch size2)}:
  1.000000      0.4485323     0.8278955E-01 0.1462999E-01
Number of Estimated Parameters {p(.)psi(patch size + patch size2)} = 4
DEVIANCE {p(.)psi(patch size + patch size2)} = 433.11174
DEVIANCE Degrees of Freedom {p(.)psi(patch size + patch size2)} = 196
c-hat {p(.)psi(patch size + patch size2)} = 2.2097538
AIC {p(.)psi(patch size + patch size2)} = 441.11174
AICc {p(.)psi(patch size + patch size2)} = 441.31687
Pearson Chisquare {p(.)psi(patch size + patch size2)} = 16387.991

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            LOGIT Link Function Parameters of {p(.)psi(patch size + patch size2)}
                                                          95% Confidence Interval
Parameter                  Beta         Standard Error      Lower           Upper
-------------------------  -----------  -----------  -----------  -----------
   1:p intercept           0.9148127      0.2123067      0.4986916      1.3309338
   2:psi intercept         0.1155327      0.2151893     -0.3062383      0.5373037
   3:patch size            0.7311997      0.1827679      0.3729746      1.0894248
   4:                     -0.1115339      0.1495027     -0.4045592      0.1814915
```
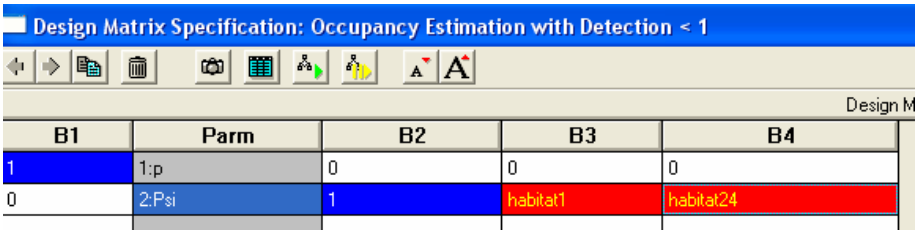

The results are consistent with the spreadsheet results from this model :

	E	F	G	H	I	J	K	L	M	N
3	Summarized Inputs					Outputs				
4	11	10	01	00	Total	Log_eL	$-2Log_eL$	K	AIC	AICc
5	50	18	22	110	200	-216.556	433.112	4	441.112	441.317
6						Model DF	C hat	P (MLE)	ψ (MLE)	
7						196	2.210	0.713983847	0.528851751	
8		B_0	B_{00}	Patch Size	Patch Size^2	Habitat 1	Habitat 2	PS*Hab1	PS*Hab2	
9	Parameter	P (Int)	Psi (Int)	Cov 1	Cov 2	Cov 3	Cov 4	Cov 5	Cov 6	
10	Estimate?	1	1	1	1	0	0	0	0	
11	Beta	0.914812	0.115535	0.731197	-0.111534	0.000000	0.000000	0.000000	0.000000	

MODEL P(.) PSI(Habitat)

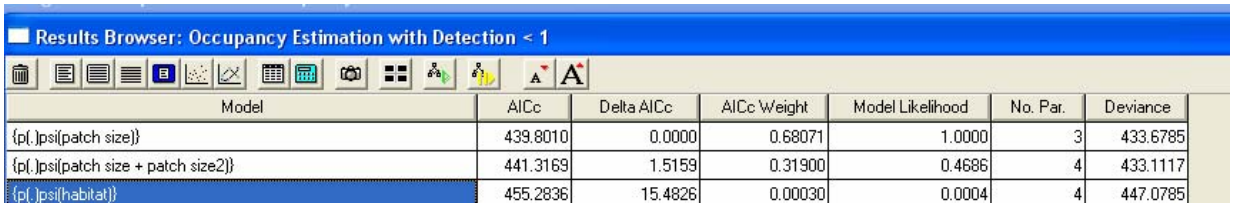
Now, let's run model p(.)psi(habitat). Start by setting up the Design Matrix with just two columns (the dot model). Go to Design | Reduced, and enter 2 for number of betas. Now, think about how many additional parameters you need to estimate to constrain psi to be a function of habitat. With three habitats, you must estimate 2 additional parameters. So right click somewhere in the design matrix and add two columns. Then, right click on the new cells in the psi row, select the individual covariates option, and add the name of the covariates to build the model: Your design matrix should look like this (except that you probably spelled habitat2 correctly!)



Let's think about what this DM specifies. First, there are two rows: the top row is where you constrain p to be a function of its intercept. The second row is where you constrain ψ to be a function of its intercept, habitat1, and habitat2. The intercept is B2, and pertains to sites in the reference category. Remember that

habitat is categorical, and each site is coded as either 1 0 (habitat 1), 0 1 (habitat 2) or 0 0 (habitat 3, the reference category). So, if a site is located in habitat 1, B3 is multiplied by 1 and B4 is multiplied by 0. And if a site is located in habitat 2, B3 is multiplied by 0 and B4 is multiplied by 1.

Go ahead and run this model, and add the results to the Results Browser, and don't forget to name the model p(.)psi(habitat):



Model	AICc	Delta AICc	AICc Weight	Model Likelihood	No. Par.	Deviance
{p(.)psi(patch size)}	439.8010	0.0000	0.68071	1.0000	3	433.6785
{p(.)psi(patch size + patch size2)}	441.3169	1.5159	0.31900	0.4686	4	433.1117
{p(.)psi(habitat)}	455.2836	15.4826	0.00030	0.0004	4	447.0785

If you ran the model in the spreadsheet, you should see that the spreadsheet betas and output match MARK.

	J	K	L	M	N
3	**Outputs**				
4	Log_eL	$-2Log_eL$	K	AIC	AICc
5	-223.539	447.078	4	455.078	455.284
6	Model DF	C hat	P (MLE)	ψ (MLE)	
7	196	2.281	0.714285626	0.529321065	

Take some time now to go through the MARK output and compare each result with the spreadsheet. Make sure you understand what MARK is reporting in the section labeled Real Parameters.

MODEL P(.)PSI(Patch Size + Habitat)

OK, two more models to go in the model set, and then we can study the Results Browser output. Set up the DM for this model, run it, and add the results to the Results Browser.

Design Matrix Specification: Occupancy Estimation with Detection < 1

Design Matrix Specification (B = Beta)

B1 p intercept	Parm	B2 psi intercept	B3 patch size	B4 habitat 1	B5 habitat 2
1	1:p	0	0	0	0
0	2:Psi	1	patchsize	habitat1	habitat24

Results Browser: Occupancy Estimation with Detection < 1

Model	AICc	Delta AICc	AICc Weight	Model Likelihood	No. Par.	Deviance
{p(.)psi(patch size + habitat)}	439.1938	0.0000	0.47976	1.0000	5	428.8845
{p(.)psi(patch size)}	439.8010	0.6072	0.35413	0.7381	3	433.6785
{p(.)psi(patch size + patch size2)}	441.3169	2.1231	0.16596	0.3459	4	433.1117
{p(.)psi(habitat)}	455.2836	16.0898	0.00015	0.0003	4	447.0785

Take a good look at the MARK output and compare your results with the spreadsheet output for this model.

MODEL P(.)PSI(Patch Size * Habitat)

OK! Our last model is the interaction model, where we estimate a unique intercept and a unique patch size slope for each of the three habitat types. See if you can set up the model and run it. Our design matrix looked like this:

Design Matrix Specification: Occupancy Estimation with Detection < 1

Design Matrix Specification (B = Beta)

B1 p intercept	Parm	B2 psi intercept	B3 patch size	B4 habitat 1	B5 habitat 2	B6 h1 * ps	B7 h2 * ps
1	1:p	0	0	0	0	0	0
0	2:Psi	1	patchsize	habitat1	habitat24	ps*h1	ps*h2

Results Browser: Occupancy Estimation with Detection < 1

Model	AICc	Delta AICc	AICc Weight	Model Likelihood	No. Par.	Deviance
{p(.)psi(patch size + habitat)}	439.1938	0.0000	0.33631	1.0000	5	428.8845
{p(.)psi(patch size * habitat)}	439.4291	0.2353	0.29899	0.8890	7	424.8458
{p(.)psi(patch size)}	439.8010	0.6072	0.24825	0.7381	3	433.6785
{p(.)psi(patch size + patch size2)}	441.3169	2.1231	0.11634	0.3459	4	433.1117
{p(.)psi(habitat)}	455.2836	16.0898	0.00011	0.0003	4	447.0785

When you are finished, let's now turn our attention to the Results Browser and the spreadsheet Model Selection table.

	V	W	X	Y	Z	AA
3	Model Selection Results Table					
4	Model	AICc	Rank	Delta	Exp(-0.5*Delta)	Weight
5	p(.)psi(patch size)	439.801	3	0.607	0.7382	0.2483
6	p(.)psi(patch size + patch size^2)	441.317	4	2.123	0.3459	0.1163
7	p(.)psi(habitat)	455.28361	5	16.089	0.0003	0.0001
8	p(.)psi(patch size + habitat)	439.194	1	0.000	1.0000	0.3363
9	p(.)psi(habitat*patch size)	439.430	2	0.235	0.8889	0.2990
10	Minimum AIC =	439.194		Sum =	2.9734	

Note that the AICc, Delta AICc, and AIC weights match. MARK conveniently sorts the model output by AIC (though you can alter how MARK orders the output by going to Order | option). Although we didn't record each model's K and -2Log$_e$L on the spreadsheet table, the results indeed match MARK. MARK includes a column called Model Likelihood, which we did not compute on the spreadsheet. What is this? Well, these are simply the ratios a model's AIC weight to the AIC weight of the best model. For instance, the second best model had an AICc weight of 0.2989, and the top model had an AICc weight of 0.3363. The likelihood of the second best model is 0.2989/0.3363 = 0.8890. The best model will always have a likelihood of 1 because its model weight is divided by itself.

MODEL AVERAGING

In the spreadsheet exercise, we briefly mentioned how to obtain model averaged parameter estimates for each site. The method that makes the most sense for models with individual covariates is to run a model, obtain a p and ψ estimate for each site, and then multiply the site-specific p and ψ by the AICc weight for that model. Then repeat this process for the remaining models in the model set, and add up the weighted p's and ψ's on a site-by-site basis. This would be very easy to do in the spreadsheet (and we probably should have added it to the exercise). I'm not sure how to do this in MARK, although model averaging is programmed and can be accessed by going to Output | Model Averaging.

CLOSING REMARKS

That's it for the covariate exercise and the introduction to the Design Matrix in MARK. In the words of Obi-Wan Kenobi, "you've taken your first steps into a larger world." Hopefully you now understand how MARK is generating parameter estimates when covariates are used to constrain a parameter. The Design Matrix takes time to learn, but it is well worth the time because you can constrain ANY parameter to be a linear combination of different variables. The chapter on Linear Constraints in MARK: A Gentle Introduction has an excellent discussion of how to implement various kinds of models in the Design Matrix.