Chapter 5 - Basic Concepts of Probability

5.1 **a.** Analytic: If two tennis players are exactly equally skillful so that the outcome of their match is random, the probability is .50 that Player A will win the upcoming match.

b. Relative frequency: If in past matches Player A has beaten Player B on 13 of the 17 occasions on which they played, then Player A has a probability of 13/17 = .76 of winning their upcoming match, all other things held constant.

c. Subjective: Player A's coach feels that he has a probability of .90 of winning his upcoming match with Player B.

5.3 **a.** *p*(that you will win 2nd prize given that you don't win 1st) = 1/9 = .111

b. *p*(that he will win 1st and you 2nd) = (2/10)(1/9) = (.20)(.111) = .022

c. *p*(that you will win 1st and he 2nd) = (1/10)(2/9) = (.10)(.22) = .022

d. *p*(that you are 1st and he 2nd [= .022]) + *p*(that he is 1st and you 2nd [= .022]) = *p*(that you and he will be 1st and 2nd) = .044

5.5 Conditional probabilities were involved in Exercise 5.3a.

5.7 Conditional probabilities: What is the probability that skiing conditions will be good on Wednesday, *given* that they are good today?

5.9 *p*(that they will look at each other at the same time during waking hours) = *p*(that mother looks at baby during waking hours) * *p*(that baby looks at mother during waking hours) = (2/13)(3/13) = (.154)(.231) = .036

5.11 A continuous distribution for which we care about the probability of an observation's falling within some specified interval is exemplified by the probability that your baby will be born on its due date.

5.13 Two examples of discrete variables: Variety of meat served at dinner tonight; Brand of desktop computer owned.

5.15 **a.** 20%, or 60 applicants, will fall at or above the 80th percentile and 10 of these will be chosen. Therefore *p*(that an applicant with the highest rating will be admitted) = 10/60 = .167.

b. No one below the 80th percentile will be admitted, therefore *p*(that an applicant with the lowest rating will be admitted) = 0/300 = .00.

5.17 Mean ADDSC score for boys = 54.29, s = 12.90 [Calculated from Data Set]

a. $z = \frac{50 - 54.29}{12.90} = -.33$

Since a score of 50 is below the mean, and since we are looking for the probability of a score *greater than* 50, we want to look in the tables of the normal distribution in the column labeled "larger portion".

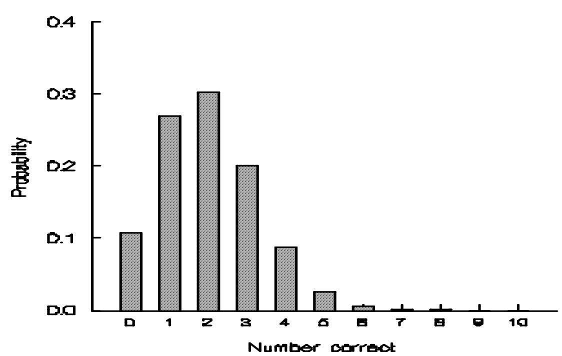
p(larger portion) = .6293

b. 29/55 = 53% > 50; 32/55 = 58% $\geq$ 50. (Notice that one percentage refers to the proportion *greater than* 50, while the other refers to the proportion *greater than or equal to* 50.)

5.19 Compare the probability of dropping out of school, ignoring the ADDSC score, with the conditional probability of dropping out given that ADDSC in elementary school exceeded some value (e.g., 66).

5.21 Plot of correct choices on trial 1 of a 5-choice task:

$p(0)$	= .1074
$p(1)$	= .2684
$p(2)$	= .3020
$p(3)$	= .2013
$p(4)$	= .0881
$p(5)$	= .0264
$p(6)$	= .0055
$p(7)$	= .0008
$p(8)$	= .0001
$p(9)$	= .0000
$p(10)$	= .0000



5.23 p(5 or more correct) $= p(5) + p(6) + p(7) + p(8) + p(9) + p(10)$
$= .0264 + .0055 + .0008 + .0001 + .0000 + .0000$
$= .028 < .05$

5.25 Probability statements about the treatment of automobile shoppers:

Simple probability:	The probability that the salesperson will make a condescending remark is .15.
Joint probability:	The probability that the salesperson will make a condescending remark and that the customer is a woman is .10.
Conditional probability:	The probability that the salesperson will make a condescending remark given that the customer is a woman is .25.

5.27 Number of subjects needed in verbal learning experiment if each is to see different classes of words in a different order:

$$P_4^4 = \frac{4!}{(4-4)!} = 24$$

5.29 Chance that subject will press correctly on first trial when learning to press three out of five buttons in a certain order:

$$P_3^5 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60$$

There are 60 possible orders to push 3 out of 5 buttons. The probability that the subject will choose the correct order on the first trial = p(1/60) = 0.017

5.31 Different ways to record from the rat's brain:

$$C_4^6 = \frac{6!}{4!(6-4)!} = \frac{6!}{4!2!} = 15$$

5.33 Probability of 16 caramels out of 60 draws sampling with replacement.

$$p(16) = C_{16}^{60} \times 15^{16} \times 85^{44}$$

$$= \frac{60!}{16!44!} \times 15^{16} \times 85^{44}$$

$$= (1.496083758 \times 10^{14})(6.568408356 \times 10^{-14})\left(7.842106418 \times 10^{-04}\right)$$

$$= .0077$$

5.35 On the theory that practice in almost anything leads to improvement, we give a sample of first year college students, who will major in the humanities (where there is a lot of reading assigned), a test for reading speed at the beginning of the fall semester. At the end of the year we again measure their reading speed. We wish to test the null hypothesis that reading speed, on average (or for most people) increased over the year.

5.37 Probability of breast cancer

$$p(H \mid D) = \frac{p(D \mid H)p(H)}{p(D \mid H)p(H) + p(D \mid \overline{H})p(\overline{H})}$$

$p(BC) = .01$

$p(+ \mid BC) = .80$

$p(+ \mid \overline{BC}) = .096$

$$= \frac{(.80)(.01)}{(.80)(.01) + (.096)(.99)}$$

$$= \frac{.008}{.008 + .095} = \frac{.008}{.103} = .078$$

5.39 Reducing the rate of false positives:

Here we can use the same calculations, but just change .096 to .05.

$$p(H \mid D) = \frac{p(D \mid H)p(H)}{p(D \mid H)p(H) + p(D \mid \overline{H})p(\overline{H})}$$

$p(BC) = .01$

$p(+ \mid BC) = .80$

$p(+ \mid \overline{BC}) = .05$

$$= \frac{(.80)(.01)}{(.80)(.01) + (.05)(.99)}$$

$$= \frac{.008}{.008 + .0495} = \frac{.008}{.103} = .139$$

The probability has nearly doubled when we nearly halved our false positive rate.