

Chapter 5 - Basic Concepts of Probability

- 5.1**
- a.** Analytic: If two tennis players are exactly equally skillful so that the outcome of their match is random, the probability is .50 that Player A will win the upcoming match.
 - b.** Relative frequency: If in past matches Player A has beaten Player B on 13 of the 17 occasions on which they played, then Player A has a probability of $13/17 = .76$ of winning their upcoming match, all other things held constant.
 - c.** Subjective: Player A's coach feels that he has a probability of .90 of winning his upcoming match with Player B.
- 5.3**
- a.** $p(\text{that you will win 2nd prize given that you don't win 1st}) = 1/9 = .111$
 - b.** $p(\text{that he will win 1st and you 2nd}) = (2/10)(1/9) = (.20)(.111) = .022$
 - c.** $p(\text{that you will win 1st and he 2nd}) = (1/10)(2/9) = (.10)(.22) = .022$
 - d.** $p(\text{that you are 1st and he 2nd [= .022]}) + p(\text{that he is 1st and you 2nd [= .022]}) = p(\text{that you and he will be 1st and 2nd}) = .044$
- 5.5** Conditional probabilities were involved in Exercise 5.3a.
- 5.7** Conditional probabilities: What is the probability that skiing conditions will be good on Wednesday, *given* that they are good today?
- 5.9** $p(\text{that they will look at each other at the same time during waking hours}) = p(\text{that mother looks at baby during waking hours}) * p(\text{that baby looks at mother during waking hours}) = (2/13)(3/13) = (.154)(.231) = .036$
- 5.11** A continuous distribution for which we care about the probability of an observation's falling within some specified interval is exemplified by the probability that your baby will be born on its due date.
- 5.13** Two examples of discrete variables: Variety of meat served at dinner tonight; Brand of desktop computer owned.
- 5.15**
- a.** 20%, or 60 applicants, will fall at or above the 80th percentile and 10 of these will be chosen. Therefore $p(\text{that an applicant with the highest rating will be admitted}) = 10/60 = .167$.
 - b.** No one below the 80th percentile will be admitted, therefore $p(\text{that an applicant with the lowest rating will be admitted}) = 0/300 = .00$.

5.17 Mean ADDSC score for boys = 54.29, s = 12.90 [Calculated from Data Set]

a.

$$z = \frac{50 - 54.29}{12.90} = -.33$$

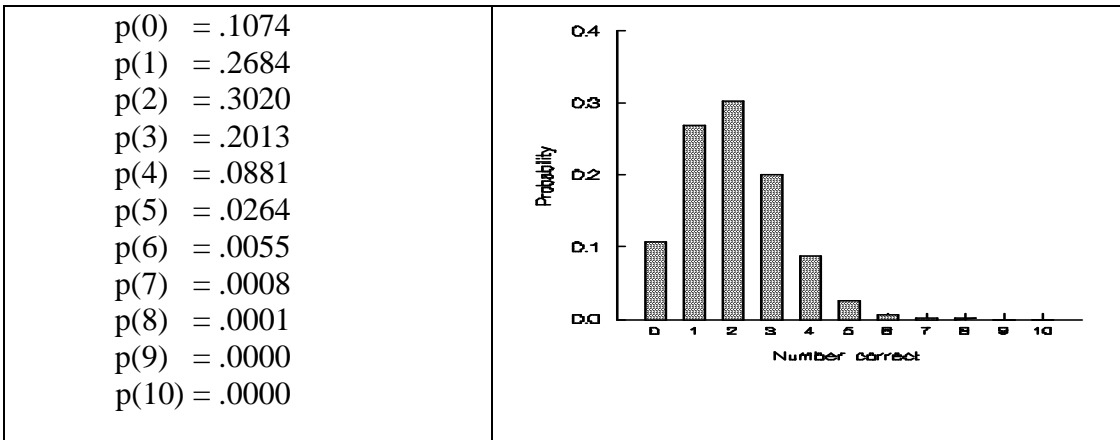
Since a score of 50 is below the mean, and since we are looking for the probability of a score *greater than* 50, we want to look in the tables of the normal distribution in the column labeled "larger portion".

$$p(\text{larger portion}) = .6293$$

b. $29/55 = 53\% > 50$; $32/55 = 58\% \geq 50$. (Notice that one percentage refers to the proportion *greater than* 50, while the other refers to the proportion *greater than or equal to* 50.)

5.19 Compare the probability of dropping out of school, ignoring the ADDSC score, with the conditional probability of dropping out given that ADDSC in elementary school exceeded some value (e.g., 66).

5.21 Plot of correct choices on trial 1 of a 5-choice task:



5.23 $p(5 \text{ or more correct}) = p(5) + p(6) + p(7) + p(8) + p(9) + p(10)$
 $= .0264 + .0055 + .0008 + .0001 + .0000 + .0000$
 $= .028 < .05$

$p(4 \text{ or more correct}) = p(4) + p(5) + p(6) + p(7) + p(8) + p(9) + p(10)$
 $= .0881 + .0264 + .0055 + .0008 + .0001 + .0000 + .0000$
 $= .1209 > .05$

At $\alpha = .05$, therefore, up to 4 correct choices indicate chance performance, but 5 or more correct choices would lead me to conclude that they are no longer performing at chance levels.

5.25 If there is no housing discrimination, then a person's race and whether or not they are offered a particular unit of housing are independent events. We could calculate the probability that a particular unit (or a unit in a particular section of the city) will be offered to anyone in a specific income group. We can also calculate the probability that the customer is a member of an ethnic minority. We can then calculate the probability of that person being shown the unit assuming independence and compare that answer against the actual proportion of times a member of an ethnic minority was offered such a unit.

5.27 Number of subjects needed in Exercise 5.26's verbal learning experiment if each subject can see only two of the four classes of words:

$$P_2^4 = \frac{4!}{(4-2)!} = \frac{4!}{2!} = 12$$

5.29 The total number of ways of making ice cream cones =

$$C_6^6 + C_5^6 + C_4^6 + C_3^6 + C_2^6 + C_1^6 = 1 + 6 + 15 + 20 + 15 + 6 = 63$$

[You can't have an ice cream cone without ice cream; exclude C_0^6].

5.31 Knowledge of current events:

If $p = .50$ of being correct on any one true-false item, and $N = 20$:

$$p(11) = C_{11}^{20} \times 5^{11} \times 5^9$$

$$C_{11}^{20} = \frac{20!}{11!(20-11)!} = \frac{20!}{11!9!} = 167,960$$

$$p(11) = C_{11}^{20} \times 5^{11} \times 5^9 = 167,960(.00048828)(.00195313) = .16$$

Since the probability of 11 correct by chance is .16, the probability of 11 or more correct must be greater than .16. Therefore we can not reject the hypothesis that $p = .50$ (student is guessing) at $\alpha = .05$.

5.33 Driving test passed by 22 out of 30 drivers when 60% expected to pass:

$$z = \frac{22 - 30(.60)}{\sqrt{30(.60)(.40)}} = 1.49; \text{ we cannot reject } H_0 \text{ at } \alpha = .05.$$

- 5.35** Students should come to understand that nature does not have a responsibility to make things come out even in the end, and that it has a terrible memory of what has happened in the past. Any “law of averages” refers to the results of a long term series of events, and it describes what we would expect to see. It does not have any self-correcting mechanism built into it.
- 5.37** It is low because the probability of breast cancer is itself very low. But don’t be too discouraged. Having collected some data (a positive mammography) the probability is 7.8 times higher than it would otherwise have been. (And if you are a woman, please don’t stop having mammographies.)