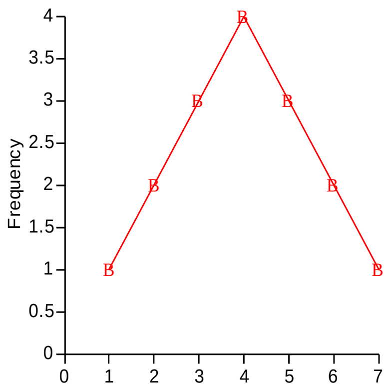
Chapter 3 - Normal Distribution

3.1 **a.** Original data:

1 2 2 3 3 3 4 4 4 4 5 5 5 6 6 7



b. To convert the distribution to a distribution of X - μ, subtract μ = 4 from each score:

-3 -2 -2 -1 -1 -1 0 0 0 0 1 1 1 2 2 3

c. To complete the conversion to z, divide each score by σ = 1.63:

-1.84 -1.23 -1.23 -0.61 -0.61 -0.61 0 0 0 0
0.61 0.61 0.61 1.23 1.23 1.84

3.3 Errors counting shoppers in a major department store:

a. $z = \dfrac{X-\mu}{\sigma}$

$= \dfrac{960-975}{15} = -\dfrac{15}{15} = -1$ between -1 and μ lie .3413

$= \dfrac{990-975}{15} = +\dfrac{15}{15} = +1$ between +1 and μ lie <u>.3413</u>

.6826

Therefore between 960 and 990 are found approximately 68% of the scores.

b. 975 = μ; therefore 50% of the scores lie below 975.

c. .5000 lie below 975
<u>.3413</u> lie between 975 and 990
.8413 (or 84%) lie below 990

3.5 The supervisor's count of shoppers:

$$z = \frac{X-\mu}{\sigma}$$

$$= \frac{950 - 975}{15}$$

$$= -1.67$$

X to ±1.67 = 2(.0475) = .095; therefore 9.5% of the time scores will be at least this extreme.

3.7 They would be equal when the two distributions have the same standard deviation.

3.9 Next year's salary raises:

a.

$$z = \frac{X - \mu}{\sigma}$$

$$-1.2817 = \frac{X - 2000}{400}$$

$$\$251268 = X$$

10% of the faculty will have a raise equal to or greater than $2,512.68.

b.

$$z = \frac{X - \mu}{\sigma}$$

$$-1.645 = \frac{X - 2000}{400}$$

$$\$1342 = X$$

The 5% of the faculty who haven't done anything useful in years will receive no more than $1,342 each, and probably don't deserve that much.

3.11 Transforming scores on diagnostic test for language problems:

X_1 = original scores	$\mu_1 = 48$	$\sigma_1 = 7$
X_2 = transformed scores	$\mu_2 = 80$	$\sigma_2 = 10$

$$\sigma_2 = \sigma_1 / C$$

$$10 = 7 / C$$

$$C = 0.7$$

Therefore to transform the original standard deviation from 7 to 10, we need to divide the original scores by .7. However dividing the original scores by .7 divides their mean by .7.

$$\bar{X}_2 = \bar{X}_1 / 0.7 = 48 / .7 = 68.57$$

We want to raise the mean to 80. 80 – 68.57 = 11.43. Therefore we need to add 11.43 to each score.

$$\bar{X}_2 = \bar{X}_1 0.7 + 11.43$$

$X_2 = X_1/0.7 + 11.43$. [This formula summarizes the whole process.]

3.13 October 1981 GRE, all people taking exam:

$$\begin{aligned} z &= \frac{X-\mu}{\sigma} \\ &= \frac{600-489}{126} \\ &= 0.88 \quad p(\text{larger portion}) = 0.81 \end{aligned}$$

A GRE score of 600 would correspond to the 81st percentile.

3.15 October 1981 GRE, all seniors and nonenrolled college graduates:

$$\begin{aligned} z &= \frac{X-\mu}{\sigma} \\ &= \frac{600-507}{118} \\ &= 0.79 \quad p = .785 \end{aligned}$$

$$\begin{aligned} z &= \frac{X-\mu}{\sigma} \\ 0.6745 &= \frac{X-507}{118} \\ 586.591 &= X \end{aligned}$$

For seniors and nonenrolled college graduates, a GRE score of 600 is at the 79th percentile, and a score of 587 would correspond to the 75th percentile.

3.17 GPA scores:

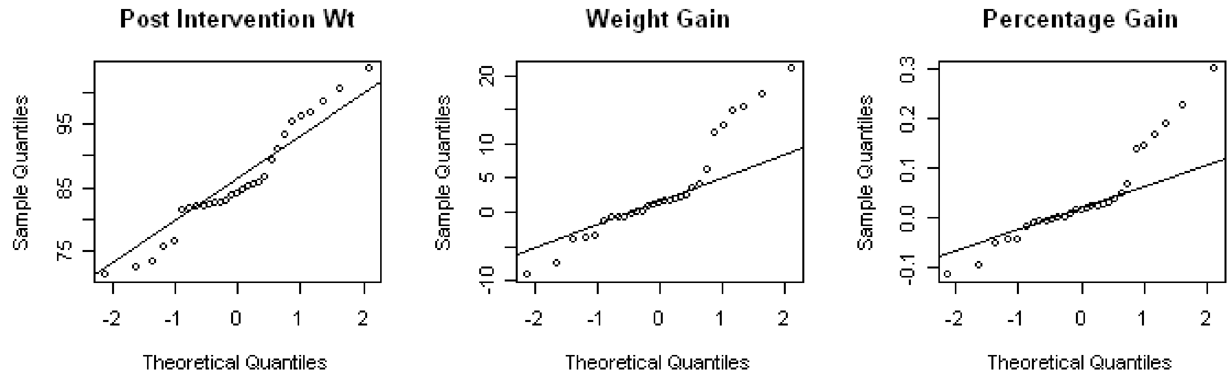
$N = 88 \;\; \bar{X} = 2.46 \;\; s = 0.86$ [calculated from data set]

$$\begin{aligned} z &= \frac{X-\bar{X}}{s} \\ 0.6745 &= \frac{X-2.46}{0.86} \\ 3.04 &= X \end{aligned}$$

The 75th percentile for GPA is 3.04.

3.19 There is no meaningful discrimination to be made among those scoring below the mean, and therefore all people who score in that range are given a *T* score of 50.

3.21 Weight gain data



None of these is very close to normal, but the post intervention weight is closest.

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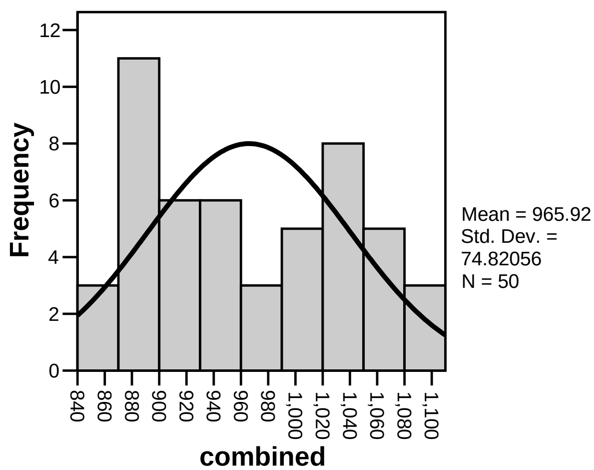
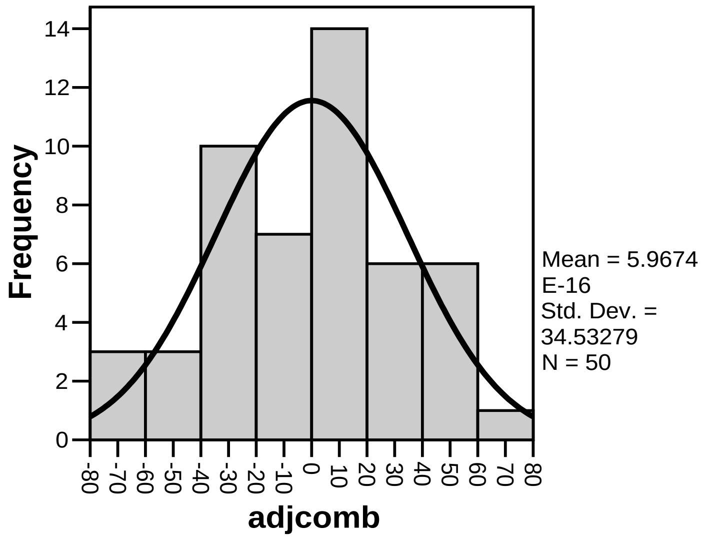
3.23 I would first draw 16 scores from a normally distributed population with $\mu = 0$ and $\sigma = 1$. Call this variable z1. The sample (z1) would almost certainly have a sample mean and standard deviation that are not 0 and 1. Then I would create a new variable z2 = z1-mean(z1). This would have a mean of 0.00. Then I would divide z2 by sd(z1) to get a new distribution (z3) with mean = 0 and sd = 1. Then make that variable have a st. dev. of 4.25 by multiplying it by 4.25. Finally add 16.3 (the new mean). Now the mean is exactly 16.3 and the standard deviation is exactly 4.25.

3.25 SAT Data



The data are actually bimodal, with probably too few scores at the extremes.



These data are much more normally distributed. As we will see in Chapter Nine, there are two kinds of students who take the SAT, depending on where they live. It the East most students take it. In the West, students applying to high ranking eastern schools take it. This leads to the bimodal distribution in the adjusted scores.