## Chapter 18 - Resampling and Nonparametric Approaches To Data

18.1 Inferences in children's story summaries (McConaughy, 1980):
a. Analysis using Wilcoxon's rank-sum test:

Younger Children
Raw $\begin{array}{llllllll}0 & 1 & 0 & 3 & 2 & 5 & 2\end{array}$
Data:
Ranks: $\begin{array}{ccccccc}1.5 & 3 & 1.5 & 6 & 4.5 & 9 & 4.5 \\ & \sum R=30 & & N=7 & & & \end{array}$

| Older Children |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | 7 | 6 | 4 | 8 | 7 |
| 7.5 | 11.5 | 10 | 7.5 | 13 | 11.5 |
| $\sum R=61$ | $N=6$ |  |  |  |  |

$$
\mathrm{W}_{\mathrm{s}}=\Sigma \mathrm{R} \text { for group with smaller } N=61
$$

$$
\mathrm{W}_{\mathrm{s}}^{\prime}=2 \overline{\mathrm{~W}}-\mathrm{W}_{\mathrm{S}}=84-61=23
$$

$W_{\mathrm{S}}<W_{\mathrm{S}}$, therefore use $W^{\prime}$ ' in Appendix $W_{\mathrm{S}}$. Double the probability level for a 2tailed test.
$\mathrm{W}_{.025(677)}=27>23$
b. Reject $H_{0}$ and conclude that older children include more inferences in their summaries.
18.3 The analysis in Exercise 18.2 using the normal approximation:

$$
\begin{aligned}
& z=\frac{\mathrm{W}_{\mathrm{S}}-\frac{n_{1}\left(n_{1}+n_{2}+1\right)}{2}}{\sqrt{\frac{n_{1} n_{2}\left(n_{1}+n_{2}+1\right)}{12}}}=\frac{53-\frac{9(9+11+1)}{2}}{\sqrt{\frac{9(11)(9+11+1)}{12}}}=-3.15 \quad \begin{array}{rc}
\underline{z} & \underline{p} \\
& 3.00 \\
.0013 \\
3.15 & .0009 \\
p(z \geq \pm 3.15)=2(.0009)=.0018<.05 & 3.25
\end{array} .0006
\end{aligned}
$$

Reject $H_{0}$, which was the same conclusion as we came to in Exercise 18.2.
18.5 Hypothesis formation in psychiatric residents (Nurcombe \& Fitzhenry-Coor, 1979):
a. Analysis using Wilcoxon's matched-pairs signed-ranks test:

| Before: | 8 | 4 | 2 | 2 | 4 | 8 | 3 | 1 | 3 | 9 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| After: | 7 | 9 | 3 | 6 | 3 | 10 | 6 | 7 | 8 | 7 |
| Difference: | -1 | +5 | +1 | +4 | -1 | +2 | +3 | +6 | +5 | -2 |
| Rank: | 2 | 8.5 | 2 | 7 | 2 | 4.5 | 6 | 10 | 8.5 | 4.5 |
|  |  |  |  |  |  |  |  |  |  |  |
| Signed |  | 8.5 | 2 | 7 |  | 4.5 | 6 | 10 | 8.5 |  |
| Rank: | -2 |  |  |  | -2 |  |  |  |  | -4.5 |

$$
\begin{aligned}
& \mathrm{T}_{+}=\Sigma(\text { positiveranks })=46.5 \\
& \mathrm{~T}_{-}=\Sigma(\text { negative ranks })=8.5 \\
& \mathrm{~T}=\text { smaller of }\left|\mathrm{T}_{+}\right| \text {or }|\mathrm{T}-|=8.5 \\
& \mathrm{~T}_{.025(10)}=8<8.5 \quad \text { Do not reject } H_{0} .
\end{aligned}
$$

b. We cannot conclude that we have evidence supporting the hypothesis that there is a reliable increase in hypothesis generation and testing over time. (Here is a case in which alternative methods of breaking ties could lead to different conclusions.)
18.7 I would randomly assign the two scores for each subject to the Before and After location, and calculate my test statistic (the sum of the negative differences) for each randomization. Having done that a large number of times, the distribution of the sum of negative differences would be the sampling distribution against which to compare my obtained result.
18.9 The analysis in Exercise 18.8 using the normal approximation:
$z=\frac{T-\frac{n(n=1)}{4}}{\sqrt{\frac{n(n+1)(2 n+1)}{24}}}=\frac{46-\frac{20(20+1)}{4}}{\sqrt{\frac{20(20+1)(40+1)}{24}}}=-2.20$
$p(z \geq \pm 2.20)=2(.0139)=.0278<.05$
Again reject $H_{0}$, which agrees with our earlier conclusion.
18.11 Data in Exercise 18.8 plotted as a function of first-born's score:


The scatter plot shows that the difference between the pairs is heavily dependent upon the score for the first born.
18.13 The Wilcoxon Matched-pairs signed-ranks test tests the null hypothesis that paired scores were drawn from identical populations or from symmetric populations with the same mean (and median). The corresponding $t$ test tests the null hypothesis that the paired scores were drawn from populations with the same mean and assumes normality.
18.15 Rejection of $H_{0}$ by a $t$ test is a more specific statement than rejection using the appropriate distribution-free test because, by making assumptions about normality and homogeneity of variance, the $t$ test refers specifically to population means.
18.17 Truancy and home situation of delinquent adolescents:

Analysis using Kruskal-Wallis one-way analysis of variance:

| Natural Home |  | Foster Home |  |  | Group Home |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{\text { Score }}{15}$ | $\frac{\text { Rank }}{}$ |  | $\frac{\text { Score }}{18}$ | $\frac{\text { Rank }}{19}$ |  | $\frac{\text { Score }}{10}$ |
| 18 | 22 | 14 | 16 | 13 | $\frac{\text { Rank }}{9}$ |  |
| 19 | 24.5 | 20 | 26 | 14 | 13.5 |  |
| 14 | 16 | 22 | 27 | 11 | 16 |  |
| 5 | 4.5 | 19 | 24.5 | 7 | 6 |  |
| 8 | 8 | 5 | 4.5 | 3 | 2 |  |
| 12 | 11.5 | 17 | 20 | 4 | 3 |  |
| 13 | 13.5 | 18 | 22 | 18 | 22 |  |
| 7 | 6.5 | 12 | 11.5 | 2 | 1 |  |
| $R_{i}=124.5$ | 170.5 | 83 | $N=27 n=9$ |  |  |  |

$$
H=\frac{12}{N(N+1)} \Sigma \frac{R_{i}^{2}}{n_{i}}-3(N+1)
$$

$$
=\frac{12}{27(27+1)}\left[\frac{124.5^{2}}{9}+\frac{170.5^{2}}{9}+\frac{83^{2}}{9}\right]-3(27+1)
$$

$$
=6.757
$$

$$
\chi_{.05}^{2}(2)=5.99 \text { Reject } H_{0}
$$

18.19 I would take the data from all of the groups and assign them at random to the groups. For each random assignment I would calculate a statistic that reflected the differences (or lack thereof) among the groups. The standard $F$ statistic would be a good one to use. This randomization, repeated many times, will give me the sampling distribution of $F$, and that distribution does not depend on an assumption of normality. I could then compare the $F$ that I obtained for my data against that sampling distribution. The result follows.

18.21 The study in Exercise 18.18 has the advantage over the one in Exercise 18.17 in that it eliminates the influence of individual differences (differences in overall level of truancy from one person to another).
18.23 For the data in Exercise 18.5:
a. Analyzed by chi-square:

|  | More |  | Fewer |
| :--- | :---: | :---: | :---: |
| Ootal |  |  |  |
|  | 10 |  |  |
| Observed | 7 | 3 |  |
|  | 10 |  |  |

$\chi^{2}=\Sigma \frac{(O-E)^{2}}{E}=\frac{(7-5)^{2}}{5}+\frac{(3-5)^{2}}{5}=1.6$
[ $\left.\chi^{2}{ }^{.05(1)}=3.84\right]$ Do not reject $H_{0}$
b. Analyzed by Friedman's test:

| Before |  | After |  |
| :---: | :---: | :--- | :---: |
| 8 | $(2)$ | 7 | $(1)$ |
| 4 | $(1)$ | 9 | $(2)$ |
| 2 | $(1)$ | 3 | $(2)$ |
| 2 | $(1)$ | 6 | $(2)$ |
| 4 | $(2)$ | 3 | $(1)$ |
| 8 | $(1)$ | 10 | $(2)$ |


| Before |  |
| ---: | :--- |
| 3 | $(1)$ |
| 1 | $(1)$ |
| 3 | $(1)$ |
| 9 | $(2)$ |
| $N$ | $=10 \quad 7$ |
|  | After <br> $\chi_{F}^{2}$ |
| $=\frac{12}{N k(k+1)} \Sigma R_{i}^{2}-3 N(k+1)$ |  |
|  | $=\frac{12}{12(2)(2+1)}\left[13^{2}+17^{2}\right]-3(10)(2+1)$ |
|  | $=1.6 \quad\left[\chi_{.05}^{2}(2)=5.99\right]$ Do not reject $\mathrm{H}_{0}$ |

These are exactly equivalent tests in this case.

