Chapter 12 Multiple Comparisons Among Treatment Means

12.1 The effects of food and water deprivation on a learning task:

a. ANOVA with linear contrasts:

Groups:	ad lib (1)	2/day (2)	food (3)	water (4)	f & w (5)	
Means:	18	24	8	12	11	
a_j:	.5	.5	-.333	-.333	-.333	$0.8333 = \Sigma a_j^2$
b_j:	1	-1	0	0	0	$2 = \Sigma b_j^2$
c_j:	0	0	.5	.5	-1	$1.5 = \Sigma c_j^2$
d_j:	0	0	1	-1	0	$2 = \Sigma d_j^2$

$$\psi_1 = (.5)(18) + (.5)(24) + (-.333)(8) + (-.333)(12)(-.333)(11) = 10.667$$

$$\psi_2 = (1)(18) + (-1)(24) + (0)(8) + (0)(12) + (0)(11) = -6$$

$$\psi_3 = (0)(18) + (0)(24) + (.5)(8) + (.5)(12) + (-1)(11) = -1$$

$$\psi_4 = (0)(18) + (0)(24) + (1)(8) + (-1)(12) + 0(11) = -4$$

$$SS_{contrast_1} = \frac{n\psi_1}{\Sigma a_j^2} = \frac{5(10.667^2)}{0.8333} = 682.667$$

$$SS_{contrast_2} = \frac{n\psi_2}{\Sigma b_j^2} = \frac{5(-6^2)}{2} = 90$$

$$SS_{contrast_3} = \frac{n\psi_3}{\Sigma c_j^2} = \frac{5(-1^2)}{1.5} = 3.333$$

$$SS_{contrast_4} = \frac{n\psi_4}{\Sigma d_j^2} = \frac{5(-4^2)}{2} = 40.000$$

Source	df	SS	MS	F
Deprivation	4	816.000	204.000	36.429*
1&2 vs 3,4,5	1	682.667	682.667	121.905*
1 vs 2	1	90.000	90.000	16.071*
3&4 vs 5	1	3.333	3.333	<1
3 vs 4	1	40.000	40.000	7.143*
Error	20	112.000	5.600	
Total	24	928.000		

$* p < .05 \quad [F_{.05(4,20)} = 2.87;\ F_{.05(1,20)} = 4.35]$

b. Orthogonality of contrasts:

Cross-products of coefficients:

$$\Sigma a_j b_j = (.5)(1) + (.5)(-1) + (.333)(0) + (.333)(0) + (.333)(0) = 0$$
$$\Sigma a_j c_j = (.5)(0) + (.5)(0) + (.333)(.5) + (.333)(.5) + (.333)(-1) = 0$$
$$\Sigma a_j d_j = (.5)(0) + (.5)(0) + (.333)(1) + (.333)(-1) + (.333)(0) = 0$$
$$\Sigma b_j c_j = (1)(0) + (-1)(0) + (0)(.5) + (0)(.5) + (0)(-1) = 0$$
$$c_j d_j = (0)(0) + (0)(0) + (.5)(1) + (.5)(-1) + (1)(0) = 0$$

c.

$$SS_{\text{treat}} = \Sigma SS_{\text{contrast}}$$
$$816.000 = 682.667 + 90.000 + 3.333 + 40.000$$

12.3 For $\alpha = .05$:

Per comparison error rate = α = .05
Familywise error rate = $1 - (1 - \alpha)^2 = .0975$.

12.5 Studentized range statistic for data in Exercise 11.2:

$\bar{X}_1 = 19.3 \quad n_1 = 10$

$\bar{X}_2 = 12.0 \quad n_2 = 10$

$$q_2 = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{MS_{error}}{n}}} = \frac{19.3 - 12.0}{\sqrt{\frac{10.56}{10}}} = \frac{7.3}{1.028} = 7.101$$

$q_2 = 7.10 = 5.023\sqrt{2} = 7.10 = t\sqrt{2}$

12.7 The Bonferroni test on contrasts in Exercise 12.2 (data from Exercise 11.1):

From Exercise 12.2: $\psi_1 = 5.25$ $\psi_2 = 2.40$ $n = 10$

$\Sigma a_j^2 = 1$ $\Sigma b_j^2 = 2$ $MS_{error} = 9.67$

$$t = \frac{\psi}{\sqrt{\frac{\Sigma a_j^2 MS_{error}}{n}}}$$

$$t_1' = \frac{5.25}{\sqrt{\frac{(1)(9.67)}{10}}} = 5.34 \qquad t_1' = \frac{2.40}{\sqrt{\frac{(2)(9.67)}{10}}} = 1.72$$

$[t_{.05}\left(df_{error} = 45; 2\,\text{comparisons}\right) = 2.32)$ Reject H_0 for only the first comparison.

12.9 A post hoc test like the Tukey or the REGWQ often does not get at the specific questions we have in mind, and, at the same time, often answers questions in which we have no interest.

12.11 Scheffé's test on the data in Exercise 12.10:

Group	1	2	3	4	5
$\bar{X}_j$	10	18	19	21	29
n_j	8	5	8	7	9
s_j^2	7.4	8.9	8.6	7.2	9.3
a_j	-16	-16	-16	21	21
b_j	-20	8	8	8	0

$$MS_{error} = \frac{\Sigma(n_j - 1)s_j^2}{\Sigma(n_j - 1)} = 8.2875$$

$$F_{contrast_1} = \frac{L^2}{\Sigma n_j a_j^2 MS_{error}} = \frac{3416^2}{(12432)(8.2875)} = 113.26$$

$$F_{contrast_2} = \frac{L^2}{\Sigma n_j b_j^2 MS_{error}} = \frac{1512^2}{(4480)(8.2875)} = 61.57$$

$$F_{crit} = (k-1)F_{.05(k-1,df_{error})} = 4F_{.05(4,32)} = 4(2.69) = 10.76$$

Thus both contrasts are significant.

12.13 Dunnett's test on data in Table 11.6:

critical value

$$\left(\bar{X}_c - \bar{X}_j\right) = t_d\sqrt{\frac{2MS_{error}}{\bar{n}_h}} = 2.58\sqrt{\frac{2(0.065)}{9.326}} = 0.305$$

The control group is significantly different from the 0.1 µg, the 0.5 µg, and the 1.0 µg groups.

12.15 They are sequentially modified because you change the critical value each time you reject another null hypothesis.

12.17 Conti and Musty (1984) recorded locomotive behavior in rats in response to injection of THC in the an active brain region. The raw data showed a clear linear relationship between group means and standard deviations, but a logarithmic transformation of the data largely removed this relationship. Mean locomotive behavior increased with dosage up to 0.5 µg, but further dose increases resulting in decreased behavior. Polynomial trend analysis revealed no linear trend but a significant quadratic trend.

12.19 If there were significant differences due to Interval and we combined across intervals, those differences would be incorporated into the error term, decreasing power.

12.21 At all three intervals there was a significant linear and quadratic trend, indicating that the effect of epinephrine on memory increases with a moderate dose but then declines with a greater dose. The linear trend reflects the fact that in the high dose condition the animals do even worse than with no epinephrine.

12.23 The first comparison calls for comparing the two control groups with the experimental groups. The solution from SPSS follows for the contrast itself. (SPSS only allows me to specify 1/3 as .33, rather than using more decimal places, which is why it complains that the coefficients don't sum to 0 and gives the contrast as 10.77 rather than 10.6

Contrast Coefficients

	conditio				
Contrast	1.00	2.00	3.00	4.00	5.00
1	.5	.5	-.33	-.33	-.33

Contrast Tests

		Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
dv	Assume equal variances	1	10.7700[a]	.96224	11.193	20	.000
	Does not assume equal	1	10.7700[a]	.99539	10.820	14.898	.000

a. The sum of the contrast coefficients is not zero.

The square root of MS_{error} = 2.366, which I will use to compute the confidence interval. I will use 10.67 as the (correct) contrast, even though that is not what SPSS reported. Then

$$
\begin{aligned}
CI_{.95} &= (\psi) \pm t_{.05} s_{error} \\
&= (10.67) \pm (2.086)(2.366) \\
&= (10.67) \pm 4.935 \\
&= 5.735 \le \mu_1 - \mu_2 \le 15.605
\end{aligned}
$$

12.25 The study by Davey et al. (2003):

The group means are Negative mood =12.6, Positive mood = 7.0, No induction = 8.7

The SPSS ONEWAY solution with one contrast comparing the Negative and Positive mood groups is shown below.

ANOVA

Things listed to check

	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	164.867	2	82.433	4.876	.016
Within Groups	456.500	27	16.907		
Total	621.367	29			

Contrast Coefficients

	Group		
Contrast	Negative	Positive	None
1	1	-1	0

Contrast Tests

		Contrast	Value of Contrast	Std. Error	t	df	Sig. (2-tailed)
Things listed to check	Assume equal variances	1	5.6000	1.83888	3.045	27	.005
	Does not assume equal	1	5.6000	2.12498	2.635	13.162	.020

The contrast between the Positive and Negative mood conditions was significant (t(27) = 3.045, p < .05). This leads to an effect size of $d = \psi / \sqrt{MS_{error}} = 5.6 / \sqrt{16.907}$ $= 5.6 / 4.11 = 1.36$. The two groups differ by over 1 1/3 standard deviations. It is evident that inducing a negative mood leads to more checking behavior than introducing a positive mood. (If we had compared the Positive and No mood conditions, the difference would not have been significant. However I had not planned to make that comparison.

12.27 This requires students to make up their own example.