Chapter 20—Nonparametric and Distribution-Free Tests

20.1 Inferences in children's story summaries (McConaughy, 1980):

a) Analysis using the Mann-Whitney test (also known as Wilcoxon's rank-sum test):

	Younger Children							Older Children					
Raw Data	0	1	0	3	2	5	2	4	7	6	4	8	7
Ranks	1.5	3	1.5	6	4.5	9	4.5	7.5	11.5	10	7.5	13	11.5
	ΣR = 30	$N = 7$						ΣR = 61	$N = 6$				

$Ws = \Sigma R$ for group with smaller $N = 61$ $\quad Ws' = 2\overline{W} - Ws = 84 - 61 = 23$

$W's < Ws$; therefore $W's$ in Appendix E. Double the probability level for a 2-tailed test.
$W_{.025}(6,7) = 27 > 23$

b) Reject the null hypothesis and conclude that older children include more inferences in their summaries.

20.3 The analysis in Exercise 20.2 using the normal approximation:

$$
\begin{aligned}
z &= \frac{Ws - \frac{n_1(n_1+n_2+1)}{2}}{\sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}}} \\
&= \frac{53 - \frac{9(9+11+1)}{2}}{\sqrt{\frac{9(11)(9+11+1)}{12}}} \\
&= -3.15
\end{aligned}
$$

$$p(z \geq \pm 3.15) = (2(.0009) = .0018 < .05$$

We will reject the null hypothesis and come to the same conclusion we came to earlier.

20.5 Hypothesis formation in psychiatric residents (Nurcombe & Fitzhenry-Coor, 1979):

Before	8	4	2	2	4	8	3	1	3	9
After	7	9	3	6	3	10	6	7	8	7
Diff.	-1	+5	+1	+4	-1	+2	+3	+6	+5	-2
Rank	2	8.5	2	7	2	4.5	6	10	8.5	4.5
Signed		8.5	2	7		4.5	6	10	8.5	
Rank	-2				-2					-4.5

T+ = Σ(positive ranks) = 46.5
T- = Σ(negative ranks) = 8.5
T = smaller of |T+| or |T-| = 8.5
n = 10
$T_{.05}(10) = 8 < 8.5$ Do not reject H_0

b) We cannot conclude that we have evidence supporting the hypothesis that there is a reliable increase in hypothesis generation and testing over time. (Here is a case in which alternative methods of breaking ties could lead to different conclusions.)

Here you might discuss how we could go about deciding how to break ties, putting the emphasis on *a priori* decisions.

20.7 Independence of first-born children:

First	12	18	13	17	8	15	16	5	8	12
Second	10	12	15	13	9	12	13	8	10	8
Diff.	2	6	-2	4	-1	3	3	-3	-2	4
Rank	4	17.5	4	11	1	8	8	8	4	11
Signed	4	17.5		11		8	8			11
Rank			-4		-1			-8	-4	

Data Cont.:

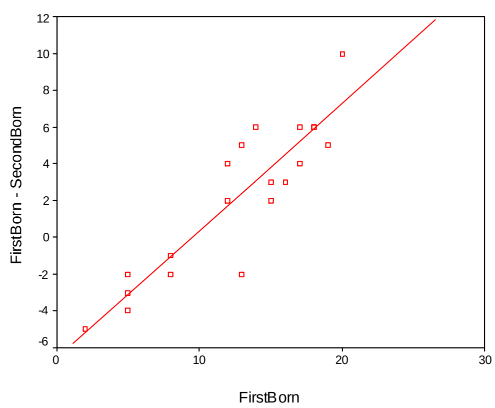
First	13	5	14	20	19	17	2	5	15	18
Second	8	9	8	10	14	11	7	7	13	12
Diff.	5	-4	6	10	5	6	-5	-2	2	6
Rank	14	11	17.5	20	14	17.5	14	4	4	17.5
Signed	14		17.5	20	14	17.5			4	17.5
Rank		-11					-14	-4		

T+ = Σ(positive ranks) = 164
T- = Σ(negative ranks) = 46
T = smaller of |T+| or |T-| = 46
n = 20
$T_{.05}(20) = 52 > 46$

b) We can reject the null hypothesis and conclude that first-born children are more independent than their second-born siblings.

Here is a good example of where we would use a "matched sample" test even though the same children do not perform in both conditions (nor could they). We are assuming that brothers and sisters are more similar to each other than they are to other children. Thus if the first-born is particularly independent, we would guess that the second-born has a higher than chance expectation of being more independent. They share a common environment.

20.9 Data in Exercise 20.7 plotted as a function of the first-born's score:



The scatterplot shows that the difference between the pairs is heavily dependent upon the score of the first-born.

20.11 The Wilcoxon matched-pairs signed-ranks test tests the null hypothesis that paired scores were drawn from identical populations or from symmetric populations with the same mean (and median). The corresponding *t* test tests the null hypothesis that the paired scores were drawn from populations with the same mean and assumes normality.

This is an illustration of the argument that you buy things with assumptions. By making the more stringent assumptions of a *t* test, we buy greater specificity in our conclusions. However if those assumptions are false, we may have used an inappropriate test.

20.13 Rejection of the null hypothesis by a *t* test is a more specific statement than rejection using the appropriate distribution-free test because, by making assumptions about normality and homogeneity of variance, the *t* test refers specifically to population means—although it is also dependent on those assumptions.

20.15 Truancy and home situation of delinquent adolescents:

Analysis using the Kruskall-Wallis one-way analysis of variance:

Natural Home		**Foster Home**		**Group Home**	
Score	Rank	Score	Rank	Score	Rank
15	18	16	19	10	9
18	22	14	16	13	13.5
19	24.5	20	26	14	16
14	16	22	27	11	10
5	4.5	19	24.5	7	6.5
8	8	5	4.5	3	2
12	11.5	17	20	4	3
13	13.5	18	22	18	22
7	6.5	12	11.5	2	1
R_i	124.5		170.5		83

$N = 27$
$n = 9$

$$H = \frac{12}{N(N+1)}\sum\frac{R_i^2}{n_i} - 3(N+1)$$

$$= \frac{12}{27(27+1)}\left[\frac{124.5^2}{9} + \frac{170.5^2}{9} + \frac{83^2}{9}\right] - 3(27+1)$$

$$= 6.757$$

$$\chi^2_{.05}(2) = 5.99$$

We can reject the null hypothesis and conclude the placement of these adolescents has an effect on truancy rates.

This analysis doesn't directly answer the question the psychologist wanted answered, because he wanted to show that the group home was better than the others. He might follow this up with Mann-Whitney tests serving in the role of multiple comparison procedures, applying a Bonferroni correction (although it might be difficult to find the necessary critical values.) Alternatively, he could just run a single Mann-Whitney between the group home and the combined data of the other two placements.

20.17 The study in Exercise 20.16 has the advantage over the one in Exercise 20.15 in that it eliminates the influence of individual differences (differences in overall level of truancy from one person to another).

20.19 For the data in Exercise 20-5:

a) Analyzed by chi-square:

	More	**Fewer**	**Total**
Observed	7	3	10
Expected	5	5	10

$$\chi^2 = \sum \frac{(O-E)^2}{E} = \frac{(7-5)^2}{5} + \frac{(3-5)^2}{5}$$

$$= 1.60$$

$\chi^2_{.05}(1) = 3.84$

We cannot reject the null hypothesis.

b) Analyzed by Friedman's test:

Before		After	
Score	Rank	Score	Rank
8	2	7	1
4	1	9	2
2	1	3	2
2	1	6	2
4	2	3	1
8	1	10	2
3	1	6	2
1	1	7	2
3	1	8	2
9	2	7	1
Totals	13		17

$N = 13 \;\; k = 2$

$$\chi^2_F = \frac{12}{Nk(k+1)} \Sigma R_i^2 - 3N(k+1)$$

$$= \frac{12}{12(2)(2+1)}[13^2 + 17^2] - 3(10)(2+1)$$

$$= 1.60$$

$\chi^2_{.05}(1) = 3.84$

These are exactly equivalent tests.

20.21 "The mathematics of a lady tasting tea;"

First Cup		**Second Cup**		**Third Cup**	
Score	Rank	Score	Rank	Score	Rank
8	3	3	2	2	1
15	3	14	2	4	1
16	2	17	3	14	1
7	3	5	2	4	1
9	3	3	4	6	2
8	2	9	3	4	1
10	3	3	1	4	2
12	3	10	2	2	1
Totals	22		16		10

$N = 8 \quad k = 3$

$$\chi^2_F = \frac{12}{Nk(k+1)}\Sigma R_i^2 - 3N(k+1)$$
$$= \frac{12}{8(3)(3+1)}[22^2 + 16^2 + 10^2] - 3(8)(3+1)$$
$$= 9.00$$
$$\chi^2_{.05}(2) = 5.99$$

We can reject the null hypothesis and conclude that people don't really like tea made with used tea bags.