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Type I, Type II and Type III Sums of Squares

Introduction

In structured experiments to compare a number of treatments, two factors are said to be *balanced* if each level of the first factor occurs as frequently at each level of the second. One example of this bsa535 is a **randomized block design**, where each level of J treatments occurs only once in each of the K blocks; thus, every possible block-treatment combination occurs once. In balanced designs, blocks and treatments are *orthogonal* factors.

Orthogonal factors are linearly independent: When graphed in n-dimensional space, orthogonal factors lie at a right angle (90°) to one another. As in other linearly independent relationships, orthogonal factors are often, but not always, uncorrelated with one another [4]. Orthogonal factors become correlated if, upon centering the factors, the angle between the factors deviates from 90°.

In balanced designs, (1) least-squares estimates bsa198 (*see* **Estimation**) of treatment parameters (each of the factor's levels) are simply the contrast of the levels' means, and (2) the sum of squares for testing the main effect of A depends only on the means of the factor's levels and does not involve elimination of blocks [3]. Property (2) implies that each level of the blocking variable contributes equally to the estimation of the main effect. Whenever properties (1) and (2) are true, the factor and blocking variable are orthogonal, and as a result, the factor's main effect is estimated independently of the blocking variable. Similarly, in a two-way ANOVA design in which cells have equal numbers of observations and every level of factor A is crossed once with every level of factor B, the A and B main effects are independently estimated, neither bsa225 one affecting the other. (*see* **Factorial Designs**)

Properties (1) and (2) also hold in another type bsa340 of orthogonal design, the **Latin square**. In an $n \times n$ Latin square, each of the n treatments occurs only once in each row and once in each column. Here, treatments are orthogonal to both rows and columns. Indeed, rows and columns are themselves orthogonal. Thus, when we say that a Latin square design is an orthogonal design, we mean that it is orthogonal for the estimation of row, column, and main effects.

In *nonorthogonal* designs, the estimation of the main effect of one factor is determined in part by the estimation of the main effects of other factors. Nonorthogonal factors occur when the combination of their levels is *unbalanced.* Generally speaking, unbalanced designs arise under one of two circumstances: either one or more factor level combinations are missing because of the complete absence of observations for one or more cells or factor level combinations vary in number of observations.

Whenever nonorthogonality is present, the effects in an experiment are **confounded** (yoked). Consider bsa131 the two 3×3 designs below in which each cell's sample size is given.

Design I

			B		
		1	2		3
	1	5	5	5	
A	2	5	5		5
	3	5	5		0

Design II

		B			
		1	2		3
	1	5	5	5	
A	2	5	5		5
	3	5	5	2	

In both designs, factors A and B are confounded, and thus, nonorthogonal. In Design I, nonorthogonality is due to the zero frequency of cell a_3b_3: (*see* **Structural Zeros**) main effect B (i.e., comparing lev- bsa656 els $B1$, $B2$, and $B3$) is evaluated by collapsing within each level of B rows $A1$, $A2$, and $A3$. However, while $A1$, $A2$, and $A3$ are available to be collapsed within $B1$ and $B2$, only $A1$ and $A2$ are available within $B3$. As a result, the main effect hypothesis for factor B cannot be constructed so that its marginal means are based on cell means that have all of the same levels of factor A. Consequently, the test of B's marginal means is confounded and dependent on factor A's marginal means. Similarly, the test of A's marginal means is confounded and dependent on factor B's marginal means. In Design II, factors A and B are confounded because of the smaller sample size of cell a_3b_3: the main effect hypothesis for factor B (and A) cannot be constructed so that each of factor

A's (and B's) levels are weighted equally in testing the B (and A) main effect.

Sum of Squares

In an orthogonal or balanced ANOVA, there is no need to worry about the decomposition of sums of squares. Here, one ANOVA factor is independent of another ANOVA factor, so a test for, say, a sex effect is independent of a test for, say, an age effect. When the design is unbalanced or nonorthogonal, there is not a unique decomposition of the sums of squares. Hence, decisions must be made to account for the dependence between the ANOVA factors in quantifying the effects of any single factor. The situation is mathematically equivalent to a multiple regression model where there are correlations among the predictor variables. Each variable has direct and indirect effects on the dependent variable. In an ANOVA, each factor will have direct and indirect effects on the dependent variable. Four different types of sums of squares are available for the estimation of factor effects. In an orthogonal design, all four will be equal. In a nonorthogonal design, the correct sums of squares will depend upon the logic of the design.

Type I SS

Type I SS are order-dependent (hierarchical) (*see*
bsa267 **Hierarchical Designs**). Each effect is adjusted for all other effects that appear earlier in the model, but not for any effects that appear later in the model. For example, if a three-way ANOVA model was specified to have the following order of effects,

$$A, B, A \times B, C, A \times C, B \times C, A \times B \times C,$$

the sums of squares would be calculated with the following adjustments:

Effect	Adjusted for
A	–
B	A
$A \times B$	A, B
C	$A, B, A \times B$
$A \times C$	$A, B, C, A \times B$
$B \times C$	$A, B, C, A \times B, A \times C$
$A \times B \times C$	$A, B, C, A \times B, A \times C, A \times B \times C$

Type I SS are computed as the decrease in the error SS (SSE) when the effect is added to a model. For example, if SSE for $Y = A$ is 15 and SSE for $Y = A \times B$ is 5, then the Type I SS for B is 10. The sum of all of the effects' SS will equal the total model SS for Type I SS – this is not generally true for the other types of SS (which exclude some or all of the variance that cannot be unambiguously allocated to one and only one effect). In fact, specifying effects hierarchically is the only method of determining the unique amount of variance in a dependent variable explained by an effect. Type I SS are appropriate for balanced (orthogonal, equal n) analyses of variance in which the effects are specified in proper order (main effects, then two-way interactions, then three-way interactions, etc.), for trend analysis where the powers for the quantitative factor are ordered from lowest to highest in the model statement, and the **analysis of covariance** (ANCOVA) in which bsa018
covariates are specified first. Type I SS are also used for hierarchical step-down nonorthogonal analyses of variance [1] and hierarchical regression [2]. With such procedures, one obtains the particular SS needed (adjusted for some effects but not for others) by carefully ordering the effects. The order of effects is usually based on temporal priority, on the causal relations between effects (an outcome should not be added to the model before its cause), or on theoretical importance.

Type II SS

Type II SS are the reduction in the SSE as a result of adding the effect to a model that contains all other effects except effects that contain the effect being tested. An effect is contained in another effect if it can be derived by deleting terms in that effect – for example, A, B, C, $A \times B$, $A \times C$, and $B \times C$ are all contained in $A \times B \times C$. The Type II SS for our example involve the following adjustments:

Effect	Adjusted for
A	$B, C, B \times C$
B	$A, C, A \times C$
$A \times B$	$A, B, C, A \times C, B \times C$
C	$A, B, A \times B$
$A \times C$	$A, B, C, A \times B, B \times C$
$B \times C$	$A, B, C, A \times B, A \times C$
$A \times C$	$A, B, C, A \times B, A \times C, B \times C$

When the design is balanced, Type I and Type II *SS* are identical.

Type III SS

Type III SS are identical to those of Type II *SS* when the design is balanced. For example, the sum of squares for A is adjusted for the effects of B and for $A \times B$. When the design is unbalanced, these are the *SS* that are approximated by the traditional unweighted means ANOVA that uses harmonic mean sample sizes to adjust cell totals: Type III *SS* adjusts the sums of squares to estimate what they might be if the design were truly balanced. To illustrate the difference between Type II and Type III *SS*, consider factor A to be a dichotomous variable such as gender. If the data contained 60% females and 40% males, the Type II sums of squares are based on those percentages. In contrast, the Type III *SS* assume that the sex difference came about because of sampling and tries to generalize to a population in which the number of males and females is equal.

Type IV SS

Type IV SS differ from Types I, II, and III *SS* in that it was developed for designs that have one or more empty cells, that is, cells that contain no observations. (*see* **Structural Zeros**) Type IV *SS* evaluate marginal bsa656 means that are based on equally weighted cell means. They yield the same results as a Type III *SS* if all cells in the design have at least one observation. As a result, with Type IV *SS*, marginal means of one factor are based on cell means that have all of the same levels of the other factor, avoiding the confounding of factors that would occur if cells were empty.

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Abstract: Whenever factorial designs have equal cell frequencies, the factors are orthogonal. Although orthogonal factors are not required, orthogonal factors permit a unique decomposition of the variance in the dependent variable. If cell frequencies vary, or there are empty cells, the factors are nonorthogonal: The variance decomposition of the dependent variable differs depending on which of Type I, II, III, or IV sums of squares is used for the analysis.

Keywords: orthogonal; balanced; sums of squares; missing data; confounded; fixed effects; random effects

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