

## **Designed Experiments**

- How many levels of X should be used?
- What should the smallest/largest values be for X?
- How should other levels be spaced along the X-axis?
- How many observations should be taken at each level?

$$\hat{\beta}_1 \sim N\left(\beta_1, \frac{\sigma^2}{\sum (x_i - \bar{x})^2}\right) \quad SE(\hat{\beta}_1) = \sqrt{\frac{S_{Y|X}^2}{\sum (x_i - \bar{x})^2}}$$

## Multiple Linear Regression (MLR)

- General Case:  $k$  Predictor variables

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_k X_{ik} + \varepsilon_i, \quad \varepsilon_i \stackrel{iid}{\sim} N(0, \sigma^2) \quad i = 1, 2, \dots, n$$

- Assumptions: *similar* to the assumptions as for Simple Linear Regression
  - **Existence** – for any combo of indep. variables  $X_i$ ,  $Y$  is a RV with finite mean & variance
  - **Independence** –  $Y_i$  are independent of each other
  - **Linearity** – the mean value of  $Y$  for each combo of  $X_i$  is a linear function of the  $\beta_i$
  - **Homoskedasticity** – the variance of  $Y$  is the same for any combo of  $X_i$
  - **Normality** – for any combo of  $X_i$ ,  $Y$  has a normal distribution
- Specific Case:  $k=2$  Predictor variables

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \varepsilon_i, \quad \varepsilon_i \stackrel{iid}{\sim} N(0, \sigma^2) \quad i = 1, 2, \dots, n$$

- Decomposition of the Total SS

$$\sum_i [Y_i - \bar{Y}]^2 = \sum_i (Y_i - \hat{Y}_i)^2 + \sum_i (\hat{Y}_i - \bar{Y})^2$$

$$SS(Total) = SS(Error) + SS(Regression)$$

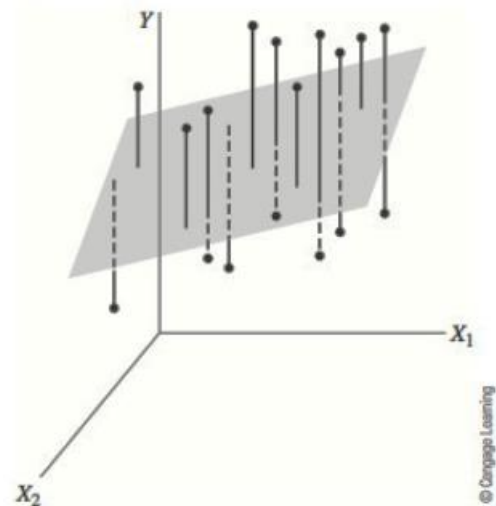
$$SSY = SSE(x_1, x_2) + SSR(x_1, x_2)$$

- ANOVA Table

Source	d.f.	Sum of Squares	Mean Square	F
Regression	2	$SSR(x_1, x_2)$	$SSR(x_1, x_2) / 2$	$MSR(x_1, x_2) / MSE$
$\begin{cases} X_1 \\ X_2 \end{cases}$	1	$SSR_1$	$SSR_1 / 1$	
	1	$SSR_2$	$SSR_2 / 1$	
Error(Residual)	$n-3$	$SSE$	$MSE = SSE / (n-3)$	
Total	$n-1$	$SSY$		

- $SSR, SSE, R^2$  depend on the model chosen

- $R^2(x_1, x_2) = SSR(x_1, x_2) / SSY$
- $R^2(x_1) = SSR(x_1) / SSY$



**FIGURE 8.2** Best-fitting plane for three-dimensional data