

• Simple Linear Regression

- Model: $Y_i = \beta_0 + \beta_1 X_i + \epsilon_i$
- Pearson Correlation Coefficient between 2 Variables (r_{yx})

$$* r_{yx} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2 \sum (y_i - \bar{y})^2}} = \hat{\beta}_1 \frac{s_x}{s_y}$$

$$* R^2 = r_{yx}^2 = \frac{SS(Regression)}{SS(Total)}$$

• Multiple Linear Regression

- Model: $Y_i = \beta_0 + \beta_1 X_{i1} + \dots + \beta_k X_{ik} + \epsilon_i$
- Lots of *Pearson Correlation Coefficients*: $r_{yx_1}, r_{yx_2}, \dots, r_{yx_k}, r_{x_i x_j}$
- Multiple Correlation Coefficient ($R_{y|x_1, x_2, \dots, x_k}$)

- * Measures overall linear association between Y and the entire set of variables $X_1, X_2, \dots, X_k$

$$* R_{y|x_1, x_2, \dots, x_k} = r_{y\hat{y}} = \frac{\sum (y_i - \bar{y})(\hat{y}_i - \bar{\hat{y}})}{\sqrt{\sum (y_i - \bar{y})^2 \sum (\hat{y}_i - \bar{\hat{y}})^2}} = \sqrt{\frac{SS(Regression)}{SS(Total)}}$$

$$* R^2 = R_{y|x_1, x_2, \dots, x_k}^2$$

- Partial Correlation Coefficients (e.g., $r_{yx_1|z}$)

- * Measures the linear association between 2 variables after controlling for the effects of other variables (called *covariates*)

$$* r_{yx_1|z} = \frac{r_{yx_1} - r_{yz}r_{x_1z}}{\sqrt{(1 - r_{yz}^2)(1 - r_{x_1z}^2)}}$$

Bivariate Normal Distribution

$$f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} e^{\frac{-1}{2(1-\rho^2)}\left(\left(\frac{x-\mu_x}{\sigma_x}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right) + \left(\frac{y-\mu_y}{\sigma_y}\right)^2\right)}$$

$$E(Y | X = x) = \mu_y + \rho \frac{\sigma_y}{\sigma_x} (x - \mu_x)$$

$$\hat{\mu}_{Y|X=x} = \bar{y} + r \frac{s_y}{s_x} (x - \bar{x})$$

