9. a H_0: $\beta_1 = \beta_2 = 0$ vs. H_A: $\beta_1 \neq 0$ and/or $\beta_2 \neq 0$ in the model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + E$.

$F = 20.03$ df: 2, 4
$P = 0.0082$
At $\alpha = 0.05$, we would reject H_0 and conclude that at least one $\beta_i \neq 0$.

b Variables-added-last tests:

i H_0: $\beta_1 = 0$ vs. H_A: $\beta_1 \neq 0$ in the model $Y = \beta_0 + \beta_1 X_1 + E$.

$$F = \frac{\text{Regression SS}(X_1)}{[\text{SSY - Regression SS}(X_1)]/5} = \frac{5732.2228}{[6305.7143 - 5732.2228]/5} = 49.98$$

df: 1, 5
$P < 0.001$
At $\alpha = 0.05$, we would reject H_0 and conclude that $\beta_1 \neq 0$.

ii H_0: $\beta_2 = 0$ vs. H_A: $\beta_2 \neq 0$ in the model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + E$.
$F(X_2|X_1) = 0.01$ df: 1, 4
$P = 0.9344$
At $\alpha = 0.05$, we do not reject H_0 and conclude that $\beta_2 = 0$.

c i H_0: $\beta_2 = 0$ vs. H_A: $\beta_2 \neq 0$ in the model $Y = \beta_0 + \beta_2 X_2 + E$.

$$F = \frac{\text{Regression SS}(X_1, X_2) - \text{Regression SS}(X_1 \mid X_2)}{[\text{SSY -(Regression SS}(X_1, X_2) - \text{Regression SS}(X_1 \mid X_2))]/5}$$

$$= \frac{5733.3213 - 1402.3153}{[6305.7143 - (5733.3213 - 1402.3153)]/5} = 10.97$$

df: 1, 5
$0.01 < P < 0.025$
At $\alpha = 0.05$, we would reject H_0 and conclude that $\beta_2 \neq 0$.

ii H_0: $\beta_1 = 0$ vs. H_A: $\beta_1 \neq 0$ in the model $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + E$.
$F(X_1|X_2) = 9.80$ df: 1, 4
$P = 0.0352$
At $\alpha = 0.05$, we reject H_0 and conclude that $\beta_1 \neq 0$.

d

Source	df	SS	MS	F
$X_1 \mid X_2$	1	1402.3153	1402.3153	9.8
$X_2 \mid X_1$	1	1.0985	1.0985	0.01
Residual	4	572.3930	143.0983	
Total	6	6305.7143		

e Based on the hypothesis tests in this problem, X_1 is the only necessary predictor. If one considered X_2 to be an important confounding variable, it may be considered a necessary predictor along with X_1 –see chapter 11 for a discussion of confounding.