A clinical research study was conducted in order to determine if there is a significant linear relationship between the amount of medication (X in grams) and the amount of time that a patient is pain free (Y in minutes).

The researcher decides to use the regression model
$$Y_i = \beta_0 + \beta_1 x_i + \epsilon_i$$
and test the following hypotheses at the .01 level of significance.
$$H_o : \beta_1 = 0$$
$$H_a : \beta_1 > 0$$

A sample of 102 individuals are recruited and given dosages of medication in such a way as to ensure that $\sum_{i=1}^{102} (x_i - \bar{x})^2 = 500$. The researcher will use a value of $s^2{}_{Y|X} = 1.8$ in the study.

Answer the following questions about the researcher's study design.

1. What is the probability that a type I error occurs?

2. If a value of $\hat{\beta}_1 = .12$ minutes/gram was estimated from the sample data, compute a p-value for the test and state a conclusion.

3. Write the rule for rejecting H_o in terms of the sample t score, t_s.

4. Write the rule for rejecting H_o in terms of $\hat{\beta}_1$ (given that $s^2{}_{Y|X}$ and $\sum_{i=1}^{102} (x_i - \bar{x})^2$ are as given above).

5. If the true relationship between the amount of medication and the amount of time that symptoms persist is such that a 42.13 gram increase of medication leads to a 5 minute increase in the amount of time that the patient is pain free, what is the probability that the researcher commits a type II error?

6. Based on your answer to the above question, what do you conclude about the researcher's study design?