Comparing Two Independent Samples

The *sampling distribution* for …

$\bar{X}_1$ is approximately Normal with *Mean*= μ_1, $SD=\dfrac{\sigma_1}{\sqrt{n_1}}$

$\bar{X}_2$ is approximately Normal with *Mean*= μ_2, $SD=\dfrac{\sigma_2}{\sqrt{n_2}}$

$\bar{X}_1-\bar{X}_2$ is approx Normal with *Mean*= $\mu_1-\mu_2$, $SD=\sqrt{\dfrac{\sigma_1^2}{n_1}+\dfrac{\sigma_2^2}{n_2}}$

$$z_s=\frac{\left(\bar{X}_1-\bar{X}_2\right)-\left(\mu_1-\mu_2\right)}{SD_{\bar{X}_1-\bar{X}_2}} \rightarrow t_s=\frac{\left(\bar{X}_1-\bar{X}_2\right)-\left(\mu_1-\mu_2\right)}{SE_{\bar{X}_1-\bar{X}_2}}$$

Example: 30 mice given one of two diets for 21 days.
Reduction in cholesterol in *mg/dl* is measured on day 22

Diet	N	Mean	StDev
Bean	15	26.46	5.90
Oat	15	32.23	9.56

$H_o:\mu_1-\mu_2=D_0$

$$t_s=\frac{(\bar{x}_1-\bar{x}_2)-D_0}{SE_{\bar{x}_1-\bar{x}_2}}$$

$\sigma_1=\sigma_2 \rightarrow$ $t_s=\dfrac{(\bar{x}_1-\bar{x}_2)-D_0}{\sqrt{s_p^2\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}}$ with $df=n_1+n_2-2$ (Equal Variance *T*-test)

Rejection Region at the α level of significance:

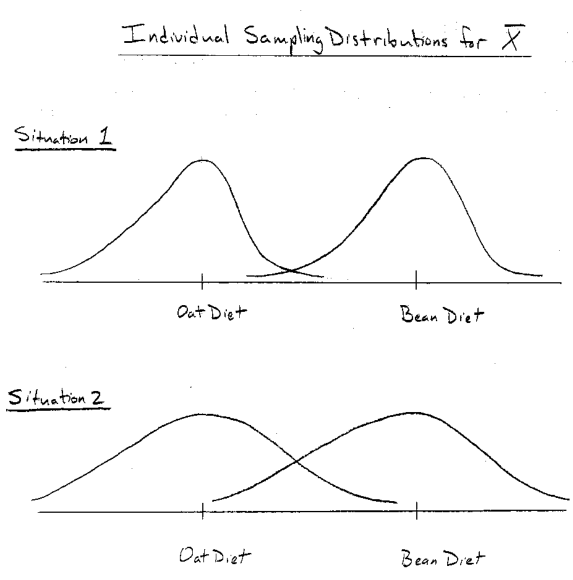
Reject Ho in favor of

$H_a:\mu_1-\mu_2>D_0 \quad if \quad t_s\ge t_\alpha$
$H_a:\mu_1-\mu_2<D_0 \quad if \quad t_s\le -t_\alpha$
$H_a:\mu_1-\mu_2\ne D_0 \quad if \quad |t_s|\ge t_{\alpha/2}$

$\sigma_1\ne\sigma_2 \rightarrow$ $t_s'=\dfrac{(\bar{x}_1-\bar{x}_2)-D_0}{\sqrt{\dfrac{s_1^2}{n_1}+\dfrac{s_2^2}{n_2}}}$ (Unpooled or Unequal Variance *T*-test)

Satterthwaite's approximation for the degrees of freedom:

$$df=\frac{\left(\dfrac{s_1^2}{n_1}+\dfrac{s_2^2}{n_2}\right)^2}{\dfrac{\left(\dfrac{s_1^2}{n_1}\right)^2}{(n_1-1)}+\dfrac{\left(\dfrac{s_2^2}{n_2}\right)^2}{(n_2-1)}}$$



Non-parametric Tests for 2 Independent Samples

- Test $H_o:\tilde{\mu}_1=\tilde{\mu}_2$ as opposed to $H_o:\mu_1=\mu_2$
- Use recoded data: the ranks of the observations as opposed to the original values.
- Do not rely on the assumption of normally distributed populations → Nonparametric

A *T*-test on the Ranks

- Rank the combined data from both samples (ties get assigned the average rank).
- Do a 2-sample *t*-test on the ranks, instead of the observed values.
  - This test is roughly equivalent to the Wilcoxon Rank Sum Test.

Wilcoxon Rank Sum Test (a.k.a. Mann-Whitney Test)

- Rank the combined data from both samples (ties get assigned the average rank).
- *T* = Sum of the ranks in sample 1 (called *W* or *S* in some software).

When H$_o$ is true, the sampling distribution for T is approx. normal $(if\ n_1>10\ \&\ n_2>10)$ with

- Mean: $\mu_T=\dfrac{n_1(n_1+n_2+1)}{2}$
- Variance: $\sigma_T^2=\dfrac{n_1n_2}{12}(n_1+n_2+1)$
- Test Statistic: $z_s=\dfrac{T-\mu_T}{\sigma_T}$

Rejection Region at the α level of significance:

$H_a:\tilde{\mu}_1>\tilde{\mu}_2 \quad if \quad z_s\ge z_\alpha$
$H_a:\tilde{\mu}_1<\tilde{\mu}_2 \quad if \quad z_s\le -z_\alpha$
$H_a:\tilde{\mu}_1\ne\tilde{\mu}_2 \quad if \quad |z_s|\ge z_{\alpha/2}$

When there are ties in the ranks there is a correction to the variance:

$$\sigma_T^2=\frac{n_1n_2}{12}(n_1+n_2+1)-\left(\frac{n_1n_2}{12}\frac{\sum_{j=1}^{k}t_j(t_j^2-1)}{(n_1+n_2)(n_1+n_2-1)}\right)$$

JMP:

- (Analyze: Fit Y by X: <red▼> Nonparametric: Wilcoxon Test)
- When there are more than 2 samples, this is known as the Kruskal-Wallis Test