

$$\sum_{x=0}^{\infty} ar^x = a/(1-r) \quad \sum_{x=0}^{\infty} xr^x = r/(1-r)^2 \text{ for } |r| < 1 \quad \sum_{k=0}^{\infty} x^k = (1-x)^{-1} \text{ for } |x| < 1$$

$$\sum_{k=1}^n k = n(n+1)/2 \quad \sum_{k=1}^n k^2 = n(n+1)(2n+1)/6 \quad e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n$$

$$\binom{n-1+r}{r} = \frac{(n-1+r)!}{r!(n-1)!} \text{ "Non-negative Ball and Urn Model"}$$

$${}_nP_r = n(n-1)(n-2) \dots (n-r+1) \quad {}_nC_r = \frac{n!}{(n-r)!r!} = \binom{n}{r} \quad P(A|B) = P(A \cap B) / P(B)$$

$$P(B) = P(B \cap A_1) + \dots + P(B \cap A_n) = P(B|A_1)P(A_1) + \dots + P(B|A_n)P(A_n)$$

$$E(X) = \sum_{x \in S} x f(x) \text{ or } \int_{x \in S} x f(x) dx \quad \sigma^2 = E[(X - \mu)^2] = E(X^2) - \mu^2$$

$$M(t) = E(e^{tX}) \quad \chi^2_{\text{GOF}} = \sum_i (O_i - E_i)^2 / E_i$$

Discrete Distributions:

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x} \quad x \in S \quad \rightarrow M(t) = [(1-p) + pe^t]^n \rightarrow \text{Binomial}(n,p) \rightarrow \mu = np$$

$$f(x) = (1-p)^{x-1} p \quad x \in S \quad \rightarrow M(t) = \frac{pe^t}{1-(1-p)e^t} \rightarrow \text{Geometric}(p) \rightarrow \mu = 1/p$$

$$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r} \quad x \in S \quad \rightarrow M(t) = \left[\frac{pe^t}{1-(1-p)e^t} \right]^r \rightarrow \text{Neg. Binom.}(r,p) \rightarrow \mu = r/p$$

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad x \in S \quad \rightarrow M(t) = e^{\lambda(e^t-1)} \rightarrow \text{Poisson}(\lambda) \rightarrow \mu = \lambda$$

$$f(x) = \frac{\binom{N_1}{x} \binom{N_2}{n-x}}{\binom{N}{n}} \quad x \in S \quad \rightarrow \text{No closed form} \rightarrow \text{Hypergeometric} \rightarrow \mu = n \left(\frac{N_1}{N} \right)$$

Continuous Distributions:

$$f(x) = \frac{1}{(b-a)} \quad a \leq x \leq b, \text{ z.o.w.} \quad \rightarrow M(t) = \frac{e^{tb} - e^{ta}}{t(b-a)} \rightarrow \text{Uniform}(a,b) \rightarrow \mu = \theta$$

$$f(x) = \frac{1}{\theta} e^{-x/\theta} \quad 0 \leq x < \infty, \text{ z.o.w.} \quad \rightarrow M(t) = \frac{1}{1-\theta t} \rightarrow \text{Exponential}(\theta) \rightarrow \mu = \theta$$

$$f(x) = \frac{1}{\Gamma(\alpha)\theta^\alpha} x^{\alpha-1} e^{-x/\theta} \quad 0 \leq x < \infty, \text{ z.o.w.} \rightarrow M(t) = \frac{1}{(1-\theta t)^\alpha} \rightarrow \text{Gamma}(\alpha, \theta) \rightarrow \mu = \alpha\theta$$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \quad -\infty \leq x < \infty \quad \rightarrow M(t) = e^{-\mu t + \sigma^2 t^2/2} \rightarrow \text{Normal}(\mu, \sigma^2) \rightarrow \mu = \mu$$