Functions of Independent RVs

X_1 and X_2 are two *independent* draws from the same RV

- X_1 , X_2 is called a *Random Sample* (RS) of size n=2
- Joint PMF: $f(x_1, x_2) = P[(X_1 = x_1) \cap (X_2 = x_2)]$
  $= P[(X_1 = x_1)] \cdot P[(X_2 = x_2)]$ since X_1 and X_2 are *independent*
  $= f_1(x_1) \cdot f_2(x_2)$
  - Example:
    X_1 = # of spots on the 1st toss of a die → $f_1(x_1)=1/6$ $x_1=\overbrace{1,2,\ldots,6}^{S_1}$, z.o.w.
    X_2 = # of spots on the 2nd toss of a die → $f_2(x_2)=1/6$ $x_2=\underbrace{1,2,\ldots,6}_{S_2}$, z.o.w.

    $f(x_1, x_2) = 1/36$ $x_1=1,2,\ldots,6$; $x_2=1,2,\ldots,6$; z.o.w.

- $E\left[h(X_1,X_2)\right] = \sum_{x\in S_1}\sum_{x\in S_2} h(x_1,x_2)\, f(x_1,x_2) = \sum_{x\in S_1}\sum_{x\in S_2} h(x_1,x_2)\, f_1(x_1) f_2(x_2)$
  - $Y = X_1 \bullet X_2$
    $E\left[Y\right] = E\left[X_1 X_2\right] = \sum_{x\in S_1}\sum_{x\in S_2} (x_1 x_2)\, f_1(x_1) f_2(x_2) =$

- Moment Generating Functions:
  - $Y = X_1 + X_2$
    $M_Y(t) = E\left[e^{tY}\right] = E\left[e^{t(X_1+X_2)}\right] =$

$$
\begin{aligned}
var(X_1+X_2) &= E\left[(X_1+X_2)-(\mu_1+\mu_2)\right]^2 \\
&= E\left[(X_1-\mu_1)+(X_2-\mu_2)\right]^2 \\
&= E\left[(X_1-\mu_1)^2 + 2(X_1-\mu_1)(X_2-\mu_2) + (X_2-\mu_2)^2\right]^2 \\
&= E(X_1-\mu_1)^2 + E\left[2(X_1-\mu_1)(X_2-\mu_2)\right] + E(X_2-\mu_2)^2 \\
&= \mathrm{var}(X_1) \quad + 2E(X_1-\mu_1)E(X_2-\mu_2) + \mathrm{var}(X_2) \qquad * \\
&= \mathrm{var}(X_1) \quad + \mathrm{var}(X_2)
\end{aligned}
$$