Gamma Function

1) $\Gamma(t)=\int_0^{\infty} y^{t-1}e^{-y}dy$ → integration by parts (*t-1*) times

2) $\Gamma(1)=\int_0^{\infty} y^{1-1}e^{-y}dy=\int_0^{\infty} e^{-y}dy=1$

3) $\Gamma(t)=y^{t-1}(-e^{-y})\Big|_0^{\infty}-\int_0^{\infty}(-e^{-y})(t-1)y^{t-2}dy$

$$=(\ \ 0\ -\ 0\ \ )\quad+(t-1)\int_0^{\infty} y^{t-2}e^{-y}dy$$

Gamma RVs $X \sim Gamma(\alpha,\theta)$

- used in a variety of modeling situations, including waiting times in a Poisson Process
- extends the Exponential RV (similar to the Negative Binomial vs. Geometric RV)
  - α is a shape parameter (represents the number of successes in a Poisson Process)
  - θ is a scale parameter (represents the mean waiting time between successes, $\theta=1/\lambda$)
  - PDF: $f(x)=\begin{cases}\dfrac{1}{\Gamma(\alpha)\theta^{\alpha}}x^{\alpha-1}e^{-\frac{x}{\theta}} & x>0\\ 0 & \text{otherwise}\end{cases}$
  - MGF: $M(t)=\int_{-\infty}^{\infty} e^{tx}f(x)dx=\dfrac{1}{(1-\theta t)^{\alpha}}$
  - $\alpha=1$ → $\dfrac{1}{\Gamma(1)\theta^{1}}x^{1-1}e^{-\frac{x}{\theta}}=\dfrac{1}{\theta}e^{-\frac{x}{\theta}}$ → Exponential PDF

Poisson Process (2)

- A large number of trials with a small *P(success)* on any trial.
- # Successes in a given time interval is a Discrete RV and is modeled by a Poisson RV.
  - λ is the mean # of successes per unit time (λ = mean rate)
  - (λt) is the mean # of successes in an interval of length t
- The time until the first successes is a continuous RV and is modeled by an Exponential RV.
- The time until the α^{th} success is a continuous RV and is modeled by a Gamma RV.

Let **T**= waiting time until there are α successes

$$F(t) = P\left(T \le t\right) = 1 - P(T > t)$$
$$= 1 - P\left(\text{less than } \alpha \text{ Successes in } [0,t]\right)$$

Chi-Square Goodness-of-fit Tests

- Used to check if data comes from a specific distribution (Probability Model)
- Compares *observed* and *expected* counts under the Probability Model in each of ***k*** categories

$$\chi^2_{GOF} = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

- When the Probability Model is correct, χ^2_{GOF} follows a $\chi^2_{(r)}$ distribution with $r=k-1-e$

- **Genetic Models**
- Hardy-Weinberg Model (independence, natural selection, genotyping errors, …)
- Dominant/Recessive Models (Punnett Squares)
  - A Hybrid cross (***Aa*** x ***Aa***)

	A	***a***
A	***AA***	***Aa***
a	***aA***	***aa***

- If ***A*** is dominant to ***a*** then we expect offspring in a ratio of 3:1
- Example: For a certain plant *Purple* flowers are thought to be dominant to *White*

A self-pollination study of 400 plants from a Hybrid cross gives
278 *Purple* & 122 *White*

1. Find the Expected numbers of *Purple* & *White* under the dominance model.
2. Determine the X^2_{GOF} value for these data.
3. Find the Probability Value (p-value) from the Chi-square table.