Uniform RVs $X \sim U(a, b)$

- used by random number generators
- used as the first step in simulating data from any distribution
- 2 parameters define the interval with equal probability
  - PDF: $f(x) = \begin{cases} \frac{1}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$
  - $M(t) = \int_{-\infty}^{\infty} e^{tx} f(x)dx = \int_a^b e^{tx} \frac{1}{b-a} dx = \left\langle \left(\frac{1}{b-a} \right) \frac{1}{t} e^{tx} \right|_a^b = \frac{e^{bt} - e^{at}}{t(b-a)} \quad t \neq 0$
  - *E(X)* is the midpoint of the interval
  - $Var(X) = \frac{(b-a)^2}{12}$

Inverse function for e^x

- $\ln(1) = 0$
- $\ln(e) = 1$
- $\ln(ab) = \ln(a) + \ln(b)$
- $\ln(a/b) = \ln(a) - \ln(b)$
- $\ln(a^n) = n\ln(a)$
- $\ln(e^x) = x\ln(e) = x \quad \leftrightarrow \quad e^{\ln x} = x$

Exponential RVs *X ~ Exp(θ)*

- used to model the time that elapses before an event occurs
  - (waiting times, lifetimes of electronic components, hard disks → RAID arrays, etc)
- one parameter, θ, describes the location and shape of the PDF
  - PDF: $$f(x)=\begin{cases}\frac{1}{\theta}e^{-\frac{x}{\theta}}, & x>0\\ 0, & \text{otherwise}\end{cases}$$
  - MGF:

$$M(t)=\int_{-\infty}^{\infty}e^{tx}f(x)dx=\int_{0}^{\infty}e^{tx}\left(\frac{1}{\theta}e^{-\frac{x}{\theta}}\right)dx=\int_{0}^{\infty}\frac{1}{\theta}e^{-x\left(\frac{1}{\theta}-t\right)}dx=\int_{0}^{\infty}\frac{1}{\theta}e^{-u}\frac{du}{\left(\frac{1}{\theta}-t\right)}=$$

$$=\lim_{b\to\infty}\left\langle\frac{-e^{-u}}{\theta\left(\frac{1}{\theta}-t\right)}\right|_0^b=\left(\lim_{b\to\infty}\frac{-e^{-b}}{\theta\left(\frac{1}{\theta}-t\right)}\right)-\left(\frac{-e^{0}}{\theta\left(\frac{1}{\theta}-t\right)}\right)=0-\left(\frac{-1}{1-\theta t}\right)=(1-\theta t)^{-1}$$

Poisson Process

- A large number of trials with a small *P(success)* on any trial.
- # Successes in a given time interval is described by a Poisson RV.
  - λ is the mean # of successes per unit time (λ = mean rate)
  - *(λt)* is the mean # of successes in an interval of length *t*
- The time between successes is a continuous RV and is modeled by an Exponential RV.

  - $P(x \text{ Successes per unit time})=\dfrac{\lambda^x e^{-\lambda}}{x!}\quad x=0,1,2,\ldots$
  - $P(x \text{ Successes in [0,t]})=\dfrac{(\lambda t)^x e^{-(\lambda t)}}{x!}\quad x=0,1,2,\ldots$

Let **T**= waiting time until the 1st success in a Poisson Process

$$F(t)=P(T\le t)$$