## Continuous Random Variables

A continuous random variable, ***X***, …

- takes any real # in *S* as a value
- Probabilities for *X* are described by the Probability Density Function (**PDF**)

Probability Density Function (**PDF**): *f(x)*

1) $f(x) > 0 \quad x \in S$

   $f(x) = 0 \quad x \notin S$

2) $\int_{-\infty}^{\infty} f(x)dx = P(S) = 1$

3) $P(a \le X \le b) = \int_a^b f(x)dx$

4) $P(X = a) = \int_a^a f(x)dx = 0$ → *f(a)* does not give the probability of a specific value

Cumulative Distribution Function (**CDF**): *F(x)*

- $F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)dt$
- $P(a \le X \le b) = F(b) - F(a)$
- The **PDF** is the derivative of the **CDF** for a continuous RV
  - $F'(x) = \frac{d}{dx} F(x) = \frac{d}{dx} \int_{-\infty}^{x} f(t)dt = f(x)$

- $M(t) = E\left[e^{tX}\right] = \int_{-\infty}^{\infty} e^{tx} f(x)dx$
- $E(X) = \int_{-\infty}^{\infty} x\, f(x)dx \qquad = M'(0)$

  $E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)dx \qquad = M''(0)$
- $Var(X) = E\left[(X - \mu_x)^2\right]$

  $= \int_{-\infty}^{\infty} (x - \mu_x)^2 f(x)dx$

  $= E(X^2) - \left[E(X)\right]^2 = M''(0) - M'(0)$