**<u>Independent Trials Model</u> (a.k.a., the Binomial Probability Model)**

- ***n*** independent trials conducted
- Success or Failure (*S* or *F*) on each trial
- *P(Success)* = *p* is the same for each trial

**X** = # of Successes in ***n*** independent trials → **X ~ Binomial(*n*, *p*)**

$$P(X=k)=\frac{n!}{k!(n-k)!}p^k(1-p)^{n-k} \quad x=1,2,\ldots,n \quad z.o.w.$$

**<u>Poisson Approximation to the Binomial</u>**

- In the 19th Century, ***n!*** is difficult to compute for large ***n***
  - Usually estimated by Stirling's Formula: $n!\approx\sqrt{2\pi n}\left(n/e\right)^n$ [note that ***e*** is involved]
- Simeon Poisson developed an approximation to the Binomial RV in 1830
  - Used for large ***n***umber of trials with small ***p***=*P(Success)* on each trial
  - Based on the parameter λ=***np***, the mean of a Binomial(*n*,*p*) RV
- Not just an approximation, but a valid ***PMF*** (i.e., $\sum_{x=0}^{\infty} f(x)=1$ )
  - Popularized by Ladislaus Bortkiewicz in 1898 who used it to model accidental deaths from horse kicks

**<u>Poisson Random Variable</u>**

$$f(x)=P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!} \quad x=0,1,2,3,\ldots$$

*Recall the MacLaurin series for* $\boldsymbol{e^y}$ [note that it involves factorials]

$$f(y)=f(0)+f'(0)\bullet y+\frac{f''(0)}{2!}\bullet y^2+\frac{f'''(0)}{3!}\bullet y^3+\ldots$$

$$e^y = 1 + y + \frac{y^2}{2!} + \frac{y^3}{3!} + \ldots = \sum_{x=0}^{\infty}\frac{y^x}{x!}$$