A function $f(x)$ possessing derivatives of all orders at $x = b$ can be expanded in the following **Taylor series**:

$$f(x) = f(b) + \frac{f'(b)}{1!}(x-b) + \frac{f''(b)}{2!}(x-b)^2 + \frac{f'''(b)}{3!}(x-b)^3 + \cdots.$$

If $b = 0$, we obtain the special case that is often called the **Maclaurin series**;

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots.$$

X is a RV with PMF $f(x)$ → $E(X) = \sum_{x=-\infty}^{\infty} x\,f(x)$

$g(x)$ is any real-valued function → e.g., $g(x) = e^{2x}$

$g(X)$ is a <u>*new*</u> RV → $E[g(X)] = \sum_{x=-\infty}^{\infty} g(x)\,f(x)$

<u>Moment Generating Function</u> (MGF) $M(t) = E\left[e^{tX}\right] = \sum_{x=-\infty}^{\infty} e^{tX} f(x)$

1) The MGF is a function of t, not a function of x
2) $M(0) = 1$
3) $M'(0) = \frac{d}{dt} M(t)\big|_{t=0} = E(X)$
4) $M''(0) = E(X^2)$
5) $Var(X) = M''(0) - [M'(0)]^2$
6) There is a 1-1 correspondence between MGFs and PMFs