Random Variable (RV):

- A real valued function defined on the sample space of an experiment.
- A variable whose value is a numerical outcome determined by chance.

  - X = # of heads in 2 tosses of a coin
  - Y = # of pips on a tossed die

Types of Random Variables

- *Continuous* Random Variables have an infinite number of possible values.
- *Discrete* Random Variables have a countable number of possible values.

Probability Distribution (for a *continuous* RV)

- represented as a *Probability Density Curve*
- Areas under a smooth curve indicate probabilities of values in a given range

Probability Distribution (for a *discrete* RV)

- Indicates how to assign probabilities for each of the possible values of a RV
- Called a Probability Mass Function (*PMF*)
- represented graphically as a *Probability Histogram*

Probability Mass Function (*PMF*) for a random variable ***X***

1. $f(x) \geq 0 \quad \text{for } x \in S$
2. $\sum_{x \in S} f(x) = 1$
3. $P(X \in A) = \sum_{x \in A} f(x)$

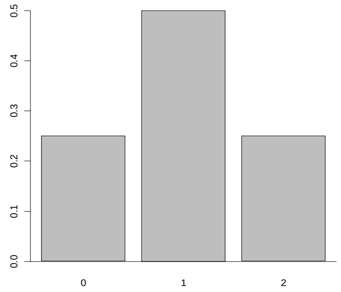
$$\boldsymbol{f(x) = P(X=x)}$$

Example: **X** = # of heads in 2 tosses of a coin

Outcome	**Probability**	***x***
TT	.5*.5=.25	0
TH	.5*.5=.25	1
HT	.5*.5=.25	1
HH	.5*.5=.25	2

⇨

x	***f(x)*= P(X=*x*)**
0	0.25
1	0.50
2	0.25



Let *f*(*x*) be the PMF for a *discrete* RV ***X*** → *f*(*x*) = P(X = *x*)

<u>Expected Value of a Random Variable</u>

- a weighted average of all possible values for *X*, weighted by the probability of each value
- $E(X) = \mu$, the mean for the RV
  - $E(X) = \sum_{x=-\infty}^{\infty} x\, f(x)$ for a *discrete* RV

<u>Example (*discrete* RV)</u>:
Y = # heads in two tosses of a fair coin

P(Y = 0) = ¼
P(Y = 1) = ½
P(Y = 2) = ¼ and zero otherwise

$$E(Y) = 0 \bullet P(Y=0) + 1 \bullet P(Y=1) + 2 \bullet P(Y=2)$$

$$= 0 \bullet \frac{1}{4} \quad + \quad 1 \bullet \frac{1}{2} \quad + \quad 2 \bullet \frac{1}{4} \quad = \quad 1$$

<u>Variance of a Random Variable</u>

- a weighted average of *squared deviations from the mean*, $[x - E(X)]^2$
- $\text{var}(X) = \sigma^2$

$$Var(X) = E[(X-\mu)^2] = \sum_{x=-\infty}^{\infty} \left(x - E(x)\right)^2 f(x)$$

<u>Example (*continued*)</u>:
Y = # heads in two tosses of a fair coin

P(Y = 0) = ¼
P(Y = 1) = ½
P(Y = 2) = ¼ and zero otherwise

$$\mu = E(Y) = 1$$

$$Var(Y) = (0-\mu)^2 \bullet P(Y=0) + (1-\mu)^2 \bullet P(Y=1) + (2-\mu)^2 \bullet P(Y=2)$$

$$= (0-1)^2 \bullet \frac{1}{4} \quad + (1-1)^2 \bullet \frac{1}{2} \quad + (2-1)^2 \bullet \frac{1}{4} \quad = \frac{1}{2}$$