P(• | B) satisfies the 3 Axioms of Probability

1. $0 \le P(A \mid B) \le 1$
2. $P(S \mid B) = 1$
3. If $A_1, A_2, \ldots, A_n$ are all mutually exclusive
   Then, $P(A_1 \cup A_2 \cup \ldots \cup A_n \mid B) = P(A_1 \mid B) + P(A_2 \mid B) + \ldots + P(A_n \mid B)$

Multiplication Rule:

$$P(A \cap B) = P(A \mid B)P(B)$$
$$= P(B \mid A)P(A)$$

Note: P(A∩B) = P(A) • P(B) only if independent

Law of Total Probability (LTP):

$$A = (A \cap B) \cup (A \cap B')$$
$$P(A) = P(A \cap B) + P(A \cap B') \quad [\textit{axiom } 3]$$
$$=$$

Ex (LTP):
An insurance company rents 35% of its cars from Cheapos Inc. (65% elsewhere)
8% of cars from Cheapos break down.
5% of cars from other companies break down.
Find the probability that a rented car breaks down.

Three events are <u>mutually independent</u> if each of the following hold:

1. $P(A \cap B) = P(A)P(B)$
2. $P(A \cap C) = P(A)P(C)$
3. $P(B \cap C) = P(B)P(C)$
4. $P(A \cap B \cap C) = P(A)P(B)P(C)$

<u>Theorem:</u> If **A** and **B** are independent, Then **A** and $\mathbf{B}^{\mathbf{C}}$ are independent.

<u>Proof:</u> $P(A \cap B') = P(B' \mid A)P(A)$ [*multiplication rule*]

Ex: 20% of the fish in a lake have a Lamprey attached (L), the others are clean (C).
Suppose we catch 5 fish (sampling with replacement), and define the following events:

A: at least one has a Lamprey attached
B: only the last 2 fish have a Lamprey attached
D: any 2 of the fish have a Lamprey attached.

Independent Trials Model (a.k.a., the Binomial Probability Model)

- ***n*** independent trials conducted
- Success or Failure (*S* or *F*) on each trial
- P(Success) = *p* is the same for each trial

Example: **Z** = # of successes in *n*=4 trials

e.g., tossing **3** Heads in **4** trials:

Outcome	Probability	z
SSSF	p*p*p*(1-p)	3
SSFS	p*p*(1-p)*p	3
SFSS	p*(1-p)*p*p	3
FSSS	(1-p)*p*p*p	3

$\Rightarrow$ **P(Z=3) = 4*p^3(1-p)**

4 **is the number of ways of ordering the S's and F's**

Binomial Probability Model:

- **X** = # of Successes in ***n*** independent trials
- $P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$
    - $\binom{n}{k}=\frac{n!}{k!(n-k)!}$ gives the number of ways to order the Successes & Failures
    - $n!=n\bullet(n-1)\cdots\bullet 2\bullet 1$ → e.g., $4!=4\bullet 3\bullet 2\bullet 1=24$