Exercise 1. *Watch your classmates' presentations (3) and write down 2 questions for each, related to graph theoretical tools.* 2 points

Chromatic polynomial

Here, we will follow some of the notes by Andrew Goodall:
`https://iuuk.mff.cuni.cz/~andrew/VKKI.pdf`
The notes here are a bit more condensed.
Recall that for the join $G \oplus H$ of two graphs G and H, we have

$$P_{G\oplus H}(z) = P_G(z) \circ P_H(z),$$

where $z^{\underline{i}} \circ z^{\underline{j}} = z^{\underline{i+j}}$ extended linearly to polynomials.

Exercise 2. *Show that for the disjoint union of two graphs $G + H$, we have* 2 points

$$P_{G+H}(z) = P_G(z) \cdot P_H(z).$$

In class and in the notes, we showed that the chromatic polynomial can be defined recursively in terms of edge deletions and contractions. For a multigraph G with at least one edge $e \in E(G)$, we have

$$P_G(z) = P_{G\backslash e} - P_{G/e}.$$

As a base case, we let $P_{E_n} = z^n$, where E_n is the empty graph on n vertices. Note that this description of the chromatic polynomial also proves that the function is indeed a polynomial of order n.

Exercise 3. *Use the recursive formula to show that, for any tree T on n vertices,* 2 points

$$P_T(z) = z(z-1)^{n-1}.$$

Exercise 4. *Use the recursive formula to show that, for any cycle C_n on n vertices,* 2 points

$$P_{C_n}(z) = (k-1)^n + (-1)^n(k-1).$$