Fundamental Theorem of Finitely Generated Abelian Groups

Exercises

Exercise 1. Show that the center of a direct product is the direct product of the centers 5.1.1

$$Z(G_1 \times G_2 \times \cdots \times G_n) = Z(G_1) \times Z(G_2) \times \cdots \times Z(G_n).$$

Deduce that a direct product of groups is abelian if and only if each of the factors is abelian.

Exercise 2. Let A and B be finite groups and let p be a prime. Prove that any Sylow p-subgroup of $A \times B$ is of the form $P \times Q$ where $P \in Syl_p(A)$ and $Q \in Syl_p(B)$. 5.1.4

Exercise 3. Give the number of non-isomorphic abelian groups of the following orders: (a) 225, (b) 1600, (c) 1155. 5.2.1

Exercise 4. Give the list of invariant factors for all abelian groups of the following orders: (a) 270, (b) 9801, (c) 165. 5.2.2

Exercise 5. Give the list of elementary divisors for all abelian groups of the following orders: (a) 270, (b) 9801, (c) 165. 5.2.3