Vectors and 2D Motion

- Vectors and Scalars
- Vector arithmetic
- Vector description of 2D motion
- Projectile Motion
- Relative Motion -- Reference Frames

Vectors and Scalars

Scalar quantities: require magnitude & unit for complete description

Examples: mass, time, temperature, speed, …… (what others?)

2.7 *kg*
57 *°C*
60 *m/s*

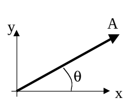
Vector quantities: require magnitude, unit & <u>*direction*</u> for complete description

Examples: displacement, velocity, acceleration …… (what others?)

500 m <u>north</u>
50 *m/s* <u>heading 040°</u>
9.8 m/s^2 <u>down</u>

Vector Notation

Vector quantities are graphically represented as arrows …….



The length of the arrow represents the magnitude of the vector and the direction is self-evident.

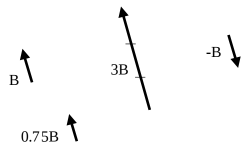
Vectors quantities are referred to by symbol, such as…
“$\vec{A}$” or “**A**” (arrow used on blackboard, boldface in the text, and on overheads)

A simple, unbold, unarrowed “A” refers to the magnitude of vector **A**.

A = |**A**|

Vector Math

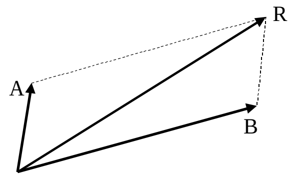
Vectors can be multiplied by scalars:



The result is that the magnitude of the vector changes, but not the direction, except in the case of muliplication by a negative number where the vector reverses direction.

Adding Vectors

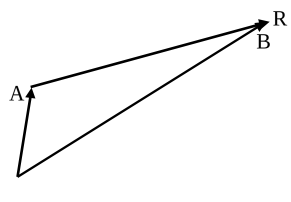
(parallelogram method)



A + B = R

Adding Vectors

("tip-to-tail" method)



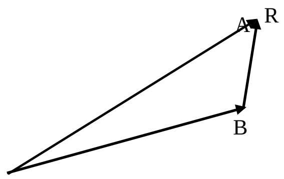
A + B = R

Adding Vectors

("tip-to-tail" method)

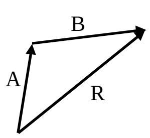
The result is independent of the order of addition!

B + A = A + B

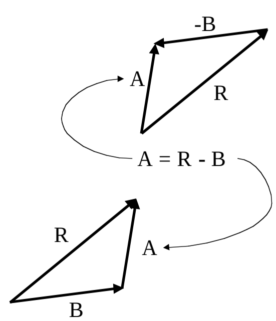


B + A = R

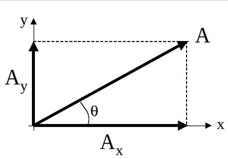
Subtracting Vectors



A + B = R



Vector Components



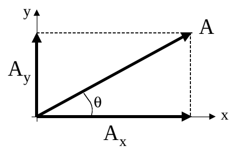
Vector A (with magnitude, A) is directed at an angle θ above the +x axis

$$A = A_x + A_y$$

A_x: vector component of A in the x-direction

A_y : vector component of A in the y-direction

Scalar Components



Given A, θ, the scalar components of A are

$A_x = A\cos(\theta)$
$A_y = A\sin(\theta)$

Scalar Components

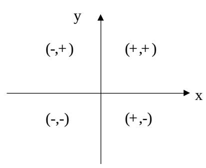
Given A_x, A_y,
how do you find A, θ ?

Magnitude:

$$A = \sqrt{A_x^2 + A_y^2}$$

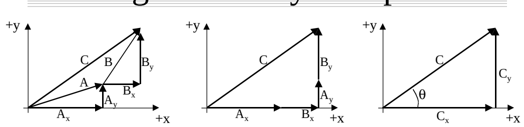
Angle:

$$\theta = \tan^{-1}(\frac{A_y}{A_x})$$



Be careful!
•Calculator in degree mode?
•Look at the signs of A_x and A_y. Does the angle make sense?
•Inverse tangent only gives back a result from -90° to +90°. How do you get the right quadrant?

Adding Vectors by Components



A+B=C

1. Choose coordinate system and draw a picture.
2. Find scalar components: A_x, A_y, B_x, B_y
3. Calculate scalar components:

   $C_x = A_x + B_x$ and $C_y = A_y + B_y$

4. Find: $C = \sqrt{C_x^2 + C_y^2}$
5. Find: $\theta = \tan^{-1}(\frac{C_y}{C_x})$

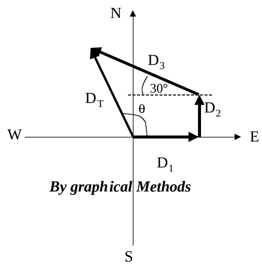
Example

Adding Displacement Vectors:

A hikers walks
- D_1 = 6.0 km east on day 1
- D_2 = 4.0 km north on day 2
- D_3 = 10.0 km at 30.0° north of west on day 3.

Find her total displacement for the Trip.

$D_T = D_1 + D_1 + D_1$



By graphical Methods

Example

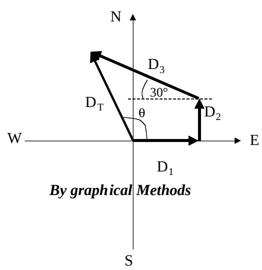
Analysis

	x-component	y-component
D_1	+6.0 *km*	0 *km*
D_2	0 *km*	+4.0 *km*
D_3	-10 cos(30°) = -8.7 *km*	+10 sin(30°) = +5.0 *km*
D_T	-2.7 *km*	+9.0 *km*

$$D_T = \sqrt{(-2.7)^2 + (9.0)^2}\, km = 9.4 km$$

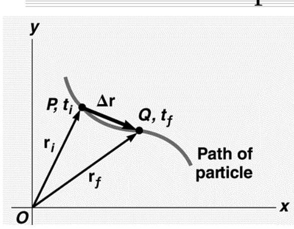
$$\theta = 180^\circ + \tan^{-1}\left(\frac{9.0}{-2.7}\right) = 180^\circ - 73^\circ = 107^\circ$$

(Needed to add 180° to get result in the correct quadrant)



By graphical Methods

Vector Description of Motion



A particle moves from P (at t_i) to Q (at t_f) along the ***trajectory*** shown above.

(A ***trajectory*** is a path of motion through space)

Average Vector Displacement:

$$\Delta r = r_f - r_i$$

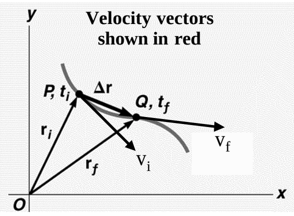
Average Vector Velocity:

$$\bar{\Delta v} = \frac{\Delta r}{\Delta t}$$

Instantaneous Vector Velocity:

$$\Delta v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t}$$

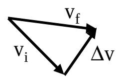
Vector Description of Motion



The velocity vector's direction is always tangent to the trajectory and in the direction of the motion.

The length of the velocity vector shows the instantaneous speed of the particle. Longer means faster.

How does velocity change?



$$\Delta v = v_f - v_i$$

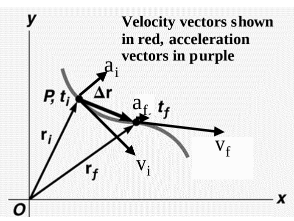
Average Vector Acceleration:

$$\overline{\Delta a} = \frac{\Delta v}{\Delta t}$$

Instantaneous Vector Acceleration:

$$\Delta a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

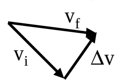
Vector Description of Motion



The velocity vector's direction is always tangent to the trajectory and in the direction of the motion.

The length of the velocity vector shows the instantaneous speed of the particle. Longer means faster.

How does velocity change?



$$\Delta v = v_f - v_i$$

Average Vector Acceleration:

$$\overline{\Delta a} = \frac{\Delta v}{\Delta t}$$

Instantaneous Vector Acceleration:

$$\Delta a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

Vector Description of Motion

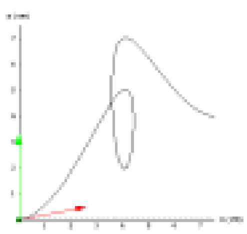
As a particle moves along its trajectory,

Acceleration vector shows how the velocity vector changes.

Velocity vector shows how the position changes

Both velocity and acceleration can change in *both magnitude and direction*

(This is a quicktime animation in the screen version)



Projectile Motion

Projectile Motion describes the motion of any object thrown into the air at any arbitrary angle.

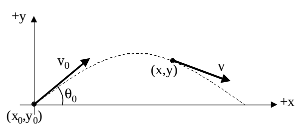
To describe projectile motion mathematically, we assume:

1. Projectile motion is uniformly accelerated motion. The acceleration vector is directed vertically downward and has a magnitude of $g = 9.8\ m/s^2$.

2. The effects or air resistance are negligible.

Quicktime Movie of a juggler

Analyzing Projectile Motion



The projectile is launched at initial velocity, v_0 at angle θ_0 above the horizontal.

Initial components are:

$$v_{x0} = v_0 \cos\theta_0$$
$$v_{y0} = v_0 \sin\theta_0$$

Subsequent to launch, *the x-component of velocity, remains constant;* only the y-component changes.

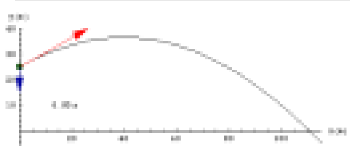
x - motion
$v_x = v_{x0} = v_0 \cos\theta_0 = \text{constant}$
$x = v_{x0}t = (v_0 \cos\theta_0)t$

y - motion
$v_y = v_{y0} - gt$
$y - y_0 = v_{y0}t - \frac{1}{2}gt^2$
$v_y^2 = v_{y0}^2 - 2g(y - y_0)$

The speed at time t is given by:

$$v = \sqrt{v_x^2 + v_y^2}$$

Example:



Find components first!

$$v_{x0} = 30\cos 30^\circ\, m/s = 26\ m/s$$
$$v_{y0} = 30\sin 30^\circ\, m/s = 15\ m/s$$

A ball is thrown off a 25 m high roof at a speed of 30 m/s at an angle of 30° above horizontal.

1) What is the maximum height?

$$v_{y0}^2 = 2g(y - y_0) \Rightarrow y = y_0 + v_{y0}^2/2g$$
$$y = 25\ m + (15\, m/s)^2/(19.6\, m/s^2) = 36\ m$$

2) At what time does it land?

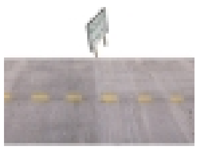
$$y - y_0 = v_{y0}t - 0.5gt^2$$
$$4.9t^2 - 15t - 25 = 0 \Rightarrow t = -1.2\ s, \quad t = +4.3\ s$$

3) How far does it travel? (What is the horizontal range?)

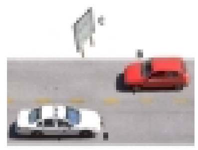
$$x - x_0 = v_{x0}t$$
$$x = 0 + (26)(4.3) = 110\ m$$

Relative Motion

How is a motion described by differerent observers in different reference frames which are in motion with respect to each other?

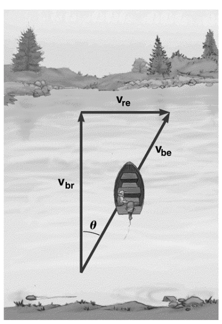


Relative Position



Relative Velocity

Relative Velocities



Consider the motion of a boat in a river.

v_{br} : velocity of b*oat* seen by r*iver*
v_{re} : velocity of r*iver* seen by e*arth*
v_{be} : velocity of b*oat* seen by e*arth*

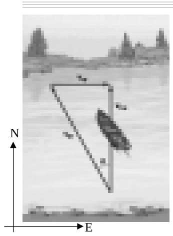
Vector equation:

$$v_{be} = v_{br} + v_{re}$$

Sign Convention:

$$v_{re} = - v_{er}$$

Example



At what angle do you point the boat to go straight across the river***?***

v_{br} : speed is 20 m/s, direction ???
v_{re} : current, 10 m/s, due east
v_{be} : speed is ???, due north

Vector equation:

$$v_{be} = v_{br} + v_{re}$$

In components:

East: $(v_{br})_x + (v_{re})_x = (v_{be})_x$
-20 m/s $\sin(\theta) + 10$ m/s $= 0$ ---> $\theta = 30°$

North: $(v_{br})_y + (v_{re})_y = (v_{be})_y$
20 m/s $\cos(30°) + 0$ m/s $= (v_{be})_y = v_{be} = 17$ m/s