Motion along a Line

- Describing Motion:
  - Displacement
  - Velocity
  - Acceleration
- Uniformly Accelerated Motion
- Free Fall

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Describing Motion along a Line

What is the position, velocity, and acceleration of the blue dot at each instant of time during its motion?

What would a graph of position vs. time look like?

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Linear kinematics variables

t:	time	$\bar{v}$:	average velocity
t_i:	initial time	$\bar{a}$:	average acceleration
t_f:	final time	a:	acceleration at t
x:	position at t	v:	velocity at t
x_0 :	position at t = 0	v_0 :	velocity at t = 0
x_i:	position at t_i	v_i:	velocity at t_i
x_f:	position at t_f	v_f:	velocity at t_f

These are the variables we use to refer to position, velocity and time for describing motion along a line.

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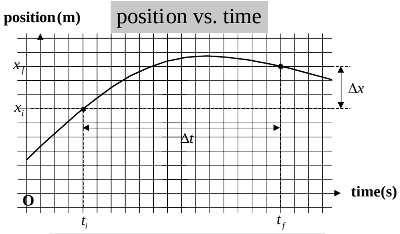
Displacement



Displacement: $\Delta x = x_f - x_i$

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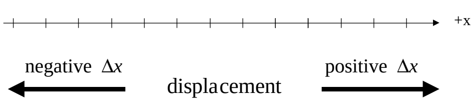
Displacement

Displacement is a ***vector quantity.*** It has a direction associated with it. The direction is implied by the algebraic sign of the displacement



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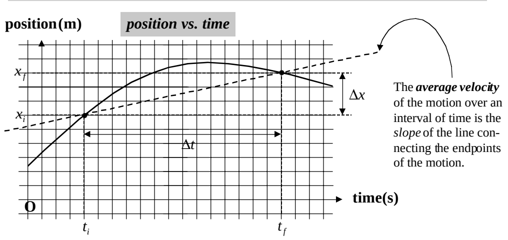
Average Velocity



average velocity: $\bar{v} = \dfrac{x_f - x_i}{t_f - t_i} = \dfrac{\Delta x}{\Delta t}$

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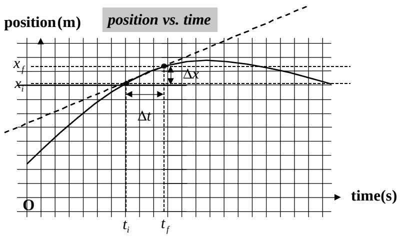
Instantaneous Velocity



If we look at successively smaller time intervals, we can obtain a better estimate of the velocity at a particular instant in time.

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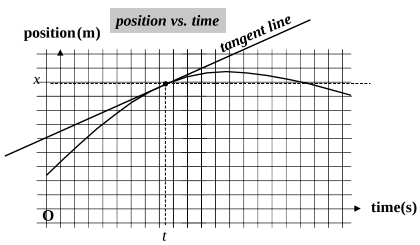
Instantaneous Velocity



Graphically, the ***instantaneous velocity***, *v*, at time, *t*, is given by the slope of the ***tangent line*** to the position vs. time graph

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Instantaneous Velocity

Mathematically, the instantaneous velocity, *v*, at time, *t*, is expressed as the limiting value of Δx/Δt as Δt shrinks to zero around a particular time, *t*

Mathematicians express this formally as …

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

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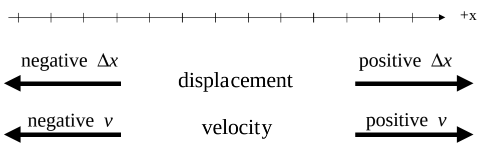
Displacement & Velocity

Like displacement, velocity is also a vector quantity**.** The direction of the velocity vector implied by the algebraic sign of the velocity



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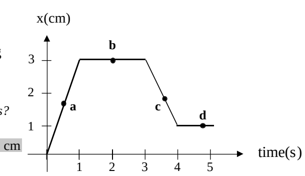
Concept Check

A ladybug walks along a meter stick. Here is a graph of the position of the ladybug as a function of time.



Q: *What is the ladybug's displacement from t =0 s to t=5 s?*

$\Delta x = 1.0\ \text{cm} - 0.0\ \text{cm} = +1.0\ \text{cm}$ →

Q: *What is the ladybug's average velocity from t =0 s to t=5 s?*

$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{+1.0\ \text{cm}}{5.0\ \text{s}} = +0.20\frac{\text{cm}}{\text{s}}$ →

Q: *What is the ladybug's instantaneous velocity at:*

point (a) $v = {}^{+3.0\ \text{cm}}/_{1.0\text{s}} = {}^{+3.0\ \text{cm}}/_{\text{s}}$ →

point (b) $v = {}^{0.0\ \text{cm}}/_{2.0\text{s}} = {}^{0.0\ \text{cm}}/_{\text{s}}$

point (c) $v = {}^{(1.0\ \text{cm} - 3.0\ \text{cm})}/_{1.0\text{s}} = {}^{-2.0\ \text{cm}}/_{\text{s}}$ ←

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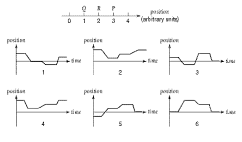
Concept Tests

A person initially at point P in the illustration stays there a moment and then moves along the axis to Q and stays there a moment. She then runs quickly to R, stays there a moment, and then strolls slowly back to P. Which of the position vs time graphs below correctly represents this motion?



An object goes from one point in sp anotherAfter it arrives at its destin displacement is:

1. either greater than or equal to
2. always greater than
3. always equal to
4. either smaller than or equal to
5. always smaller than
6. either smaller or larger

than the distance it traveled.

Pi Pi

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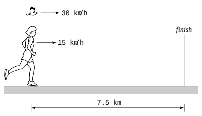
Concept Tests

A marathon runner runs at a stead km/hr. When the runner is 7.5 km from finish, a bird begins flying from the ru the finish at 30 km/hr. When the bird reach-es the finish line, it turns around and fl back to the runner, and then turns around again, repeating the back-and-forth until the runner reaches the finish. How many kilometers does the bird travel



1. 10 km
2. 15 km
3. 20 km
4. 30 km

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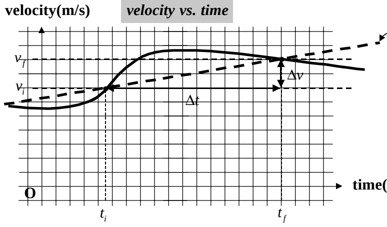
Average Acceleration



The ***average acceleration*** of the motion over an interval of time is the *slope* of the line connecting the endpoints of the motion on the v vs. t graph

average acceleration: $\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$

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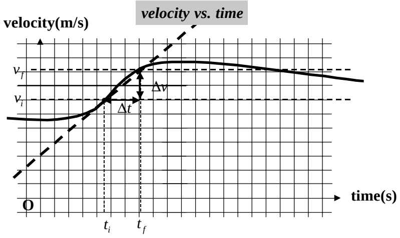
Instantaneous Acceleration



If we look at successively smaller time intervals, we can obtain a better estimate of the acceleration at a particular instant in time.

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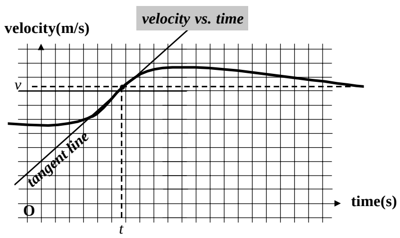
Instantaneous Acceleration



Graphically, the ***instantaneous acceleration***, *a*, at time, *t*, is given by the slope of the tangent line to the velocity vs. time graph

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Instantaneous Acceleration

Mathematically, the instantaneous velocity, *a*, at time, *t*, is expressed as the limiting value of $\Delta v/\Delta t$ as Δt shrinks to zero around a particular time, *t*

Mathematicians express this formally as …

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

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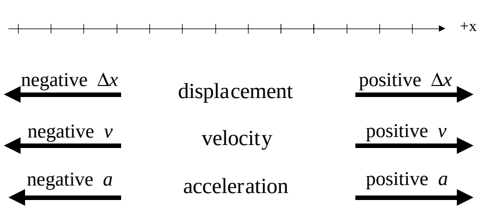
Displacement, Velocity & Acceleration

Like displacement and velocity, acceleration is also a ***vector quantity.*** The direction of the acceleration vector is implied by the sign of the acceleration



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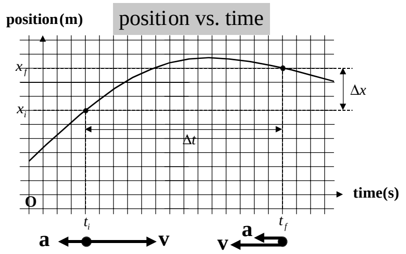
Interpreting position vs. time graphs



It is possible to look at the slope of the position vs. time graph and how the slope changes with time to determine the velocity and acceleration.

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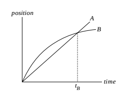
Concept Tests

1. If the acceleration is in the same direction as the velocity, the object is____________
   (A) speeding up.
   (B) slowing down.

2. If the acceleration is in the opposite direction to the velocity, the object is______
   (A) speeding up.
   (B) slowing down.

3. "Negative acceleration" and "deceleration" always mean the same thing.
   (A) True.
   (B) False.

The graph shows position as a funct time for two trains running on pa tracks. Which is true:



1. At time t_B, both trains have the same velocity.
2. Both trains speed up all the time.
3. Both trains have the same velocity some time before t_B.
4. Somewhere on the graph, both trains have the same acceleration.

Pi

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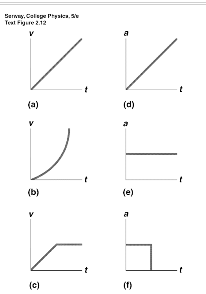
Concept Test



Match the correct acceleration vs. time graph to each velocity graph.

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Uniformly Accelerated Motion

Constant acceleration means a = ā doesn't change with time, therefore:

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t}$$

And so:

$$v = v_0 + at$$

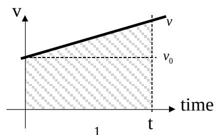
$$\Delta x = \frac{(v_0 + v)}{2}t = \frac{(v_0 + (v_0 + at))}{2}t$$

$$= v_0 t + \frac{1}{2}at^2$$

$$\Delta x = \frac{(v_0 + v)}{2}t = \frac{(v_0 + v)}{2}\frac{(v - v_0)}{a}$$

$$= \frac{(v^2 - v_0^2)}{2a} \quad \Rightarrow v^2 = v_0^2 + 2a\Delta x$$

Displacement is equal to the shaded area under the v vs. t graph, shown below.



$$\Delta x = v_0 t + \frac{1}{2}(v - v_0)t$$

$$\Delta x = \frac{(v_0 + v)}{2}t = \bar{v}t$$

$$\therefore \bar{v} = \frac{(v_0 + v)}{2}$$

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Uniformly Accelerated Motion

Equations of uniformly accelerated motion (after Serway, Table 2-3)

	Equation	*missing variable*
(1)	$v = v_0 + at$	$x - x_0$
(2)	$x - x_0 = \frac{1}{2}(v + v_0)t$	a
(3)	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
(4)	$v^2 = v_0^2 + 2a(x - x_0)$	t

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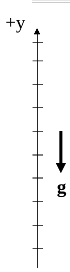
Free Fall



A freely falling object is an object moving under the influence of gravity only, regardless of its initial motion. Objects thrown upward or downward and those objects released from rest are all falling freely once they are released.

Once they are in free fall, all objects have a downward acceleration towards the ground which is the free fall acceleration, **g.**

Magnitude of **g** = 9.8 m/s² (or nearly so, in most places)

If we erect a linear y-coordinate axis so that y increases as one moves upward from the surface of the earth, then the free fall acceleration has a negative sign in this coordinate system.

a = -g = -9.8 m/s²

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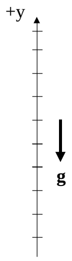
Free Fall



Free Fall Equations

	Equation	missing variable
(1)	$v = v_0 - gt$	$y - y_0$
(2)	$y - y_0 = \frac{1}{2}(v + v_0)t$	g
(3)	$y - y_0 = v_0 t - \frac{1}{2}gt^2$	v
(4)	$v^2 = v_0^2 - 2g(y - y_0)$	t

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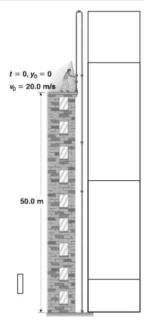
Free Fall- example



A stone is thrown straight up from a 50.0 m tall building at 20.0 m/s.

A) How much time is required for the stone to reach the highest point of motion

B) How high does the stone go?

C) How much time until the stone passes its launch point on the way down?

D) What is the stone's velocity when it passes its launch point on the way down?

E) What is the stone's velocity at t=5.0 s ?

F) What is the stone's velocity at the instant before it contacts the ground?

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