THE IDEAL STRING

Natural Modes

Natural Frequencies

Vibration Modes for an Ideal Stretched String

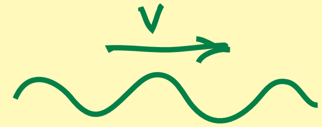
The Model:

- An ideal string has no stiffness and has uniform linear density (μ) throughout.
- The restoring force for transverse vibrations is provided solely by the tension (T) in the string.

Consequences:

- Can support travelling transverse waves with a characteristic speed.

speed of tranverse waves on a string: $$v = \sqrt{\frac{T}{\mu}}$$



T is the tension in Newtons (N)
μ is the linear density of the string (kg/m)
v is the speed in m/s

- The travelling waves can interfere with each other to form standing waves between the points of support of the stretched string.

- ONLY CERTAIN SPECIAL WAVELENGTHS WILL ALLOW CONSTRUCTIVE INTERFERENCE OF THE TRAVELLING WAVES. EACH OF THESE WAVELENGTHS CORRESPONDS TO A NATURAL MODE OF THE STRING, AND EACH NATURAL MODE HAS ITS OWN NATURAL FREQENCY.

Vibration Modes for an Ideal Stretched String

Natural Modes:

- **Standing waves occur only when the string length, L, is a whole number of half-wavelengths.**

Allowed wavelengths: $$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3, 4, \ldots\ldots\ldots$$

- **Each one of these wavelengths has its own particular frequency, given by $v = f\lambda$**

Allowed frequencies: $$f_n = \frac{v}{\lambda_n} \qquad n = 1, 2, 3, 4, \ldots\ldots\ldots\ldots$$

- **Each one of these allowed frequencies is the natural frequency of one of the natural vibration modes of the string. Note that these frequencies form a harmonic series based on $f_1 = v/2L$**

- **The natural modes each have a node at each end of the string, and n-1 additional nodes along the length of the string.**

Vibration Modes for an Ideal Stretched String

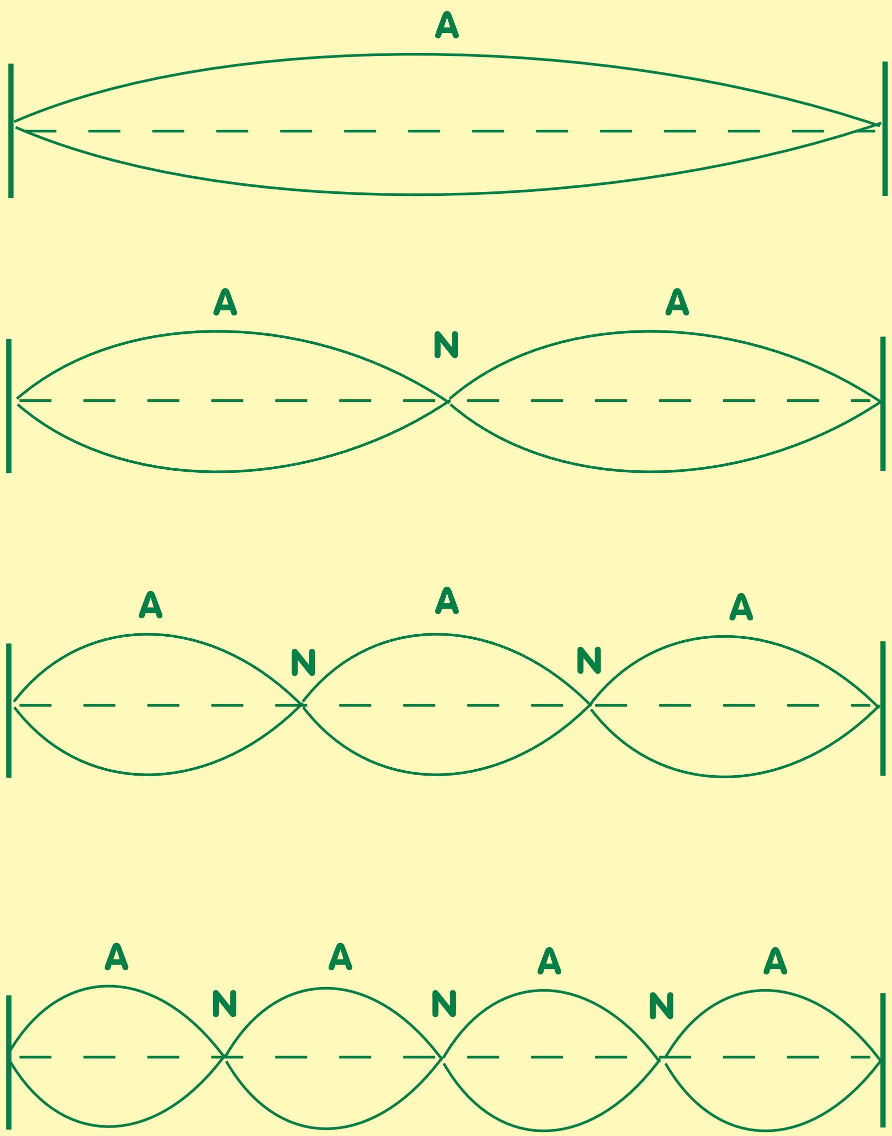
- **A right moving transverse wave can interfere with a left moving wave to give a standing wave so long as the interference pattern produces a node at each end.**

right moving wave

+ left moving wave

= standing wave

Vibration Modes for an Ideal Stretched String



Mode 1: $f_1 = v/2L$, $\lambda_1 = 2L$

Mode 2: $f_2 = 2f_1$, $\lambda_2 = L$

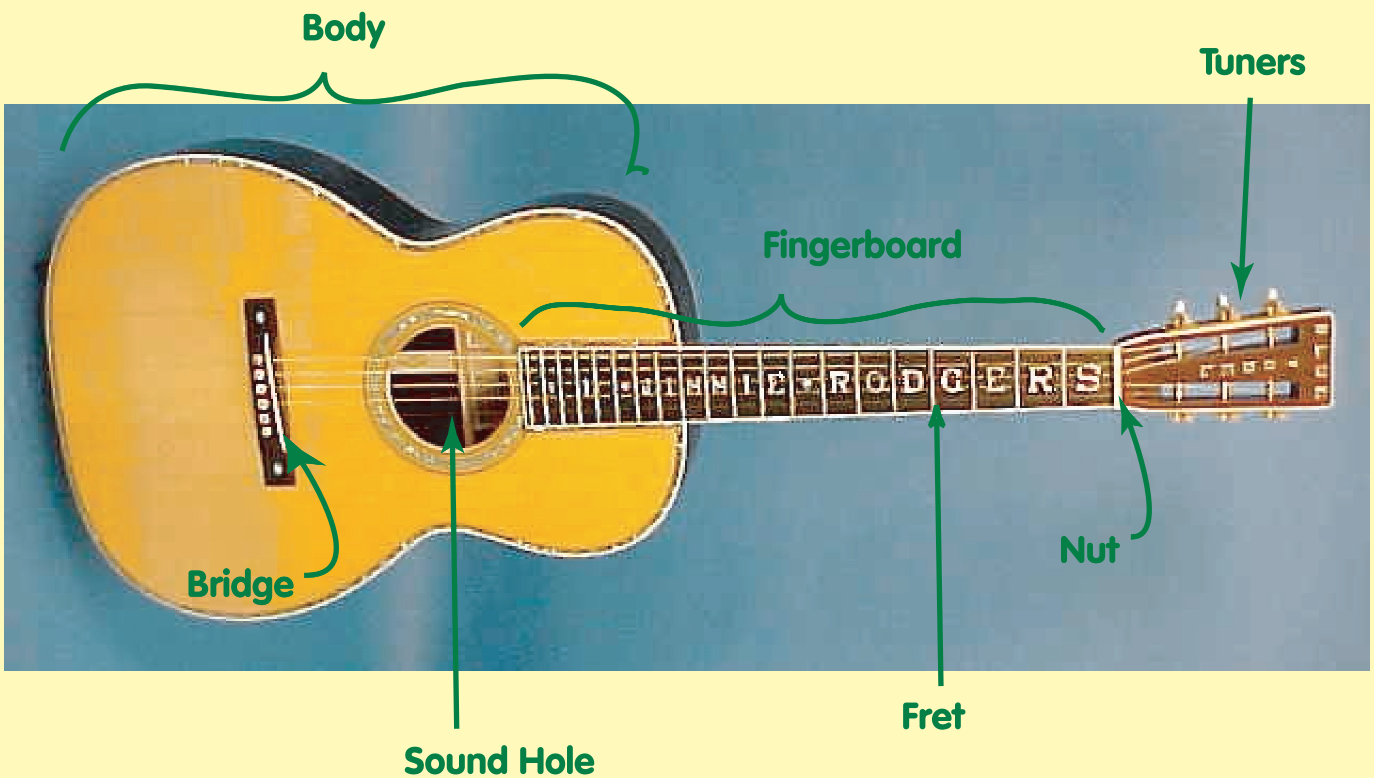
Mode 3: $f_3 = 3f_1$, $\lambda_3 = 2L/3$

Mode 4: $f_4 = 4f_1$, $\lambda_4 = L/2$

..... infinite number of modes possible

The Acoustic Guitar

Basic components of a Guitar:



- The guitar has six strings that run along the fingerboard and are free to vibrate between the nut and the bridge.
- The strings or usually pitched in standard tuning -- E3 A3 D4 G4 B4 E5 (top to bottom in the picture)
  [Mine is in open G tuning -- D3 G3 D4 G4 B4 D5]
- The frets are spaced to admit half tone intervals in the fundamental modes of each fretted string.
- The body of the guitar is designed to efficiently turn the vibrational energy of the strings into sound.

Acoustic Guitar Modes

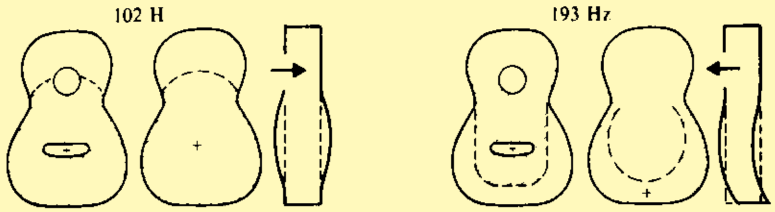


FIG. 10.28
Vibrational motion of a freely-supported Martin D-28 folk guitar at three strong resonances in the low frequency range.

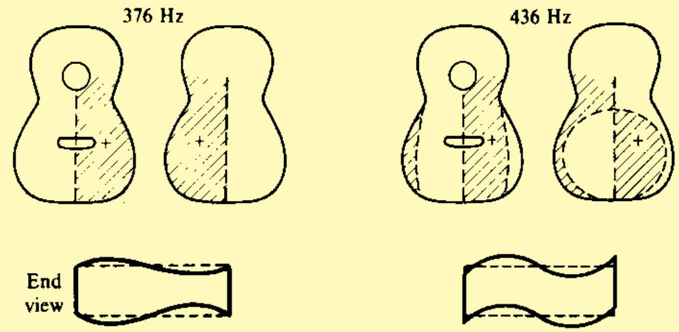


FIG. 10.29
Vibrational configurations of a Martin D-28 guitar at two resonances resulting from "see-saw" motion of the (1,0) type.

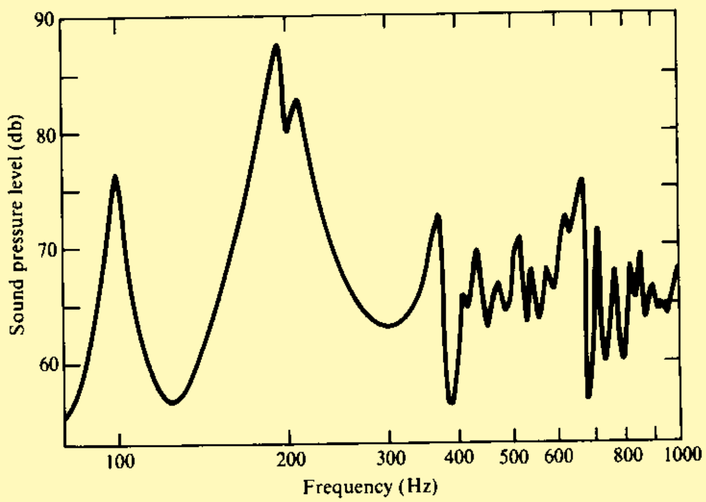


FIG. 10.31
Sound pressure level one meter in front of a folk guitar (Martin D-28) driven by a force of constant amplitude applied to the treble side of the bridge.

From Rossing: "The Science of Sound"

THE PIANO

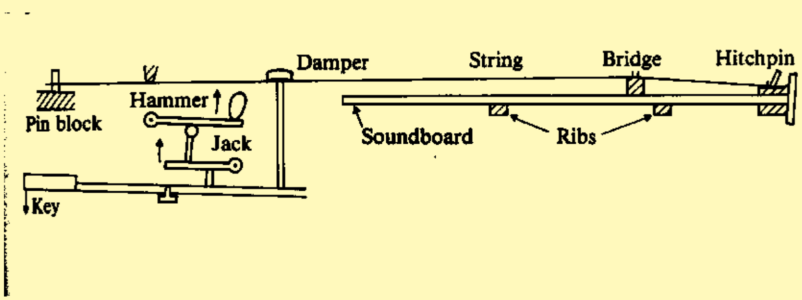


FIG. 14.1
A simplified diagram of the piano. When a key is depressed, the damper is raised, and the hammer is "thrown" against the string. Vibrations of the string are transmitted to the soundboard by the bridge.

PIANO SOUNDS

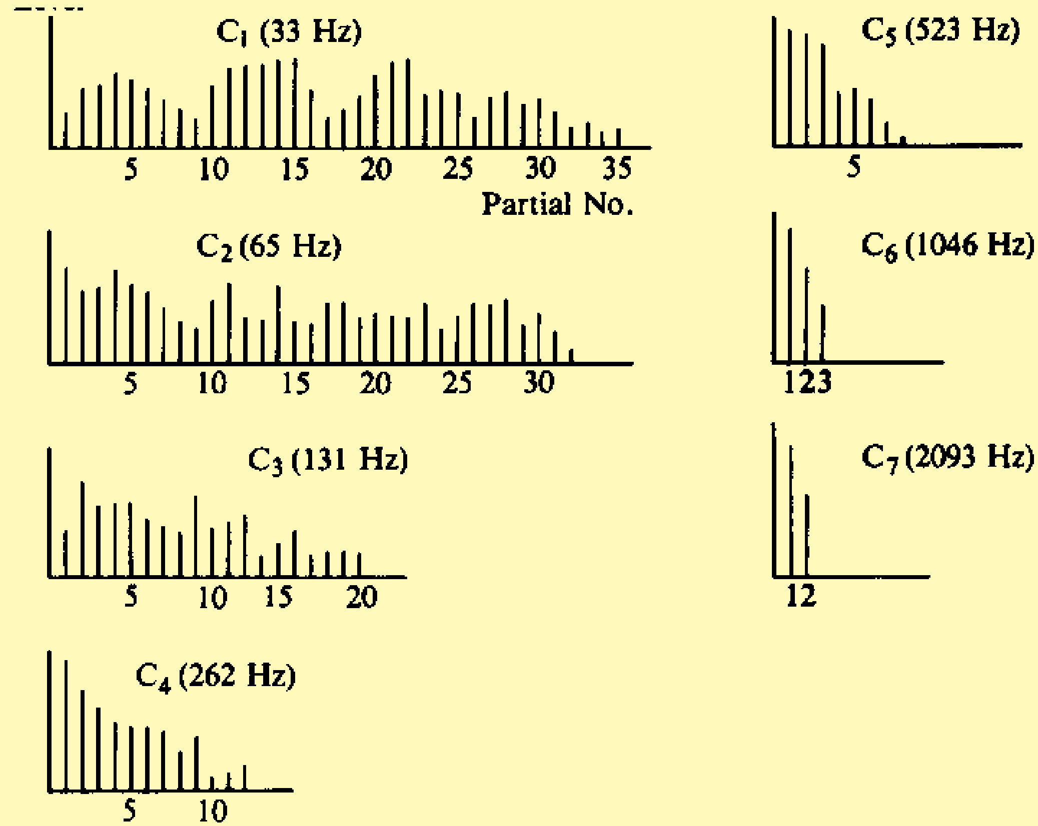


FIG. 14.5
The spectrum of high and low piano notes. (After Fletcher et al., 1962).

The spectrum of each note depends on where the hammer strikes, the mass of the hammer compared to the strings it hits and the body resonances of the piano's soundboard.

PIANO TUNING

Effects of string stiffness

$$f_n = n\, f_1\, (1 + (n^2 - 1)\, A)$$

$$A = \frac{\pi^3\, r^4\, E}{T\, L^2}$$

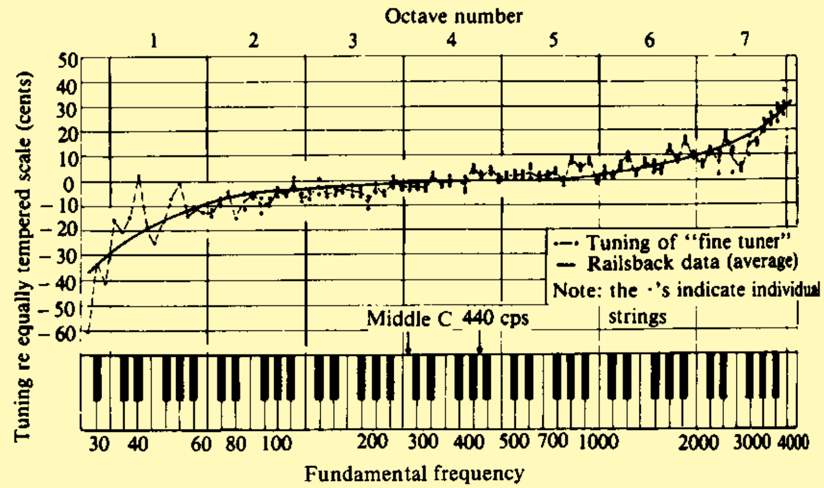
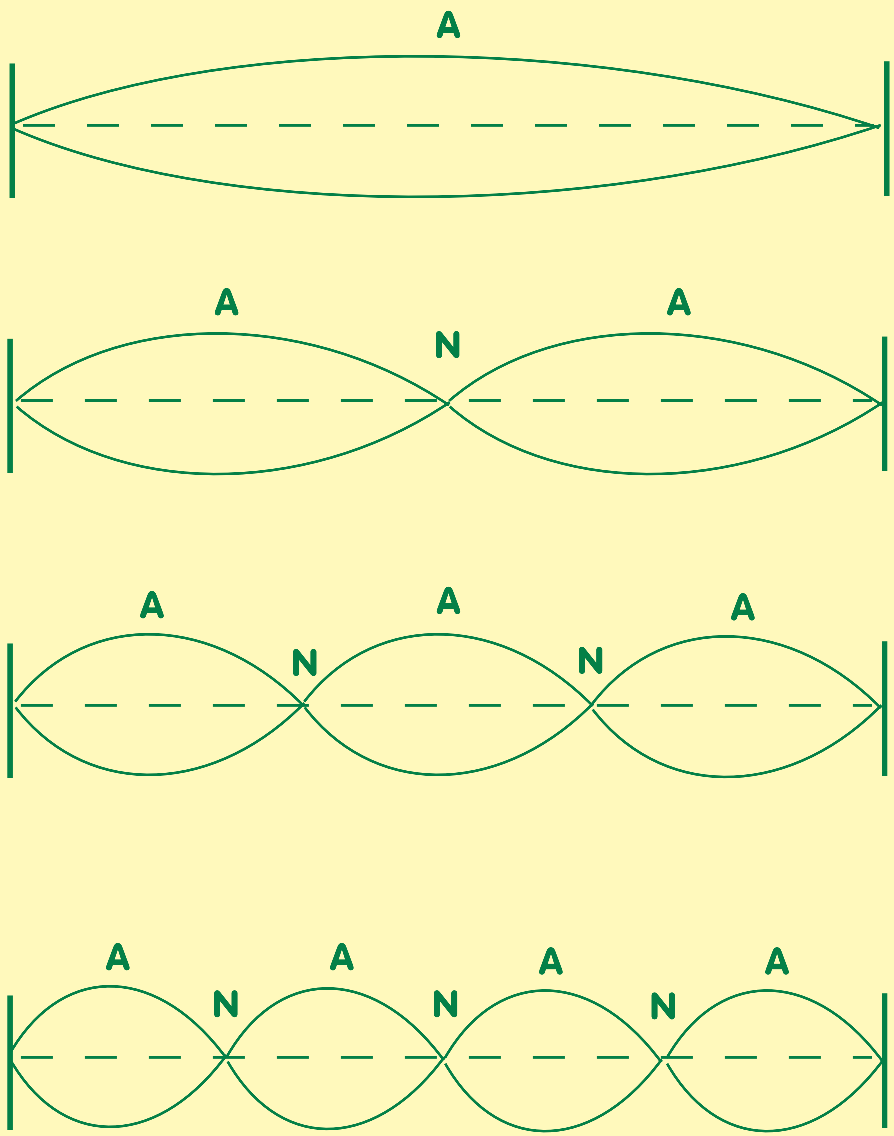


FIG. 14.4
Deviations from equal temperament in a small piano. (From Martin and Ward, 1961.)

Piano strings act somewhat like beams as well as like ideal strings. The thicker the string, the more its higher harmonic frequencies are sharpened from the value you would expect from a harmonic series. Because of this, pianos are tuned in "stretch tuning" so that the fundamental frequencies of the higher notes will not beat against the sharpened harmonics of the lower note.

In addition, the three strings that act together to sound a note are actually detuned from each other by a couple of cents so that the note's decay time will be longer.

Vibration Modes for an Ideal Stretched String



Mode 1: $f_1 = v/2L$, $\lambda_1 = 2L$

Mode 2: $f_2 = 2f_1$, $\lambda_2 = L$

Mode 3: $f_3 = 3f_1$, $\lambda_3 = 2L/3$

Mode 4: $f_4 = 4f_1$, $\lambda_4 = L/2$

..... infinite number of modes possible