Modal Logic and Contingentism: A Comment on Timothy Williamson's *Modal Logic as Metaphysics*

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Logic, as standardly conceived, is the science of consequence: a logic tells us which claims follow from which. Quantified modal logic (QML) is supposed to tell us which claims follow from which in a language which provides the means to express both metaphysical modality and quantification.[1] One might think on this basis that we can determine which is the correct QML independently of substantive questions of science and metaphysics: modal logic, on this view, cannot tell us whether it is possible that there be no numbers; nor can it tell us whether, necessarily, everything is at bottom physical. At most, it might be held, logic can tell us what follows from these claims. We might dub this view *neutralism about QML*, since it holds that the correct QML must be neutral on substantive disputes in modal metaphysics.

It is difficult to find an extended, full-throated defense of neutralism in the literature.[2] But Timothy Williamson's *Modal Logic as Metaphysics* provides an extended, full-throated criticism. Williamson aims to show how, in particular, the model theory of QML bears on a substantive, if highly abstract, dispute in modal metaphysics.

Consider one of the most obvious things about the Sun: it is such that there is something identical to it. More briefly, it is something. Being something, of course, does not make it unique. It shares this feature with everything. Is

[1]Of course, there are applications of the study of the formally specified languages of QML in which '$\Box$' is interpreted as something other than metaphysical modality. Those applications of the formalism are irrelevant to our discussion.

[2]Perhaps [Kaplan, 1989, pp. 42-43] offers a defense.

the Sun, however, such that, possibly, *nothing* is identical to it? Is it possible, that is, that it have failed to be anything at all? Plausibly, the answer is *yes*. Plausibly, the physical constants characterizing the laws of our universe might have been just different enough so that there were no stars at all, and so no Sun. Consider another, equally obvious thing about you: you are such that, actually, something is identical to you.[3] You share this feature with everything. But might there have been something which, in contrast with you and everything else there actually is, is actually nothing? Again, plausibly, the answer is *yes*. There might, it seems, have been a magma plume below Vermont, so that there was a volcano where instead my university actually stands. But, it would seem, if there were such a volcano, it would also be such that, actually, nothing is identical to it.

The view Williamson calls *necessitism* answers *no* to both of these questions. Necessitists hold that, necessarily, everything is such that, necessarily, something is identical to it. More briefly, necessitism is the view that, necessarily, everything is necessarily something. So, necessitists hold that the Sun is necessarily something, and thus would be something even if there were no stars.[4] Similarly, necessitists hold that, necessarily, any volcano in Vermont is necessarily, hence actually, something. The question of whether necessitism or its negation, *contingentism*, is true is a substantive metaphysical dispute. Nevertheless, Williamson argues, one can fruitfully bring the model theory of QML to bear on this dispute, thereby providing reasons for favoring necessitism.

The book starts with an explanation of necessitism and contingentism and related views (Ch. 1) and a history of the development of QML and modal metaphysics (Ch. 2). Williamson then explores a proposal concerning how possible worlds semantics bears on the necessitism-contingentism dispute (Ch. 3). In later chapters, he considers a particular contingentist approach to possible worlds semantics (Ch. 4), and offers an explanation and defense of higher-order QML (Ch. 5). He then lays out two main arguments that necessitism is preferable to contingentism (Chs. 6-7). Briefly, Williamson contends that (*i*) standard contingentists cannot accept higher-order comprehension principles

[3]The 'actually' operator here is the operator defined by Kaplan [1989]: in particular, ϕ is simply true iff 'necessarily actually ϕ' is.

[4]In a situation in which there are no stars, a necessitist would of course say that the Sun would not be a star; Williamson suggests that she also say that the Sun would not be concrete (pp. 7-8). Perhaps a necessitist might be tempted to say that the Sun does not exist in such a situation; Williamson counsels against this strategy (p. 19).

of the strength necessary for ordinary mathematical purposes; and (*ii*) that necessitists but not contingentists of a certain stripe can make distinctions, using the resources of higher-order QML, that the contingentist ought to regard as legitimate. He closes the book by describing some of the philosophical upshots of necessitism and making some methodological remarks (Ch. 8 ff.). I will focus in what follows on Williamson's Ch. 3 proposal for bringing possible worlds semantics to bear on the debate over necessitism.

1 Kripke-Style Semantics for QML

Since Frege and Russell at least, the study of logic has focused on artificial, formally specified languages, whose syntax and semantics are particularly easy to state and study, and whose primitive symbols, with the exception of a few distinguished connectives, operators, *etc.*, are often uninterpreted. These uninterpreted strings are then given a semantics which specifies in set-theoretic terms an interpretation for each complex expression on the basis of stipulated interpretations for the simpler expressions from which it is derived. In the broadly Kripkean possible worlds semantics for QML that is now standard, each such model presumes that, in the background, we are given a non-empty set of indices W, a distinguished member of that set of indices w, a relation R on W, and a function $\mathcal{D}$ that maps each member w^* of W to a set $\mathcal{D}(w^*)$.[5] Williamson calls any tuple $\langle W, R, w, \mathcal{D}\rangle$ meeting these conditions an *inhabited model structure* (p. 121).

The details of the semantic clauses specifying conditions for truth in a model are a matter of some difficulty. For now, let's use an extension of the system proposed in [Kripke, 1963]. In particular, we follow that treatment by considering a language with no individual constants.[6] We will extend Kripke's treatment by allowing a distinguished identity predicate '='. In a given model M, predicates are assigned extensions at every world; the extension of an n-place predicate at a world is a set of n-tuples of objects, where each object is in the domain of some world. Open formulae are evaluated relative to both a world w and an assignment a. Any of the values of a may be members of the domain of any

[5]Some work in broadly Kripkean possible worlds semantics does not assume that we are given a distinguished member w of the set of indices W. I will ignore this wrinkle in what follows.

[6]This is the one respect in which the present setup differs from Williamson's (p. 121). I will revisit the question of individual constants later.

world whatsoever. The clauses for atomic formulae, conjunctions, and negations are just what one would expect them to be. Quantifier ranges, however, are restricted to the domains of the world of evaluation: a quantified formula $(\forall x)\phi$ is true at a world w on an assignment a iff ϕ is true at w on every x-variant of a that assigns a member of $\mathcal{D}(w)$ to x. Truth for a sentence (a closed formula) at a world can be de-relativized from assignments by quantification in any of a number of ways: we might, for instance, say that a sentence is true at w iff it is true at w on every assignment, or we might instead say that a sentence is true at w iff it is true at w on some assignment. It doesn't matter, since a sentence true at w on any assignment is true at w on all of them.[7] Williamson chooses the universal form of de-relativization (p. 120), and extends it to all formulae: a formula ϕ is true at w iff it is true at w on all assignments. We will revisit this choice in our discussion below.

Here, for convenient reference, are the semantic clauses for the recursive definition of the truth of a formula ϕ in a model M at a world w on an assignment a ($M, w, a \vDash \phi$). V is the model's assignment of extensions in every world to the non-logical predicates (*i.e.*, the predicates other than '=') of the language.

1. $M, w, a \vDash Fx_1, x_2, ..., x_n$ iff $\langle a(x_1), a(x_2), ..., a(x_n)\rangle \in V(F)(w)$;

2. $M, w, a \vDash x_1 = x_2$ iff $a(x_1) = a(x_2)$;

3. $M, w, a \vDash \neg\phi$ iff $M, w, a \nvDash \phi$

4. $M, w, a \vDash (\phi \wedge \psi)$ iff $M, w, a \vDash \phi$ and $M, w, a \vDash \psi$;

5. $M, w, a \vDash (\forall x)\phi$ iff $(\forall d \in \mathcal{D}(w))M, w, a[x/d] \vDash \phi$; and

6. $M, w, a \vDash \Box\phi$ iff $(\forall w^*)(wRw^* \Rightarrow M, w^*, a \vDash \phi)$.

2 Metaphysical Universality

With these technical details on the table, it's easy to see that studying the formal properties of the language of QML using possible worlds semantics is an

[7] Here I assume that $\mathcal{D}(w^*)$ is non-empty for all $w^* \in W$. Technically, the definition of an inhabited structure imposes no such requirement. In the present context, the assumption is harmless because intended structures will verify the assumption: it is necessary, for instance, that something is the empty set. If we drop the assumption, then the point in the main text can be made by appeal to a de-relativization which says that a sentence is true at w iff there are at least as many assignments on which it is true at w as those on which it is false at w.

interesting, powerful, but ultimately purely mathematical enterprise. After introducing the notion of a model structure, it is traditional to offer an elucidatory remark like, "intuitively, W is a set of *possible worlds*, w is the *actual world*, R is an *accessibility relation*, and $\mathcal{D}(w^*)$ contains the objects that *exist* at w^*." But these elucidatory remarks are just supposed to help the reader cotton on to the roles the various elements are supposed to play in the semantics; they're not officially part of the mathematical story. Officially, any entity whatsoever can figure in a Kripke-style model. You, for instance, are an index of some inhabited model structure. How, then, are we to get the math to bear on the metaphysics?

Standardly, it is thought that we can bring model theory to bear on metaphysics by isolating an *intended class of models*. Models in the intended class interpret the expressions of the language of QML in the way we intend when we are asking after the features of modal reality. So, the sentences verified by all models in such a class may reasonably be taken accurately to characterize features of modal reality. But which classes of models are intended? Williamson's strategy for answering this question proceeds in two steps. First, isolate what one might plausibly take to be the *logical truths* concerning metaphysical necessity:[8] a set of formulae in the language of QML that, when '$\Box$' is interpreted as metaphysical necessity, '$=$' is interpreted as identity, quantifiers are interpreted as absolutely unrestricted, and connectives are interpreted in the standard way, are true independently of the interpretations of other symbols.[9] Second, specify a class of *intended model structures* by appeal to the set of logical truths: a structure is intended iff its logic – the formulae verified by all models with that structure – coincides exactly with the set of logical truths concerning metaphysical necessity. Then a class of models is intended if it is the class of models of an intended structure.

What are the logical truths of metaphysical modality? We will isolate only those truths which may be expressed in the language of first-order QML, though

[8] I am ignoring here the distinction between logical truth and logical consequence; see p. 94. This distinction will not substantially affect the discussion.

[9] The *non-logical truths* of metaphysical necessity are those whose truth depends, in part, on the intended interpretations of other vocabulary. So, for instance, if 'it is necessary that Socrates is not a world war' is true, then its truth depends on the intended interpretation of, among other things, 'is a world war'; if 'is a world war' were interpreted so as to express the property *being a philosopher* while other expressions retain their actual interpretations, then the erstwhile truth (or its orthographic duplicate) would be false.

Williamson's criterion will appeal to an extension of that language.[10] Suppose we are given a formula of the language of QML, and an infinite stock of fresh variables of the following types: invididual variables, and n-place predicate variables, for each natural number n. (We think of sentence letters as 0-place predicates.) Uniformly replace each of the non-logical constants by variables of the appropriate type, then close the result by prefixing universal quantifiers for each free variable. Call this the *universal generalization* of original formula. Suppose, to illustrate, that the formula is $\Box(\exists x)(Fx \Leftrightarrow x = y)$. We obtain its universal generalization by first replacing the only occurrence of the only non-logical constant 'F' in the formula by a variable 'Y' to yield $\Box(\exists x)(Yx \Leftrightarrow x = y)$, and then prefixing the result with appropriate universal quantifiers to yield $(\forall Y)(\forall y)\Box(\exists x)(Yx \Leftrightarrow x = y)$. Notice that occurrences of quantifiers, variables, modal operators, connectives, and the identity predicate remain.

The universal generalization of a formula ϕ will be a sentence (*i.e.*, a closed formula) of a second-order extension of the language of QML. Williamson assumes that we have an understanding of the higher-order quantifiers that is primitive in the sense that there is no satisfactory interpretive account of those quantifiers that does not itself deploy quantifiers of just the same sort (p. 258). Suppose we agree. Then we can understand what the universal generalization of any formula says. Moreover, as long as we count '$\Box$' as a logical operator, the universal generalization of a formula makes a purely logical claim: a formula's universal generalization deploys no non-logical vocabulary at all. If it is true, then its truth is independent of the interpretation of any vocabulary other than the quantifiers, '$\Box$', the connectives, and '='. It captures, it would seem, some general, structural aspect of modal reality, so long as '$\Box$' is interpreted to express metaphysical necessity and the quantifiers are interpreted as absolutely unrestricted.[11]

Williamson concludes that this makes it a good candidate for a logical truth concerning metaphysical modality. And, he suggests, this status should be reflected back onto the formulae of QML whose universal generalization it is: each of those formulae should also be reckoned logical truths concerning metaphysical modality. A formula is *metaphysically universal* iff its universal generalization is true.

[10] See pp. 95, 258.

[11] I will take the provisos concerning the interpretation of '$\Box$' and the absence of restrictions on quantifiers to be understood in what follows.

The class of metaphysically universal formulae MU is Williamson's proposed candidate for the class of logical truths concerning metaphysical modality (pp. 92-4, 131-2). This completes the first step in Williamson's specification of a class of intended model structures: we have specified the class of logical truths. The second step is now straightforward. The logic of a model structure is the set of formulae true in every model of that structure. Such a logic L is *sound for MU* iff L is a subset of MU; L is *complete for MU* iff MU is a subset of L (95). Williamson proposes that an inhabited model structure is intended iff it is sound and complete for MU, *i.e.*, its logic contains exactly the metaphysically universal formulae. This, then, is how Williamson suggests that we bring the tools of modal logic to bear on modal metaphysics. In particular, he immediately deploys these tools to shed light on the dispute between necessitism and contingentism.

3 Necessitism, Contingentism, and MU

Recall that necessitism holds that, necessarily, everything is necessarily something. Standard forms of contingentism are motivated in part by the implausibility of certain consequences of this claim. They are motivated, for instance, by the implausibility of the idea that, necessarily, any volcano in Vermont is necessarily something, hence actually something. Standard forms of contingentism must for this reason hold that the Barcan Formula

BF $(\Diamond(\exists x)\phi \Rightarrow (\exists x)\Diamond\phi)$

can have false instances on an intended interpretation. Replace the schematic sentence letter ϕ in **BF** with a formula Fx to yield

(1) $(\Diamond(\exists x)Fx \Rightarrow (\exists x)\Diamond Fx)$

where we intend that 'F' be interpreted as expressing the property of *actually being nothing*. Then its antecedent is true according to the standard forms of contingentism, since it is possible that there be something – *e.g.*, a volcano – that is such that, actually, nothing is identical to it. But its consequent is false, since everything is, of course, necessarily actually identical to something. So, a standard contingentist must deny that (1) is metaphysically universal.

Williamson argues that, if there is an inhabited structure that is sound and complete for MU, then that structure also validates **BF** (pp. 134-5). The argument relies on a technical result connecting **BF** to necessitism. Call a structure $\langle W, R, w, \mathcal{D}\rangle$ *non-increasing* iff $\mathcal{D}(w^*) \subseteq \mathcal{D}(w)$ for all w^* accessible from w. Intuitively, a non-increasing structure is one in which every possible object is a member of the domain of the actual world. If, for instance, a certain possible volcano is a member of the domain of a world in a non-increasing structure, then it is also a member of the domain of the actual world of that structure. The technical result (well known, but proved on pp. 124-5) is that any inhabited structure that is non-increasing validates **BF**. Williamson shows that, if an inhabited structure is sound and complete for MU, then it is non-increasing. Consider the open formula $(\exists y)x = y$. It is metaphysically universal, since its universal generalization, $(\forall x)(\exists y)x = y$, is true. If an inhabited structure validates this open formula, then, in every model, every member d^* of the domain of any world accessible from the actual world w is such that the assignment of d^* to x satisfies the formula. But d^* satisfies the formula only if it is a member of $\mathcal{D}(w)$. Thus, every member d^* of the domain of any world accessible from w is also a member of w. So, the structure is non-increasing and satisfies **BF**.

This is bad news for standard contingentists, who reject **BF**. If, as Williamson urges, an inhabited structure is intended only if it is sound and complete for MU, then any intended structure validates **BF**, and hence all instances of **BF** are metaphysically universal. But closed metaphysically universal formulae are clearly true.[12] Thus, if there is an intended inhabited structure, then standard contingentism is false. Standard contingentists, it appears, must reject the idea that there is an intended inhabited structure.

4 Intended Structures for Contingentists

Appearances are misleading, however, for the contingentist has other alternatives available to her. For instance, she may locate the fault in the definition of validity on a structure; if she says that a formula ϕ is valid on a structure iff it is true in every model of that structure on all assignments of elements of the domain of the actual world to variables, then the argument does not get off the ground (p. 136).[13]

[12] Keep in mind that the language we are working with right now is assumed to be free of individual constants. When constants are introduced, there is reason to think that some closed metaphysically universal formulae are false. We will revisit this issue below.

[13] Williamson argues that a similar restriction needs to be imposed on the evaluation of formulae containing individual constants. Since the language under study contains no individual

Williamson contends that minor alterations of this sort to the semantics are "unmotivated ad hoc complications" (p. 136). From a standard contingentist point of view, of course, the complications are not unmotivated: they are motivated, contends the standard contingentist, by the need to specify the class of intended structures, together with whatever motivation he thinks supports contingentism. This sort of contingentist response, however, amounts to little more than insisting on the co-validity of *modus ponens* and *modus tollens*. So, let's set this response aside in favor of something more interesting and more powerful.

An alternative contingentist response locates the problem in the fact that the semantic clause de-relativizing the truth of an open formula to assignments a. According to that de-relativization, ϕ is true (at a world in a model) iff ϕ it is true relative to all assignments a which map x to an object from the domain of any world whatsoever. One proposal would be to amend that clause so that the truth condition for an open formula is identical to the truth condition for its closure: any formula is true at a world w iff it is true at w on every assignment of members of $\mathcal{D}(w)$ to variables.[14] Again, Williamson's argument would not get off the ground. This proposal is very far from ad hoc and unmotivated. When Kripke introduced the model-theoretic setting in which we are working, he faced the problem that **BF** and its converse appear to be implied by standard modal and quantificational principles. He solved this problem by giving open formulae the *generality interpretation*, which is exactly the interpretation offered by the proposal at hand [Kripke, 1963, pp. 68-9].

Still, I think it's more natural to leave the model-theoretic semantics alone, and revise the criterion for logical truth. The generality interpretation, after all, is just one of the permissible interpretations of open formulae. Moreover, open formulae seem, on their face, to be unattractive candidates for logical truths. As Williamson notes in another context, an open formula "... is unsuitable for independent use in a speech act" (p. 346), presumably because, absent

constants, this additional restriction is unnecessary in the present context. I will revisit issues concerning individual constants below.

[14]This emendation of the de-relativization of truth to assignments would not materially affect results concerning the truth or validity of formulae containing the open formula, since the basic notion is *truth relative to a world and an assignment*, and the definition of that notion is unaltered. To be sure, one cannot summarize the upshot of an argument to the effect that an open formula is not true at a world on every assignment by saying that the formula is "not true" at the world. In general, however, that particular way of summing things up is not necessary.

some further interpretation, it makes no claim of truth or falsity. We can, in the formal semantics, stipulatively define a notion of truth (at a world in a model) for an open formula that is assignment-independent. But we should recognize that stipulation for what it is: a matter of defining things so that (hopefully) our proofs aren't as wordy, rather than an attempt to capture a pre-theoretically compelling idea. The de-relativization of truth for formulae is more a matter of formal bookkeeping than a deep metaphysical insight. Because open sentences are unattractive candidates for truths, they are, likewise, unattractive candidates for logical truths. Otherwise, Williamson's specification of the class of logical truths is very attractive. So, let's use that specification but exclude the open formulae. Say that a formula is *metaphysically unlimited* iff it is closed and metaphysically universal. Call the class of metaphysically unlimited formulae MU^*. An inhabited structure is *sound for* MU^* iff the sentences (*i.e.*, closed formulae) it validates are all in MU^*. It is *complete for* MU^* iff it validates every sentence in MU^*. An inhabited structure is intended iff it is sound and complete for MU^*. This is a criterion for logical truth and a corresponding criterion for an intended inhabited structure that standard contingentists can easily endorse.

If MU^* is the class of logical truths concerning metaphysical modality, then Williamson's argument that standard contingentism must reject the idea that there is an intended inhabited structure is unsound. In particular, $(\exists y)x = y$ is not metaphysically unlimited, so there is no reason to think that any intended inhabited structure should validate it. Moreover, MU^* is at least as plausible and natural a candidate for the class of logical truths as MU. I have argued that, in fact, it is more plausible and natural because it eschews the unattractive assumption that open formulae are truths. Suppose, however, that I am wrong about that. We would still lack any support for favoring the claim that MU is the class of logical truths concerning the metaphysics of modality over the equally plausible and natural claim that MU^* enjoys that status instead. Without such support, Williamson's argument is incomplete.

5 Williamson's Construction

Williamson provides a construction of an inhabited model structure, and shows that, if necessitism is true, it meets the necessitist criterion for an intended structure: its logic is sound and complete for MU (pp. 102ff., 140-2). Can the contingentist do the same with respect to the neutral criterion for an intended structure, which requires that its logic be sound and complete for MU^*? Again, the news for contingentism is good: if Williamson's argument shows that the inhabited structure he describes is sound and complete for MU, then a similar construction and argument will establish a similar result for the contingentist.

Williamson's construction assumes that the universe of *propositions* form a boolean algebra under the operations of infinitary conjunction (Π), infinitary disjunction (Σ), and negation ($\sim$). This boolean algebra contains exactly one contradiction, 0, and exactly one tautology, 1. The algebra is partially ordered by the relation $\leq$, where $p \leq q$ iff $\Pi(p, q) = p$. Intuitively, $\leq$ is the entailment relation. Williamson assumes that this boolean algebra is *complete*, in that every set of propositions has a $\leq$-least upper bound and a $\leq$-greatest lower bound. Intuitively, the $\leq$-least upper bound of a set of propositions is the strongest proposition entailed by each member of the set, and its greatest lower bound is the weakest proposition that entails each of them. So, the l.u.b. will be the (perhaps infinitary) disjunction of propositions in the set, and the g.l.b. wil be their (perhaps infinitary) conjunction. A proposition is an *atom* just in case it is not the contradiction, but is otherwise strong enough to to entail, for every proposition p, either p or $\sim p$. Thus, an atom is *consistent* because it is not 0, and it is *maximal* because it entails or excludes every proposition. Williamson assumes that the boolean algebra of propositions is *atomic* in the sense that every proposition other than the contradiction is entailed by some atom.

The construction so far leaves out modality. Modality is handled by assuming that there is an operation L of necessitation on the propositions. Intuitively, for instance, if p is the proposition that Socrates is not a world war, then Lp is the proposition that, necessarily, Socrates is not a world war.[15] L is assumed

[15] This informal explanation of L is crucial for the adequacy of Williamson's construction. The two-element boolean algebra containing only 0 and 1 is a complete, atomic boolean algebra. None of Williamson's other assumptions excludes the hypothesis that there are only two propositions, The True and The False, and thus that there is only one possible world in the intended structure. The informal explanation excludes this hypothesis about the universe of propositions because it is obvious that, though the proposition that Shaquille O'Neal is 3 inches taller than Lebron James is true, its necessitation is false. Thus, no two-proposition universe satisfies the assumption that L is necessitation, and so no one-world structure is intended. For this reason, the variant approach described on p. 104, on which the operation L is defined in the obvious way in terms of the boolean algebra, is not viable, unless we offer further constraints on the universe of propositions. In general, Williamson's construction of an intended inhabited structure would benefit from a clearer articulation of the assumptions concerning the universe of propositions on which it relies.

to distribute over conjunctions: $L(\Pi(S)) = \Pi(\{Ls|s \in S\})$. Given Williamson's assumptions, the atoms of the boolean algebra are good candidates for worlds of the intended structure, since they are maximal, consistent sets. We may think of the propositions entailed by an atom as those that are "true at" that atom. The atom which entails only true propositions is thus a good candidate for the actual world of the structure, since it is maximal, consistent, and entails all and only the true propositions. We let an atom w^* be accessible from an atom w iff, for all propositions p, if w entails Lp, then w^* entails p. Intuitively, the worlds accessible from w verify all of the propositions that w "says" are necessary. Equivalently, all propositions true at a world accessible from w are "said" by w to be possible.[16]

The construction so far leaves out quantification. This is handled by adding a domain function. We are assuming necessitism, so the domain function is constant, mapping each atom of the boolean algebra to a set $\mathcal{D}$, which we are to "... temporarily pretend is the intended domain of the first-order quantifiers" (p. 140).[17] Williamson assumes that n-place predicates are interpreted by propositional functions from $\mathcal{D}^n$ into the propositions. Suppose, for instance, that we intuitively interpret the predicate 'F' as expressing the property *being a world war*. We can represent this by assigning to 'F' the function which takes any individual d to the proposition that d is a world war. Given an assignment a of objects from the constant domain $\mathcal{D}$ of an intended structure, this interpretation of each of the n-place predicates as propositional functions nails down what Williamson calls a "faithful interpretation" for each sentence of the language of QML. The faithful interpretation of an atomic sentence $Fx_1, x_2, ..., x_n$ is the value of the propositional function mapped to 'F' by the interpretation at the sequence $\langle a(x_1), a(x_2), ..., a(x_n)\rangle$. The faithful interpretation of a conjunction is the conjunction of the faithful interpretations of the conjuncts, and the faithful interpretation of the negation and necessitation of a formula is constrained to be the negation and necessitation, respectively, of the faithful interpretation of the formula.

This leaves only the specification of the notion of a "faithful interpretation" of the universal generalization of a formula. One might expect at this point for Williamson to add a "generalization" operation on propositions, which somehow

[16]Thanks are due to Paul Hovda for information and discussion about boolean algebras.

[17]We have to pretend because, as Williamson notes (pp. 139-40), there is no universal set. Thus, for any set, our unrestricted quantifiers range over something not in the set.

takes one, for instance, from the proposition that Socrates is a world war to the proposition that everything is a world war. This is not what he does. Instead, the faithful interpretation of a universal generalization $(\forall x)\phi$ on a is defined to be the (perhaps infinitary) conjunction of the interpretations of ϕ on any x-variant of a that assigns a member of $\mathcal{D}$ to x. So, for instance, if the interpretation maps 'F' to a function from any individual in $\mathcal{D}$ to the proposition that that individual is a world war, then the corresponding faithful interpretation of $(\forall x)Fx$ on a is the conjunction of the propositions that Socrates is a world war, that Plato is a world war, *etc.*, for each individual in $\mathcal{D}$.

Here, for convenient reference, is the recursive definition of a faithful interpretation of a formula relative to an interpretation I of predicate letters and an assignment a of members of $\mathcal{D}$ to the variables. We assume that I is a function from n-place predicates to n-ary propositional functions, and we will denote the faithful interpretation relative to I and a as I_a.

1. $I_a(Fx_1, x_2, ..., x_n) = I(F)(\langle a(x_1), a(x_2), ..., a(x_n)\rangle)$.
2. $I_a(x_1 = x_2) = \begin{cases} 1 & \text{if a(x}_1\text{)} = a(x_2)\text{; and} \\ 0 & \text{otherwise.} \end{cases}$
3. $I_a(\neg\phi) =\sim I_a(\phi)$.
4. $I_a(\phi \wedge \psi) = \Pi\{I_a(\phi), I_a(\psi)\}$.
5. $I_a(\forall x)\phi = \Pi\{I_{a[x/d]}(\phi)|d \in \mathcal{D}\}$.
6. $I_a(\Box\phi) = L(I_a(\phi))$.

Williamson then claims that "... the universal generalization of [a formula] A is true, and so A is metaphysically universal, if and only if $I_a(A)$ is true for every faithful interpretation I and assignment a" (p. 141). This is not true if we impose no further constraints on $\mathcal{D}$. Suppose $\mathcal{D}$ is a singleton $\{d\}$, and consider the Heraclitean claim that all is one:

H $(\forall x)(\forall y)x = y$.

Every faithful interpretation on every assignment a is such that I_a interprets (H) (which is its own universal generalization) as the tautologous proposition 1, *i.e.*, the proposition that d is identical to d. So, of course it is true on every faithful interpretation. And yet (H) is not metaphysically universal.

What has gone wrong? The problem is that the proposed "faithful interpretation" of quantificational sentences like (H) does not interpret them as quantificational propositions: the "faithful interpretation" of (H) is, in fact, consistent with the obvious truth that there are at least two things. The proposition that Williamson's argument shows is MU is not the proposition we intend by (H). In summary, the "faithful interpretation" of (H) is not an intended interpretation.

Williamson concedes (p. 141) that the interpretation of quantification as conjunction does not appropriately capture the modes of presentation of universal claims. He argues that capturing modes of presentation is beyond the purview of the logical and metaphysical investigation we are presently pursuing. But the problem in this instance is not merely a matter of the interpretation for (H) failing to express the right mode of presentation; rather, the problem is that the proposed interpretation for the universal claim gets both the truth value and the consequences of the sentence (on its intended interpretation) wrong.

Now, this objection to Williamson's claim is not obviously fair, since it depends on the assumption that the set we are pretending is the intended domain of absolutely unrestricted quantification is a singleton. One would have to be really good at pretending to play along with this assumption. I'm sure that I am not capable of such a feat. Given that the set-theoretic background for the semantics includes an axiom of infinity, it is plausible to think that $\mathcal{D}$ should be constrained to be infinite. If we impose this constraint on Williamson's construction, then there is no danger of validating (H) and its ilk. One might, nevertheless, have two remaining reservations. First, adding the constraint on $\mathcal{D}$ seems on its face a bit of a kludge. It's a prima facie ad hoc constraint that nevertheless manages to prevent trouble. The second, related reservation is that adding this constraint does not really go to the root of the problem. The root of the problem is that the proposed interpretation of quantificational claims makes no contact at all with the logical structure of ostensibly quantificational propositions on which Williamson's construction is based. Thus, the claims that Williamson's argument shows belong to MU are not the claims we intended to express using the quantifiers of QML. This point remains even if every inhabited structure of the sort that Williamson describes that has an infinite domain happens to validate exactly the metaphysically universal formulae on a "faithful interpretation."

6 A Contingentist Construction

So, we should be unsure, I think, whether Williamson's construction really works as advertised. Suppose, however, that it does. Then it is easy for the standard contingentist to follow the trail that Williamson has blazed. Assume with Williamson that the propositions form a complete, atomic boolean algebra. Let the set of worlds W, the actual world w, and the accessibility relation R be defined just as Williamson does. We add a domain function $\mathcal{D}$, constrained so that $\mathcal{D}(w^*)$ is infinite for all $w^* \in W$. Since we are assuming standard contingentism, we may require that $\mathcal{D}$ be such that the inhabited structure is neither non-increasing nor non-decreasing.[18] We let assignments map variables to any member of the union of the sets $\mathcal{D}(w^*)$ for $w^* \in W$. Let $C(p, q)$ abbreviate $\sim \Pi(p, \sim q)$; intuitively, $C(p, q)$ is the material conditional whose antecedent is the proposition p and whose consequent is the proposition q. Let $I_a^{w^*}(\phi) = \Pi\{I_{a[x/d]}(\phi) | d \in \mathcal{D}(w^*)\}$; $I_a^{w^*}(\phi)$ is the (perhaps infinitary) conjunction of the interpretations of ϕ on the x-variants of a that assign a member of $\mathcal{D}(w^*)$ to x. Replace clause (5) of the definition of I_a above with

5. $I_a((\forall x)\phi) = \Pi\{C(w^*, I_a^{w^*}(\phi)) | w^* \in W\}$.

It is then easy to argue, along essentially exactly the same lines as Williamson's argument, that the universal generalization of a sentence (*i.e.*, a closed formula) ϕ is true, and so ϕ is metaphysically unlimited, if and only if $I_a(A)$ is true for every faithful interpretation I and assignment a.[19] If Williamson's argument succeeds in showing that an inhabited structure of the sort he describes (with the additional constraint needed to exclude (H) and its fellow travelers) is sound and complete for MU, then this argument succeeds in showing that the inhab-

[18] A structure $\langle W, R, w, \mathcal{D}\rangle$ *non-decreasing* iff $\mathcal{D}(w^*) \supseteq \mathcal{D}(w)$ for all w^* accessible from w.

[19] Recall that I is a function from n-place predicates to n-ary propositional functions. Any such I determines a model $M_I = \langle W, R, w, \mathcal{D}, V_I\rangle$, where $\langle d_1, d_2, ..., d_n\rangle \in V_I(F)(w)$ iff $w \leq I(F)(\langle d_1, d_2, ..., d_n\rangle)$. The key lemma says that for any basic interpretation I, formula ϕ, assignment a, and world w, $M_I, w, a \vDash \phi$ iff $w \leq I_a(\phi)$. This is proved by induction on the complexity of the formula ϕ. The cases of atomic formulae, conjunctions, negations, and necessitations are handled by Williamson's argument at p. 106n. Suppose, then, that we are given a formula $(\forall x)\psi$. $I_a((\forall x)\psi)$ is the conjunction of conditionals $C(v, \Pi\{I_{a[x/d]}(\psi) | d \in \mathcal{D}(v)\})$ for $v \in W$. $w \leq \sim v$ for each atom v other than w, so it's enough to show that, $M_I, w, a \vDash (\forall x)\psi$ iff, for all $d \in \mathcal{D}(w), w \leq I_{a[x/d]}(\psi)$. Now, $M_I, w, a \vDash (\forall x)\psi$ iff, for all $d \in \mathcal{D}(w)$, $M_I, w, a[x/d] \vDash \psi$. But the inductive hypothesis yields that, for all $d \in \mathcal{D}(w)$, $M_I, w, a[x/d] \vDash \psi$ iff $w \leq I_{a[x/d]}(\psi)$. QED. On the basis of this lemma, an adaptation of the argument on p. 107 will show that the model structure is sound and complete for MU^* so long as a similar adaptation of that argument shows that Williamson's construction is sound and complete for MU.

ited structure I have described is sound and complete for MU^*. In short, if Williamson shows that there is an intended inhabited structure by the necessitist's lights, then his argument also, suitably amended, shows that there is an intended inhabited structure by the standard contingentist's lights. Contrary to Williamson's claim, contingentists need take no more jaundiced a view of the import of Kripke-style models than necessitists.

Williamson acknowledges that contingentists might manage to isolate a class of intended inhabited structures (p. 137). He argues, however, that the standard contingentist still cannot take the model theory in a fully realistic spirit. According to standard contingentism, there might have been things that are, actually, identical to nothing. In an intended inhabited structure, such things are represented by elements of the domain of a non-actual world in the structure that are not also elements of the domain of the actual world of the structure. Each representative, of course, is actually identical to something. So, those denizens of non-actual domains are actual things that go proxy for things that might have been something but are actually nothing. Call something *merely possible* just in case it is possibly something, but actually nothing. Nothing is merely possible.[20] Thus, the standard contingentist is committed to the idea that some elements of the inhabited structure go proxy for merely possible things without there *really* being any merely possible things. In this sense, the standard contingentist holds that no model of the intended inhabited structure is in every respect an accurate representation of modal reality.

The charge is correct. Not every aspect of any model of an intended structure corresponds to an aspect of modal reality if standard contingentism is true. We cannot read features of modal reality off the features shared by all models of an intended structure.[21] It seems, however, that the necessitist is in the same boat. The reason we have to pretend that, in a necessitist intended structure, $\mathcal{D}$ is the intended domain of quantification, is that, as Williamson notes, there is no set of all objects. Thus, any model of any intended Kripke-style structure has a feature that does not accurately reflect modal reality: the model contains only set-many individuals, but there are really more than set-many individuals. Williamson conjectures that we can find some higher-order analogue of the construction of an intended inhabited structure for necessitists, where the domain $\mathcal{D}$ (and,

[20] Williamson agrees; see pp. 22-3.

[21] It should be said, however, that this point is independent of Williamson's argument that standard contingentism must reject the idea that there is an intended inhabited structure.

for that matter, the boolean algebra of propositions) need not be set-sized.[22] Similarly, the contingentist might hope for some higher-order construction which appeals to an algebraic structure of *relations* to serve as interpretations of open formulae, as opposed to a Tarski-style satisfaction semantics.[23] Suppose these hopes are realized. In neither case can either the necessitist or the contingentist take every aspect of the Kripke-style intended inhabited structure accurately to characterize the features of modal reality.

7 Adding Individual Constants

There is, however, one final challenge for the contingentist that we have not yet discussed. We have been discussing a formal language for QML that lacks individual constants. If we allow individual constants, then, it seems, Williamson's argument that a contingentist cannot accept that there is an intended inhabited structure can be revived. The most natural semantic clause for constants treats them analogously to variables: an interpretation may specify any element of the domain of any world as the interpretation of an individual constant. Suppose c is an individual constant, and consider the sentence $(\exists y)c = y$. It is metaphysically universal, since its universal generalization, $(\forall x)(\exists y)x = y$, is true. Moreover, it is metaphysically unlimited, since it is both closed and metaphysically universal. If an inhabited structure validates this sentence, then, in every model, every member d^* of the domain of any world accessible from the actual world w is such that the sentence is true when d^* is assigned as the referent of c. But the sentence is true when c refers to d^* only if d^* is a member of $\mathcal{D}(w)$. Thus, every member d^* of the domain of any world accessible from w is also a member of w. So, the structure is non-increasing, and satisfies **BF**. Since the formula is a member of MU^*, it would seem to follow that any structure sound and complete for MU^* validates **BF**. If this argument is sound, then the contingentist must concede that there is no intended inhabited structure according to her own criterion (p. 136).

There are a number of responses to this new, rehabilitated problem available to a contingentist. One thing she might do is impose a restriction on the

[22] Williamson sketches the outlines of a construction of this sort on p. 117 and notes some potential pitfalls.

[23] I have in mind something along the lines of Bealer's [1983] construction, though Bealer proposes to define necessity rather than rely on a primitive necessitation relation.

interpretation of c so that it must be assigned a referent from the domain of the actual world of the inhabited structure. This is one of those measures characterized by Williamson as "unmotivated ad hoc complications" (p. 136). I am inclined to agree that, other things being equal, the restriction on the interpretation of individual constants is unmotivated by contingentist lights. It would make $(\exists y)c = y$ into a logical truth concerning the metaphysics of modality, and it seems implausible to me, at least, that a contingentist should accept that that is a logical truth.

Matters are not so clear, however, given a stipulation that Williamson makes concerning the language we are treating. He contends that a language is "well-designed for expressing good scientific theories [only if] the denotation of any constant is one of the values over which variables of the same type range" (p. 131). He stipulates that the language we are considering is well-designed by this standard. This stipulation is needed to ensure that the members of MU (and, for that matter, MU^*) are all true, and so are appropriate candidates for logical truth. Suppose our constant c had as its denotation something outside the range of the quantified variables. As we have seen, $(\exists y)c = y$ is both metaphysically universal and metaphysically unlimited; but it is true only if the denotation of c is in the range of the quantifier '$(\exists y)$'. According to the contingentist, the (actual) range of individual quantified variables is represented in an inhabited structure by the domain of the actual world of the structure. Thus, the restriction of denotations of individual constants to the domain of the actual world is required by the content of Williamson's stipulation. Absent the stipulation, the restriction is unmotivated; in the present context, however, the restriction is motivated by the imposition of the stipulation.

I have shown that necessitism and standard contingentism can each offer a well-motivated criterion for an inhabited structure to be intended. I have argued, briefly, that the contingentist's criterion is preferable on independent grounds. We have reviewed Williamson's argument that his construction meets the necessitist's criterion for an intended inhabited structure. I have contended that, if that argument is sound, then a similar argument shows that a similar construction will meet the contingentist's criterion for an intended structure. I conclude that Williamson is not correct to say that "necessitism has a theoretical advantage over contingentism in giving a clearer, simpler, and more satisfying account for quantified modal languages of the relation between truth in a model and truth" (p. 139). Necessitism and standard contingentism are, rather, on a par in this respect: either both can give a very clear, very simple account of the relation between truth in a model and truth, or neither can.

We should not let the critical character of my remarks obscure the extent of Williamson's achievements. Williamson offers a clear and plausible criterion for an inhabited structure to be intended. This criterion offers a precise means for bringing the power of the model theory of QML to bear on questions concerning features of modal reality; see, for instance, pp. 95-7. My contingentist-friendly criterion is a mere tweak. Williamson's construction of an inhabited structure that meets his criterion, if successful, adds further insights; see, for instance, pp. 109-11. My proposed contingentist construction involves only a slight adjustment. His argument that contingentists must deny that there is an intended structure poses an interesting challenge to contingentism. It is a testament to the clarity, richness, and power of the book that meeting that challenge draws so significantly on resources Williamson himself has provided.

Moreover, I have only discussed material covered in a single chapter. I have focused almost exclusively on Williamson's contention in Chapter 3 that standard contingentists must regard Kripke-style possible worlds semantics with a jaundiced eye. Each chapter of the book deserves extended scrutiny that considerations of space prevent me from providing here. Williamson develops and discusses the broadly logical case for necessitism with breathtaking erudition and care. Here, then, I have done little more than scratch the surface of one small part of a large, rich, and interesting book. I hope to have said enough nonetheless to convince you that this book will reward careful study.[24]

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[24]Thanks to Paul Hovda and Mark Moyer for discussion and comments on earlier drafts.

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