MICROECONOMIC THEORY OLD AND NEW

A Student's Guide

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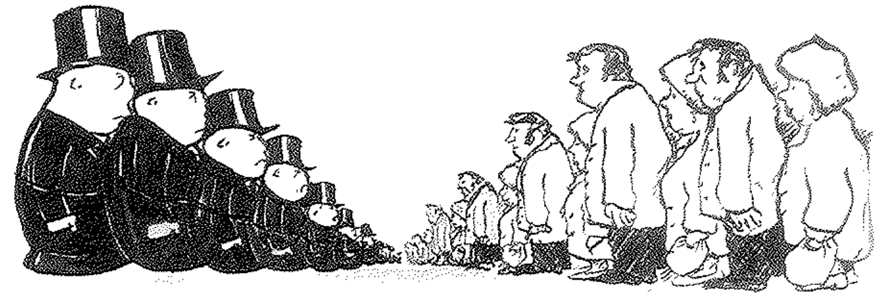
PART ONE

THE WALRASIAN SYSTEM



INTRODUCTION AND OVERVIEW

The umbrella covering the various pieces of economic theory is called *welfare economics*. It provides the basic framework for applying the tools of economics to problems such as estimating the benefits of trade, valuing environmental features, and determining the criteria for sustainability. Far from being an esoteric footnote to economic theory, welfare economics provides the basic worldview of economists, giving answers to fundamental questions regarding the ultimate purpose of economic activity and the best policies to promote human well-being. The validity of some of the most widely used tools of economics—cost-benefit analysis, measures of total factor productivity, and Pareto efficiency—depends critically on the validity of the underlying welfare economic model.

For more than half a century, economic theory and policy has been dominated by a type of welfare economics called *Walrasian* economics, named after the political economist Léon Walras (1834–1910). The cornerstone of the Walrasian system is the characterization of human behavior embodied in "economic man," or *Homo economicus*, whose preferences are assumed to be stable, consistent, and independent of the preferences of others. With this starting point, the leading figures in the "marginalist revolution" of the 1870s—William Stanley Jevons, Vilfredo Pareto, and Walras—constructed a mathematical model of an economy in equilibrium that defined the science of economics as "the optimal allocation of scarce resources among alternative ends." In the decades following World War II this model not only came to dominate microeconomic analysis but also became the starting point for macroeconomics—the so-called microfoundations approach.

The starting point of the Walrasian system is the exchange of a fixed collection of goods among individuals bargaining directly with one another. The end result of free and voluntary exchange is that no further trading will make one person better off without making someone else worse off. This result is called *Pareto efficiency*, and it establishes one of the key ideas in modern economics, namely, the welfare benefits of trade. The next step is to introduce prices into the basic model and show that a perfectly operating market economy will duplicate the results of face-to-face bartering. The last part of the puzzle is to recognize that prices may be "imperfect" but that it is possible for enlightened intervention to correct these *market failures* and reestablish the conditions for Pareto efficiency.

THE THREE BUILDING BLOCKS OF THE WALRASIAN SYSTEM

The first building block of the Walrasian system is to establish that the free exchange of commodities will lead to Pareto efficiency in a pure barter economy. Individuals with a predetermined amount of commodities are allowed to directly and freely trade valuable goods with each other, and Pareto efficiency is achieved when no further trading can increase the well-being of one person without decreasing the well-being of another. The second building block is the demonstration that if market prices correctly reflect individual preferences, then a perfectly competitive market economy will lead to Pareto efficiency (the First Fundamental Theorem of Welfare Economics). That is, competition in free markets will exactly duplicate the Pareto efficient outcome that would result from direct negotiations and exchange in a barter economy. The third and final piece of the system is the recognition that the prices of market goods may be distorted for a variety of reasons. These price distortions, called market failures, include the broad categories of externalities, market power, and public goods. In these cases, governments have a legitimate role to play in correcting the failures of markets in order to establish

the proper value (price) of things such as environmental services (the Second Fundamental Theorem of Welfare Economics). The underlying assumption is that people rationally and consistently respond to price signals.

To summarize the Walrasian system:

1. Trading in a barter economy—Unfettered bartering by agents with stable preferences will lead to Pareto efficiency, a situation in which no further trading can make one person better off without making another person worse off.
2. Adding prices—If prices correctly reflect consumer preferences, then competitive markets are always Pareto efficient. Free markets will exactly duplicate the results of a direct barter system.
3. Adjusting prices—When market failures are present, enlightened government intervention can adjust market prices so that a socially efficient Pareto outcome can be established.

These three building blocks provide the worldview of most economists. The ultimate source of value and the ultimate arbiter of efficiency in the Walrasian system are the preferences of *Homo economicus*, whatever these preferences might be and however they are formed. These building blocks hold together only if all the assumptions defining *Homo economicus* and perfect competition are met.

Today, welfare economics is undergoing a revolution that promises to fundamentally change the way economists see the world. Walrasian welfare economics is being challenged by a new economics, grounded in behavioral science, that recognizes the social and biological context of decision making and the complexity of human behavior. The current sea change in economic theory offers a unique opportunity for economists, working together with other behavioral scientists, to move mainstream economic theories and policies toward an empirical, science-based approach unbounded by a priori assumptions.

This book has two goals. The first is to present clearly and precisely how the internal logic of the Walrasian model works. What is the starting point for the model, how do the pieces fit together, and what are the policy implications? The second is to present the current revolution in welfare economics and the theoretical and empirical challenges to Walrasian theory.

FURTHER READING

An essential source for background on economics concepts, definitions, and the history of economic thought is *The New Palgrave Dictionary of Economics*, 4 vols., ed. J. Eatwell, M. Milgate, and P. Newman (London and New York: Macmillan, 1987).

Recent Microeconomic Texts

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Mas-Colell, A., M. Whinston, and J. Green. 1995. *Microeconomic Theory.* New York: Oxford University Press.

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Not-So-Recent (but Very Useful) Microeconomic Texts

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Classic Texts

Pareto, V. [1906] 1971. *Manual of Political Economy.* New York: Augustus Kelley.

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Walras, L. [1926] 1977. *Elements of Pure Economics.* Fairfield, CT: Augustus Kelley.

1

THE NEOCLASSICAL THEORY OF THE CONSUMER

Let us return to the state of nature and consider men as if . . . sprung out of the earth, and suddenly, like mushrooms, come to full maturity without any kind of engagement to each other.

—*Thomas Hobbes,* De Cive; or, The Citizen *[1651], edited with an introduction by Sterling P. Lamprecht (New York: Appleton-Century-Crofts, 1949), 100*

THE FOUNDATION OF UTILITY THEORY—DIRECT EXCHANGE IN A PURE BARTER ECONOMY

Imagine you are driving along a highway behind a truck loaded with merchandise. A box falls out and lands on the side of the road and you stop to take a look and examine the contents. The box is full of CDs (compact discs) of all sorts—classical music, country and western, hip-hop, jazz, Hawaiian, and blues. There is nothing in the box to indicate ownership—no invoice, no name on the box—and you did not notice the name on the truck. You are on your way to your economics class and, feeling slightly guilty about taking the box, you decide to distribute the CDs to your classmates. Suppose there are twenty people in your class and you have 500 CDs to hand out. You start handing them out randomly, not necessarily evenly—some people end up with lots of CDs and some with only two or three. So now there is a group of twenty people sitting around a table with 500 CDs randomly distributed and unevenly divided among them. This sets the stage for learning about how economists think about prices, markets, free trade, social welfare, **utility**, and efficiency.

The Exchange of Goods in a Pure Barter Economy

The three starting points for the analysis that follows are:

1. The number of CDs to trade (500) is fixed before trading starts.
2. The distribution of these CDs among the twenty people is given before trading starts.
3. The (musical) preferences of the twenty individuals do not change during the bargaining process.

Now the fun begins. Your economics teacher seizes the opportunity to teach the class about market exchange and devotes the class time to establishing a "market equilibrium." Your fellow students start trading CDs—Brittney Spears for Bad Religion, Shostakovich for Metallica, or Green Day for Dale Watson. Trading goes on for most of the class as people haggle, barter, trade, and retrade to get the CDs they want. After an hour or so, things get quiet as no one is willing to make another deal. The students have done the best they can, *given their different musical preferences and their initial endowment of CDs*. This situation is called **Pareto efficiency,** an essential concept in neoclassical **welfare economics**.

Pareto Efficiency in the Exchange of Goods—*A situation in which no further trading of goods can make one person better off without making another person worse off.*

The process of haggling and bartering in trade is what Adam Smith had in mind when he talked about the "invisible hand" of the market economy; that is, the push-and-shove, give-and-take, dynamic vitality of capitalism. Direct bartering allows for face-to-face interaction and for nonpecuniary motives such as altruism and envy, and of course old-fashioned greed. Perhaps you refuse to trade with some people because you resent the fact that they have more CDs than you do and you do not want them to be better off than they already are. Maybe you trade five CDs with someone for one CD you do not particularly want because you are trying to get a date with him or her. All these factors may affect the "well-being" you get from the CDs and they can be incorporated into your trading decisions.

Those of you who have learned the economic model of "perfect competition" (discussed in detail in Chapter 4) might recognize that some of the conditions of that model are fulfilled in this simple barter case. For example:

perfect information—everyone knows exactly how many CDs everyone else has and who the artists are

perfect resource mobility—trade can take place almost instantaneously and at negligible cost

homogeneous product—among the CDs there might be four brand-new, identical copies of Pink Floyd's *Evolution* CD

This simple example illustrates some of the most basic ideas that economists dearly cling to.

Trade is good. All trade is assumed to be voluntary, so why would people trade if it did not make them better off?

Restrictions on trade are bad. What if the instructor limited trades to two per person? Or collected a tax for each Radiohead CD traded? This would hinder or even prevent the achievement of Pareto efficiency.

The simple model of exchange in a barter economy is in the back of most economists' minds as they make policy recommendations on everything from international trade to global warming. A question to keep in mind is: How closely does this face-to-face barter situation resemble a modern market economy with prices, distant markets, complex social institutions, and limited information?

A GRAPHICAL ANALYSIS OF BARTER AND TRADE

As useful as the verbal description of exchange is, it has limitations in terms of its analytical power. Economists deal with data about economic activity, and to interpret this data it is necessary to examine it in an analytical, meaning mathematical, framework. By adding a few more assumptions to the barter model we can reexamine our exchange example using graphs, then mathematics.

One of the most basic and critical tools of Walrasian analysis is the **indifference curve**, as depicted in Figure 1.1. The points on a particular indifference curve show all the combinations of two commodities (X and Y in Figure 1.1) that yield the same level of utility. In fact, it might be called an iso-utility curve, analogous to isothermals on a weather map. According to Figure 1.1, the consumer is just as happy with the combination of goods X and Y given by point 1 as he or she is with the combination given by point 2. The consumer is just as happy with 2X and 4Y as he or she is with 4X and 2Y.

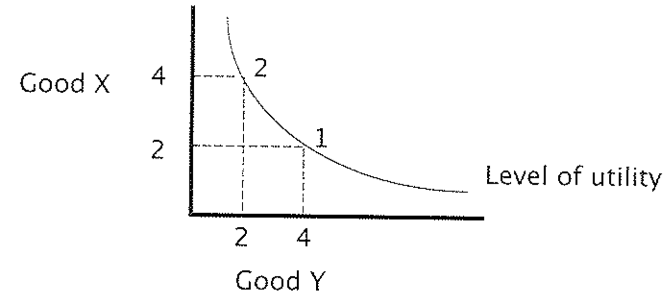


Figure 1.1. An indifference curve

The indifference curve in Figure 1.1 embodies a number of assumptions about human behavior. The economic analysis of consumer behavior is based on a conception of human nature defined by the assumptions of ***Homo economicus***, or "economic man" (sometimes called the rational actor model). Economic man has well-defined preferences that are stable over time. Individual welfare (utility) is equated with the consumption of market commodities, as shown by the axes of the diagram—goods X and Y. More is preferred to less, so higher indifference curves, those farther away from the origin, represent more total utility to the consumer than ones closer to the origin. Commodities are subject to substitution, as indicated by the downward slope of the indifference curve. Indifference curves have the mathematical property of being "smooth and continuous," meaning there are no "jumps" in utility as one commodity is substituted for another as we move along the curve.

Axioms of Consumer Choice Defining *Homo economicus* (*economic man* or the *rational actor*)

1. Non-satiation—More is preferred to less. A commodity bundle on a higher indifference curve is preferred to one on a lower indifference curve.

2. Transitivity—If commodity bundle A is preferred to bundle B, and bundle B is preferred to C, then bundle A is preferred to C. This implies consistency in consumer choice.
3. Preferences are stable and complete—For any pair of commodity bundles A and B, the consumer either prefers A to B, B to A, or is indifferent between the two bundles. These preferences are stable over time.
4. Diminishing marginal rates of substitution—As a consumer has more of one commodity relative to another one, he/she is willing to give up more of it for a unit of the second commodity.
5. Continuity—This is a mathematical property, meaning that any point on a line drawn between two points on an indifference curve is an *interior point*. As we will see later, this assumption is necessary to ensure a unique solution to any *constrained maximization* problem.
6. Exogenous preferences—The preferences of one consumer are unaffected by the preferences of others.

The slope of an indifference curve at a particular place along a curve, $\Delta X / \Delta Y$ or the "rise over the run," indicates the **marginal rate of substitution** (MRS) of one commodity for another. For example, as we go between points 1 and 2, the marginal rate of substitution of good Y for good X ($MRS_{Y\ for\ X}$) is -1, $\Delta X / \Delta Y = -2/2 = -1$. The consumer is willing to give up 1 X to get 1 Y without changing total utility. The shape of the curves, becoming steeper or flatter as they approach the X or Y axes, indicates a *diminishing marginal rate of substitution*. As a consumer has more and more of good X (or good Y), he or she is willing to give up more and more of X (or Y) to get another Y (or X).

It is important to recognize the assumptions invoked in the basic model of exchange in a barter situation and those that are added as we move from a verbal to a graphical and then to a mathematical representation of exchange. Remember that this model is meant to be a plausible representation of actual human behavior. When we move to a graphical representation of utility, what assumptions are added to the three we started with in the pure barter case?

▶ ASSUMPTION ALERT! *Critical behavioral assumptions we have added to move from a verbal to a graphical analysis of exchange:*

1. *The utility of one individual can be determined independently of the utility of others.*
2. *Utility or well-being is equated with consumption of the market goods X and Y.*
3. *More consumption is always preferred to less.*
4. *All items giving an individual "utility" are subject to substitution and trade.* ◀

FROM INDIFFERENCE CURVES TO EXCHANGE: THE EDGEWORTH BOX DIAGRAM

Armed with our model of human behavior and our goal of efficiency, we can develop a set of rules about how two people (or more than two people using mathematics) will engage in barter and trade to make themselves better off. The diagram in Figure 1.2 is called an *Edgeworth box*, named after the economist and mathematician Francis Edgeworth (1845–1926).

Figure 1.2 is actually two indifference curve diagrams put together, one for consumer A and one for consumer B. The origin for consumer A is at the

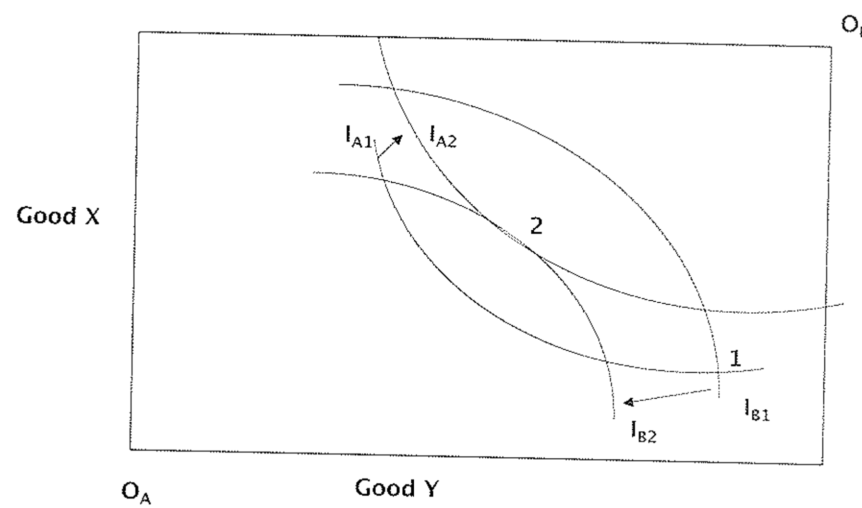


Figure 1.2. Exchange as depicted in an Edgeworth box diagram

lower left-hand corner so that his utility increases steadily as we move up and to the right in the Edgeworth box (because he has more of goods A and B). The origin for consumer B is at the upper right-hand corner so that her utility increases as we move down and to the left in the Edgeworth box. Any point inside the Edgeworth box shows the distribution of the two goods among the two consumers.

Notice that if we move from point 1 to point 2, the utility for both consumers increases. Consumer A moves from indifference curve I_{A1} to the higher indifference curve I_{A2} and consumer B moves from indifference curve I_{B1} to the higher indifference curve (farther from the origin for B) I_{B2}. At point 2 both consumers have been made better off by trading. A movement from point 1 to point 2 in Figure 1.2 is a graphical representation of a voluntary trade of one CD for another in the barter example we began with.

Notice that at point 2 no further trading can take place without making one of the consumers worse off. If we move away from point 2 to anywhere else on indifference curve I_{A2}, consumer B moves to an indifference curve with less utility. If we move from point 2 to any other point on indifference curve I_{B2}, consumer A is on a lower indifference curve with less total utility. Point 2 is a Pareto-efficient point. Now looking at the indifference curves for the two consumers at point 2 we can see something very important. The indifference curves are just tangent to one another, indicating that their slopes (their marginal rates of substitution of X for Y) are the same.

Pareto Condition I. *The condition for Pareto efficiency in exchange in a barter economy is that the marginal rates of substitution between the two goods is the same for the two consumers. When this occurs, no further trading can increase the utility of one consumer without decreasing the utility of the other.*

▶ ASSUMPTION ALERT!

1. *This is a model of the static exchange of a fixed amount of goods among consumers with stable preferences, and each consumer has (implicitly) perfect information about the characteristics of the goods and the preferences of the other consumer.*

2. *The particular Pareto-efficient outcome depends on the initial distribution of the goods among the two consumers. Look at Figure 1.2 and convince yourself that a different initial distribution of X and Y will result in a different Pareto-efficient distribution.* ◀

CRITICAL THINKING—The rationale for the benefits of exchange depicted in Figure 1.2 is perhaps the single most important concept in contemporary economic theory and policy. Answer the following questions based on Figure 1.2 and then critically examine your answers in light of the assumptions underlying the figure. Think of real-world examples and real-world complications.

1. Why do most economists advocate free trade?
2. Why do most economists insist that the optimal amount of pollution is greater than zero?
3. So far we have said nothing about prices; all trade is the result of direct negotiations. How would using prices as indicators of value change your answers to questions 1 and 2?

REMEMBER, THINK CRITICALLY!

ONE MORE THING BEFORE MOVING ON—THE MANY PARETO EFFICIENCIES

For any particular initial distribution of goods X and Y among consumers A and B, there will be only one Pareto-efficient outcome of trade. Each different initial distribution of X and Y will yield a different Pareto-efficient outcome. A line connecting all the Pareto points in an Edgeworth box is called a **contract curve**, and such a curve is shown in Figure 1.3. We will return to the contract curve later when we discuss the notion of a **social welfare function**. Given the preferences of the two consumers A and B, for the two goods X and Y, as depicted by the shapes of the indifference curves, the contract curve shows the Pareto-efficient allocations of the two goods *for all possible* initial distributions of the two goods between the two consumers.

Figure 1.3 illustrates a very important concept lying at the base of Walrasian economic policy. By altering the initial distribution of goods X and Y (this is called a **lump-sum transfer**), any particular Pareto-efficient outcome

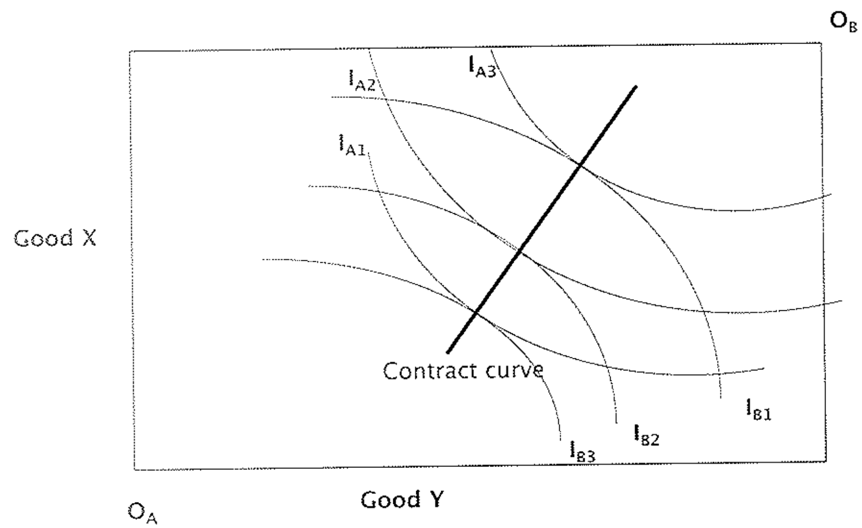


Figure 1.3. A contract curve showing **all Pareto-efficient possibilities**

can be reached. This has important implications for economic policy. In this framework the ideal policy to correct inequality, for example, is to let the political process set the parameters (the initial distribution of goods) and let the "market" determine the final outcome. This preserves the efficiency of the trading process.

THE MATHEMATICAL INTERPRETATION OF UTILITY

Mathematically, the indifference curve may be stated as a **utility function** of the form:

(1.1) $U_A = f(X,Y)$

The utility of consumer A is a function of (depends on) the amounts of commodities X and Y consumed. Much of Walrasian analysis uses the mathematics of **constrained optimization**. This mathematics is very simple but it can be intimidating to the uninitiated. Most of the mathematics in economics deals with marginal change, which is expressed by the concept of the derivative. For example, the change in utility of consumer A that results from a

change in the amount of good X is expressed as dU/dX (or using the Greek letter delta, $\Delta U/\Delta X$ or $\partial U/\partial X$). It shows the effect of a small change in the amount of good X on total utility *with the amounts of all other goods possessed by consumer A held constant*. This is called the **marginal utility** of X. Likewise, dU/dY is the marginal utility of good Y.

Referring to equation (1.1), we can perform a mathematical operation called total differentiation to examine the change in utility resulting from changes in the amounts of goods X and Y.

(1.2) $dU = (dU/dX)\,\Delta X + (dU/dY)\,\Delta Y$

The change in total utility in the simple two-good economy is equal to the marginal utility of X, that is, how a one-unit change in X (or dX) changes the utility of the consumer (or dU), multiplied by the actual unit change in X (or ΔX) plus the marginal utility of Y multiplied by the actual unit change in Y. For example, if the marginal utility of X is 2 "utils" and the marginal utility of Y is 3 "utils," and we give the consumer 2 more X's and 3 more Y's, the consumer's utility goes up by $2 \cdot 2 + 3 \cdot 3 = 13$ utils.

By definition, utility does not change along an indifference curve, so $dU = 0$. Thus we can rewrite equation (1.2) as:

(1.3) $(dU/dX)\,\Delta X = -(dU/dY)\,\Delta Y$ or

(1.4) $(dX/dY) = -(dU/dY)/(dU/dX) = -(MU_Y/MU_X) = MRS_{Y \text{ for } X}$

Along an indifference curve, the ratio of marginal utilities is equal to the (negative) slope of the indifference curve, (dX/dY). So the slope of the indifference curve at any particular point shows the rate at which the consumer can substitute one good for another and keep her utility constant. Stated another way, the ratio of marginal utilities of the two goods is equal to the marginal rate of substitution of one of those goods for the other.

Terms and Concepts to Know Before Moving On (see the glossary at the end of the chapter)

Diminishing marginal rate of substitution
Exogenous preferences

Homo economicus
Indifference curve
Marginal rate of substitution
Marginal utility
Pareto efficiency in exchange
Utility
Utility function
Welfare economics

CONSTRAINED OPTIMIZATION

Economic analysis is dominated by models of constrained optimization. Models are constructed to maximize one thing (utility, production, profit) subject to some constraint (income, production costs). The mathematics may seem complicated at first blush, but once you learn the principle of constrained optimization you can apply it to a wide variety of economic problems.

The CD trading example is a constrained optimization problem. Each person playing the game attempts to maximize the satisfaction gained from his or her collection of CDs given the following initial constraints:

1. The total collection of CDs to be traded is given.
2. The initial distribution of these CDs among consumers is given.
3. The preferences of all participants are consistent and stable.

By trading with each other, consumers A and B maximize the utility they derive from consuming goods X and Y given the three initial conditions. They trade goods until they reach a point where any further trading would make at least one of them worse off. At this point the slopes of the indifference curves of the two consumers are the same, which means that the marginal rates of substitution of good X for good Y is the same for both consumers.

Mathematically, the constrained optimization problem looks like this:

$$(1.5) \quad Z_A = U_A(X_A, Y_A) + \lambda[U_B(X^0 - X_A, Y^0 - Y_A) - U_B^0]$$

Equation (1.5) is called a *Lagrangian* equation and is an indispensable tool of Walrasian welfare economics. By convention, we use Z for the Lagrange

equation rather than L so as not to confuse it with equations for labor. The utility of consumer A (U_A) is maximized subject to the available amounts of the goods—total amounts of the goods minus those consumed by consumer B. The total amounts of the two goods are given as $X^0 = X_A + X_B$ and $Y^0 = Y_A + Y_B$. λ is the *Lagrangian multiplier*, and it is discussed in more detail in Chapter 4.

The combination of goods X and Y that give the highest possible utility to consumer A can be found by taking the partial derivatives (denoted by Δ) of Z_A with respect to X_A, Y_A, and λ.

$$(1.6) \quad \partial Z_A / \partial X_A = \partial U_A / \partial X_A - \lambda(\partial U_B / \partial X_B) = 0$$

$$(1.7) \quad \partial Z_A / \partial Y_A = \partial U_A / \partial Y_A - \lambda(\partial U_B / \partial Y_B) = 0$$

$$(1.8) \quad \partial Z_A / \partial \lambda = U_B(X^0 - X_A, Y^0 - Y_A) - U_B^0$$

Dividing equation (1.6) by equation (1.7) yields the condition for maximizing the utility of consumer A, given the fixed utility of consumer B:

$$(1.9) \quad (\partial U_A / \partial X_A) / (\partial U_A / \partial Y_A) = (\partial U_B / \partial X_B) / (\partial U_B / \partial Y_B)$$

This is exactly the same condition for Pareto efficiency that we saw earlier in Figure 1.2. The ratios of the marginal utilities of goods X and Y (the marginal rates of substitution) have to be the same for both consumers A and B. When this condition is fulfilled, no further trading of the goods can make one person better off without making the other person worse off.

THE NECESSITY OF THE INDEPENDENT UTILITIES ASSUMPTION IN WALRASIAN THEORY

Notice that the utility function for consumer A (equation [1.5]) does not depend directly on the utility of consumer B but only on the amounts of X and Y he consumes. His utility is unaffected by the amounts of X and Y that consumer B has. The assumption of independent utilities is critical to the result shown in equation (1.9). To ensure Pareto efficiency, the rates of commodity substitution, Y for X, have to be the same for both consumers. As Chapter 3 shows, this result is critical for establishing the conditions for general equilibrium in a competitive economy. If the consumption of one consumer is directly affected by the level of utility of the other consumer, as in the utility functions,

$$(1.10) \quad U_A = U_A(X_A, Y_A, X_B, Y_B) \text{ and } U_B = U_A(X_A, Y_A, X_B, Y_B)$$

then the first condition for Pareto efficiency does not hold, that is, $MRS^A_{Y \text{ for } X} \neq MRS^B_{Y \text{ for } X}$, and the conditions of general equilibrium (see Chapter 3) cannot be established (for a mathematical proof of this, see Henderson and Quandt 1980, 297). Numerous experiments in the fields of behavioral economics, neuroscience, and psychology have established that preference formation is in fact "other regarding," that is, the utility of one person is affected by the utility of others. The implications of these findings for utility theory are explored in Part Two.

APPENDIX

Convexity Tests

The assumption of the convexity of indifference curves is necessary to ensure a unique solution to any consumer maximization problem. Convexity ensures that the indifference curves for the two consumers are tangent at only one point. Graphically, convexity means that all the points on a line drawn between any two points on the indifference curve are interior points. Without convexity we could have multiple tangency points between two indifference curves. We also need to establish that utility is being maximized, not minimized. If utility is at a maximum level, then any movement away from that point subtracts from total utility (a negative number).

Mathematically, convexity and utility maximization can be established by starting with the utility function:

$$(1.11) \quad U^0 = f(X, Y), \text{ where utility is constant at } U^0.$$

The total differential of this function is $dU = (\partial U / \partial X)dX + (\partial U / \partial Y)dY$. The change in utility (dU) is equal to the change in utility from an additional unit of good X ($\partial U / \partial X$) times the change in the number of units of good X (dX) plus the change in utility from an additional unit of good Y ($\partial U / \partial Y$) times the change in the number of units of good Y (dY).

We are moving along an indifference curve, meaning that utility is unchanged, $dU = 0$, so we can write:

$$(1.12) \quad dU = (\partial U / \partial X)dX + (\partial U / \partial Y)dY = 0, \text{ and}$$

$$(1.13) \quad dY / dX = -(\partial U / \partial X) / (\partial U / \partial Y)$$

The slope of the indifference curve (dY / dX) equals the ratio of marginal utilities of X and Y, which is the marginal rate of substitution of Y for X. Either ∂X or ∂Y must be negative to offset the positive effect of a change in the amount of the other good.

We can further differentiate equation (1.13), yielding the second differentiation of equation (1.11), to get:

$$(1.14) \quad d^2Y / dX^2 = [-1 / (\partial U / \partial Y)^3] \, [(\partial^2 U / \partial X^2)(\partial U / \partial Y)^2 - 2 (\partial^2 U / \partial X \partial Y)(\partial U / \partial X)(\partial U \partial Y) + (\partial^2 U / \partial Y^2)(\partial U / \partial X)^2]$$

Equation (1.14) shows the rate of change of the slope of the indifference curve.

In order for the indifference curve to have a negative slope, the function must be differentiable (smooth and continuous) and the value of the term within the brackets of equation (1.14) must be negative:

$$(1.15) \quad (\partial^2 U / \partial X^2)(\partial U / \partial Y)^2 - 2 (\partial^2 U / \partial X \partial Y)(\partial U / \partial X)(\partial U \partial Y) + (\partial^2 U / \partial Y^2)(\partial U / \partial X)^2 < 0$$

Equation (1.15) can be used to test the convexity of specific forms of the utility function. For example, consider the function $U = XY$. Does this function pass the convexity test?

$\partial U / \partial X = Y$

$\partial U / \partial Y = X$

$\partial^2 U / \partial X^2 = 0$

$\partial^2 U / \partial X \partial Y = 1$

$\partial^2 U / \partial Y^2 = 0$

$\partial^2 U / \partial Y \partial X = 1$

Substituting these results into equation (1.15) yields:

(1.16) $(0)(X^2) - 2(1)(Y)((X) + (0)(Y^2) + (0)(Y^2) = -2YX < 0$

This function is convex so long as positive amounts of each good are consumed.

Discounting

As we have seen, the basic Walrasian model is one of static exchange—it depicts a one-shot exchange of a fixed set of goods. In real market situations the time factor is critical. In our CD example, suppose we want to trade one CD for the promise of another to be delivered at some point in the future. We know that, in general, we would rather have something now than later, so a CD received a year from now is worth less than one received now. This difference in value is captured by a discount rate. For example, if something received a year from now is worth only 90 percent of what it is worth if received now, this implies a discount rate of 10 percent.

Using a discount rate allows us to take the time dimension into account without changing the basic framework of analysis. Referring to Figure 1.4, if good Y is delivered in the future, all we have to do is apply the discount rate r according to the discount formula $(1+r)^{-t}$ and proceed as usual.

GLOSSARY

Constrained optimization—The maximization or minimization of an objective function subject to constraints imposed on the independent variable. For example, maximizing utility subject to an income constraint or minimizing costs subject to an output constraint.

Contract curve for exchange—The locus of all Pareto-efficient points in an Edgeworth box diagram, each point representing a different initial distribution of goods. At each point on the contract curve, the marginal rates of substitution between the two goods are the same for the two consumers.

Convexity test—A mathematical determination of whether a function (a utility function in this chapter) is smooth and continuous. Convexity is neces-

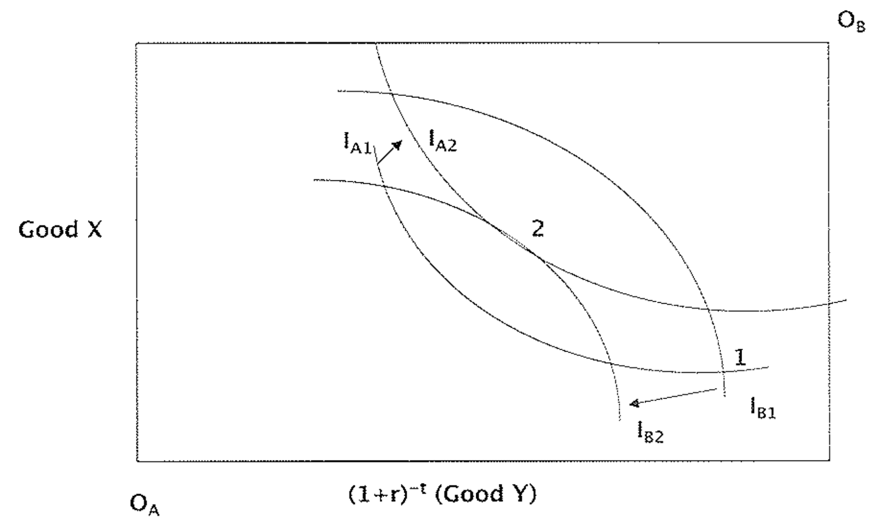


Figure 1.4. Discounting in an Edgeworth box diagram

sary to demonstrate that there exists a unique combination of goods that maximizes a consumer's utility.

Diminishing marginal rate of substitution—As the amount of one good increases relative to another good, the more of the first good a consumer is willing to give up in exchange for the second good, keeping total utility constant.

Diminishing marginal utility—As a consumer obtains more and more of one good, the amounts of all other goods held constant, the point will be reached where the utility from an additional unit begins to decline.

Discounting—The assumption is made in economic theory that a good delivered in the future is worth less than that same good delivered presently. Discounting determines how much less goods delivered in the future are worth, valued from the point of view of the present.

Exogenous preferences—Assumption that human preferences are entirely self-regarding, that is, commodity bundles are evaluated independently of what other people have or choose. Preferences can be evaluated outside (they are exogenous to) social context.

Homo economicus—Rational economic man, whose preferences are consistent, insatiable, and independent of the preferences of others.

Indifference curve—A curve showing all the combinations of two goods yielding the same amount of utility.

Lump-sum transfer—In general equilibrium analysis, the transfer of some initial endowment of goods from one person to another. Equilibrium will still be attained, but the Pareto-efficient distribution of goods will be changed.

Marginal rate of substitution—The rate at which a consumer can substitute one good for another without changing his or her level of total utility. Also called the rate of commodity substitution.

Marginal utility—The additional utility obtained from one additional unit of a commodity, the amounts of all other commodities held constant.

Pareto efficiency in exchange—In consumption, a situation in which no further trading of goods can make one person better off without making another person worse off.

Social welfare function—A graph or curve showing all the possible combinations of individual utilities where social welfare is the same. The social welfare function is based on given preferences, technology and resource endowment, and some specific ethical assumption about the fair distribution of goods among consumers.

Utility—The amount of satisfaction derived from consuming market goods and services.

Utility function—Expresses utility or well-being as a function of the quantities of market goods consumed. In this chapter, we have seen the general form $U_A = f(X,Y)$. Utility functions may also be written to indicate specific functional relationships between the commodities, such as the general Cobb–Douglas ($U = AX^aY^b$), linear ($U = aX + bY$), or fixed proportions (Leontief) ($U = \min(aX, bY)$) utility functions.

Welfare economics—The branch of economics dealing with the welfare or well-being of human society.

REFERENCE

Henderson, J., and J. Quandt. 1980. *Microeconomc Theory: A Mathematical Approach*. New York: McGraw-Hill.

2

THE NEOCLASSICAL THEORY OF PRODUCTION

Assuming equilibrium, we may even go so far as to abstract from entrepreneurs and simply consider the productive services as being, in a certain sense, exchanged directly for one another.

—*Léon Walras,* Elements of Pure Economics *[1874] (London: George Allen and Unwin, 1954), 225*

THE FOUNDATION OF PRODUCTION THEORY—INPUT EXCHANGE IN A BARTER ECONOMY

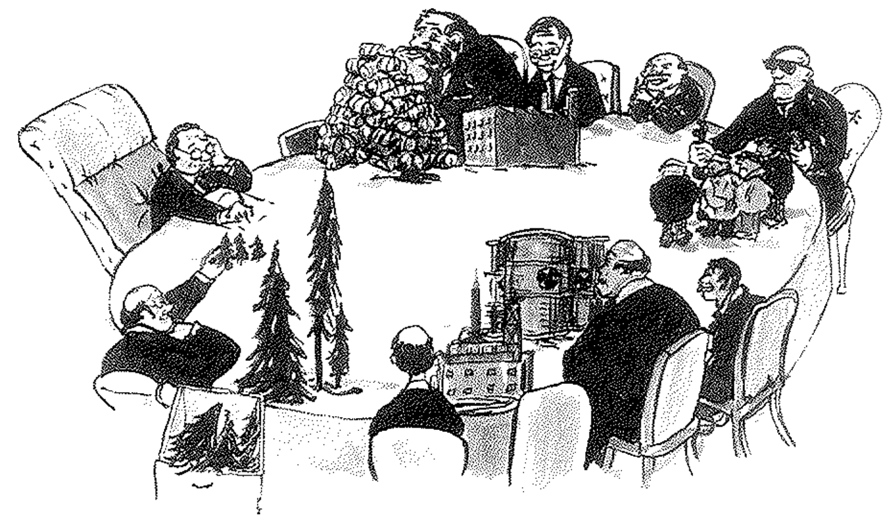
We saw in Chapter 1 how two consumers in a barter economy allocate their scarce resources (the goods in their possession) in order to maximize their well-being. The other side of the coin in a modern economy is production, and the neoclassical analysis of production uses exactly the same framework as does the model of consumers exchanging goods. Consumers are the locus of "consumption" and firms are the locus of "production." In the pure model of production, firms directly exchange productive inputs with other firms in order to increase output.

In our analysis, we will assume there are only two firms that use two inputs, capital (K) and labor (L), to produce two goods, X and Y.

Input Exchange in a Pure Barter Economy

The three starting points for the analysis that follows are:

1. The inputs to be exchanged—the amounts of capital and labor—are given at the start of the analysis.
2. The initial distribution of the inputs among the firms is given at the start of the analysis.



3. Technology—the way in which capital and labor are used to produce the output of goods X and Y—does not change during the period of analysis.

Just as consumers exchanged goods so as to maximize utility, so too do firms exchange inputs so as to maximize output. Consumers have different tastes (different utility functions, in economic jargon) and firms have different technologies (different **production functions**). Firms keep trading inputs until no further trading can increase the output of one firm without decreasing the output of another firm. This is exactly the same concept of **Pareto efficiency in exchange** described in Chapter 1, only now applied to production.

Pareto Efficiency in Input Allocation—*A situation in which no further trading of inputs can increase the output of one firm without decreasing the output of another firm.*

THE GRAPHICAL ANALYSIS OF INPUT ALLOCATION

The production equivalent of the indifference curve is the **isoquant**, or "same quantity," showing the different combinations of capital and labor that can be used to produce the same amount of output. Figure 2.1 shows how a firm uses

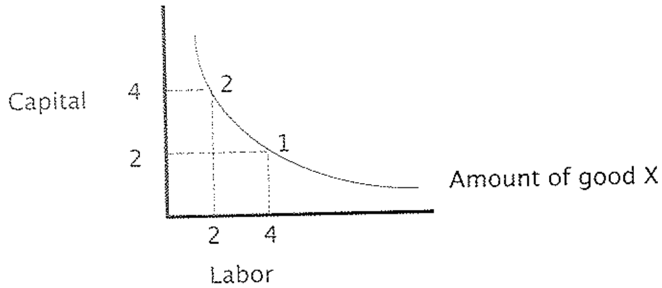


Figure 2.1. An isoquant

productive inputs, labor and capital, to produce a particular good X. The same amount of good X can be produced using either four units of capital and two units of labor or by using two units of capital and four units of labor. In the same way that commodities are subject to substitution in consumption, so too are inputs subject to substitution in production as indicated by the downward slope of the isoquant. Higher isoquants, farther away from the origin, represent more total output than ones closer to the origin. Isoquants, like indifference curves, have the mathematical property of being smooth and continuous; there are no "jumps" in output as one input is substituted for another. Keep in mind that the shape of isoquants says something about the physical nature of production. That is, when we assume smooth and continuous isoquants, we are saying that capital and labor are perfectly malleable. If we want to produce 10 units of X per day, for example, there is a machine available that is exactly the right size to do that, and a slightly larger one to produce 11 units of X, and so on.

The slope of an isoquant at a particular place along a curve, $\Delta K / \Delta L$, again the "rise over the run," indicates the **marginal rate of technical substitution** (MRTS) of one input for another. For example, between points 1 and 2 in Figure 2.1 the marginal rate of technical substitution of input K for input L ($MRTS_{L\ for\ K}$) is 1. At that point the firm can reduce the input of K by two units and add two units of L without changing total output. The shape of the isoquants, becoming steeper or flatter as they approach the K or L axes, indicates a **diminishing marginal rate of technical substitution**. As shown in

Figure 2.1, as a firm uses more and more of input K, it takes more of input K relative to input L to keep production at the same level.

▶ ASSUMPTION ALERT! *Critical assumptions made so far about production:*

1. *All inputs used to produce a particular good are substitutable for one another.*
2. *Inputs are malleable, that is, there is no "lumpiness" in the production process.*
3. *The shapes of isoquants may vary, but whichever one is used is taken to be an adequate representation of the physical and technological reality of producing the good in question.* ◀

PARETO EFFICIENCY IN INPUT ALLOCATION

The trading of productive inputs among firms can also be examined using an Edgeworth box diagram. Figure 2.2 depicts the trade of two inputs (K and L) between two firms producing goods X and Y. Any point in the Edgeworth box represents the allocation of the two inputs between the two firms. Notice that if we move from point 1 to point 2 in the Edgeworth box, the production of both goods X and Y increases. The production of good X shifts from isoquant

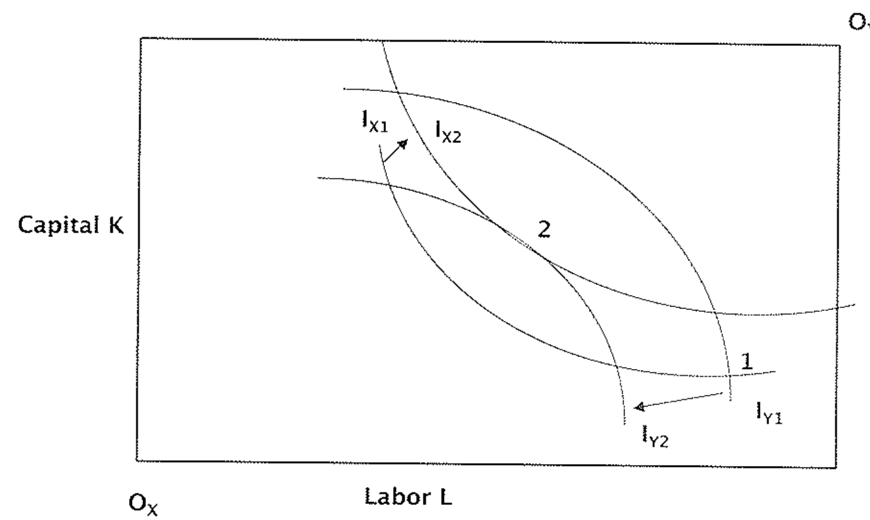


Figure 2.2. Exchange of inputs as depicted in an Edgeworth box diagram

I_{X1} to the higher isoquant I_{X2} and the production of good Y moves from isoquant I_{Y1} to the higher isoquant (farther away from the origin for Y) I_{Y2}.

Notice that at point 2 no further trading of inputs can take place without reducing the output of at least one good. If we move away from point 2 to anywhere else on isoquant I_{X2}, the production of good Y has to occur on an isoquant closer to the origin with lower output. Likewise, if we move from point 2 to any other point on isoquant I_{Y2}, the production of good X is reduced because we are on a lower isoquant with less total output. Point 2 is a *Pareto-efficient* point for the production of goods X and Y. At this point the isoquants for the production of both goods are tangent to one another indicating that their slopes are the same, that is $MRTS^X_{L\ for\ K} = MRTS^Y_{L\ for\ K}$.

> Pareto Condition II. *The condition for Pareto efficiency in production in a barter economy is that the marginal rates of technical substitution for the two inputs are the same for the production of both goods. When this occurs, no further trading can increase the production of one good without decreasing the production of another good.*

THE MATHEMATICAL INTERPRETATION OF THE ISOQUANT

Mathematically, the isoquant curve may be stated as a production function of the general form:

(2.1) $Q_X = f(K,L)$

The quantity of good X produced is a function of the amounts of capital and labor used. As in the problem of consumer choice discussed above, Walrasian analysis uses the mathematics of constrained optimization to describe production. The change in the output of good X resulting from a change in the amount of capital used is expressed as dQ_X / dK, $\Delta Q_X / \Delta K$ or $\partial Q_X / \partial K$. It shows the effect of a small change in the amount of K on the total output of X. This is called the **marginal product** of capital. Likewise, dQ_X / dL is the marginal product of labor.

Just as we did for the utility function in Chapter 1, we can totally differentiate the production function depicted in equation (2.1) to examine the change in output resulting from changes in the amounts of inputs K and L.

(2.2) $dQ_X = (dQ_X / dK)\ \Delta K + (dQ_X / dL)\ \Delta L$

The change in the total output of X is equal to the marginal product of capital (the effect of a one unit change in K on Q_X) multiplied by the actual unit change in K times the marginal product of L multiplied by the actual unit change in L. For example, if the marginal product of K is 2 X and the marginal product of L is 3 X, and we give the producer two more K's and three more L's, the output of X goes up by $2 \cdot 2 + 3 \cdot 3 = 13$ X.

By definition, the amount produced does not change along an isoquant, so $dQ_X = 0$. Thus we can rewrite equation (2.2) as:

(2.3) $(dQ_X / dL)\ \Delta L = -(dQ_X / dK)\ \Delta K$

Because either dL or dK must be negative along an isoquant, we can write this using positive signs as:

(2.4) $(dQ_X / dL) / (dQ_X / dK) = (MP_L / MP_K) = MRTS_{L\ for\ K} = (dK / dL)$

Along an isoquant, the ratio of marginal products (the marginal rate of technical substitution of L for K) is equal to the (negative) slope of the isoquant, $-(dK / dL)$. So the slope of the isoquant at any particular point shows the rate at which labor can be substituted for capital and keep output constant. The marginal rate of technical substitution of one input for another is equal to the ratio of the marginal products of those inputs.

Terms and Concepts to Know Before Moving On (see the glossary at the end of the chapter)

Diminishing marginal productivity
Diminishing marginal rate of technical substitution
Isoquant
Marginal rate of technical substitution
Pareto efficiency in production
Production function

CONSTRAINED OPTIMIZATION

As in the case of consumers trading goods, the problem of firms trading inputs can also be examined using a constrained optimization approach. Each firm attempts to maximize its output by trading inputs with other firms, given the initial constraints:

1. The total amounts of inputs are given.
2. The initial distribution of these inputs between the firms is given.
3. The technology used by the firms does not change.

In Figure 2.2, firms X and Y maximize the output of the goods they produce given their technology (production functions) and initial endowment of K and L. This occurs when the slopes of the isoquants of the two firms are the same, which means that the marginal rates of technical substitution of inputs K and L are the same for both firms.

Mathematically, the constrained optimization problem looks like this:

$$Z_X = Q_X(K_X, L_X) + \mu[Q_Y(K^0 - K_A, L^0 - L_A) - Q_Y^0] \quad (2.5)$$

The output of firm $X(Q_X)$ is maximized subject to the given available amounts of the inputs—which would be the total amounts of the inputs K and L minus those used by firm Y. The total amounts of the two inputs are given as $K^0 = K_X + K_Y$ and $L^0 = L_X + L_Y$. The symbol μ is the Lagrangian multiplier.

The combination of inputs K and L yielding the highest possible output for firm X can be found by taking the partial derivatives of Q_X^* with respect to K_X, L_X, and λ.

$$\partial Z_X / \partial K_X = \partial Q_X / \partial K_X - \mu\,(\partial Q_Y / \partial L_Y) = 0 \quad (2.6)$$

$$\partial Z_X / \partial L_X = \partial Q_X / \partial L_X - \mu\,(\partial Q_Y / \partial L_Y) = 0 \quad (2.7)$$

$$\partial Z_X / \partial \mu = Q_Y(K^0 - K_X, L^0 - L_X) - Q_Y^0 \quad (2.8)$$

Dividing equation (2.6) by equation (2.7) yields the condition for maximizing the output of firm X, given the fixed output of firm Y:

$$(\partial Q_X / \partial K_X) / (\partial Q_X / \partial L_X) = (\partial Q_Y / \partial K_Y) / (\partial Q_Y / \partial L_Y) \quad (2.9)$$

This is exactly the same condition for Pareto efficiency that we saw in Chapter 1, except now we are dealing with marginal products instead of marginal utilities. The ratios of the marginal products of inputs K and L (the marginal rates of technical substitution) have to be the same for both firms X and Y. When this condition is fulfilled, no further trading of the inputs can increase the output of one firm without decreasing the output of the other.

MORE ON PRODUCTION FUNCTIONS

The production function in equation (2.1) is written in what is called a "general form." It merely states that the production of good X depends on some amounts of the inputs K and L. In empirical studies of substitution possibilities among inputs, the production function must be given a specific mathematical form. Different forms make different assumptions about the nature of substitution among inputs, that is, about the nature of production technology. For example, a linear production function assumes that output is an additive function of the inputs used.

In the case of a linear production function (Figure 2.3), inputs are very easily substituted for one another. In fact, good X can be produced using only labor or only capital. The input substitution possibilities are infinite.

At the other end of the scale is the Leontief or fixed proportion production function (Figure 2.4), named after the economist Wassily Leontief, who received a Nobel Prize in economics for his pioneering work in input-output analysis.

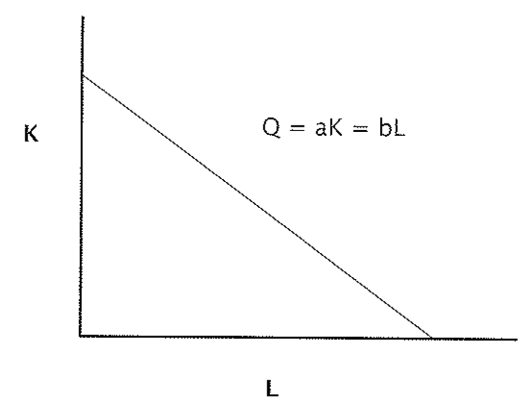


Figure 2.3. A linear production function

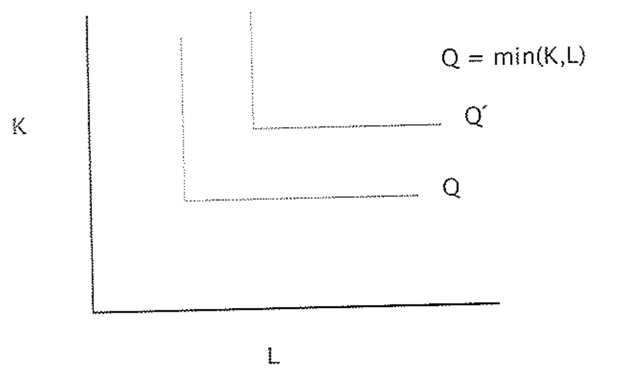


Figure 2.4. A Leontief or fixed proportions production function

In this case there are no substitution possibilities among inputs. To produce more output, both labor and capital are needed, and they are needed in exact proportions. No substitution is possible among inputs.

One of the most widely used functional forms for the production function is the Cobb–Douglas production function (Figure 2.5), proposed by Charles Cobb and Paul Douglas in 1927 (and apparently it was used even earlier). It describes output as a function of the inputs capital and labor and a technological parameter "A." In its simple form, technology is assumed to be *exogenous* so that a technology advance (an increase in A) will allow more output to be produced with given amounts of capital and labor.

The shape of the Cobb–Douglas function is a rectangular hyperbola. This implies that as the amount of one input increases relative to the amount of the other, past a certain point the marginal product of the first input will decline. This brings up an important concept in production theory known as the **elasticity of substitution.**

$$(2.10) \quad \sigma = \Delta(K/L)/(K/L) \div \Delta(MRTS_{L\text{ for }K})/MRTS_{L\text{ for }K}$$

Since in competitive equilibrium $MRTS_{L\text{ for }K} = MP_L/MP_K$ (see Chapter 4), the elasticity of substitution is usually written:

$$(2.11) \quad \sigma = \%\ \Delta(K/L) \div \%\ \Delta[(MP_L/MP_K)]$$

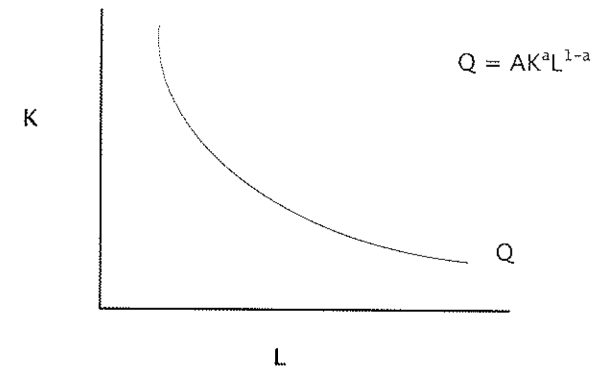


Figure 2.5. A Cobb–Douglass production function

The elasticity of substitution is the percent change in the ratio of the inputs used, divided by the percent change in the ratio of their marginal products. It shows how a change in the relative productivities of two inputs changes the relative amount used of those inputs.

When we bring prices into our basic model we will see that, assuming competitive equilibrium in the input market, the ratio of marginal products of the inputs will equal the ratio of their prices. So the elasticity of substitution shows how easy it is to substitute one input for another as their relative prices (in competitive equilibrium, this is equal to their relative marginal products) change. The three functional forms above have very different elasticities of substitution, reflecting the assumptions about production technology built into the production functions. A linear production function has an almost infinite elasticity of substitution because output can be easily increased by increasing either input without using more of the other. In fact it is possible to produce the good using only capital or only labor. The Leontief function has an elasticity of substitution of zero because an increase in both inputs in the same proportions is necessary to increase output. The Cobb–Douglas function has an elasticity of substitution equal to one, meaning that if the marginal product of labor increases by 10 percent relative to the marginal product of capital, then the amount of labor used increases by 10 percent relative to the amount of capital used.

It is worth going into a little more detail about the mathematics of the Cobb–Douglas function because it illustrates the power and convenience of the production function approach, and also the hidden dangers of building in assumptions about the nature of the production process based primarily on mathematical convenience. A typical way to write the Cobb–Douglas production function is:

(2.12) $Q=AK^{a}L^{1-a},\ 0<a<1$

In this form, the Cobb–Douglas function exhibits **constant returns to scale**. If capital and labor are both increased by the same percentage, output also increases by the percentage. A 10 percent increase in both capital and labor means that output will increase by 10 percent. In mathematical jargon, this function is *linearly homogeneous*.

Another property of the Cobb–Douglas function is that the average products of capital and labor (Q / L) and (Q / K) and their marginal products (dQ / dL) and (dQ / dK) depend on the ratio in which capital and labor are used (K / L). To prove this, we can rewrite equation (2.7) as:

(2.13) $Q=AK^{a}L^{1-a}=A(K/L)^{a}L$

Then the average products of labor and capital are:

(2.14) $Q/L=A(K/L)^{a}(L/L)=A(K/L)^{a}$

(2.15) $Q/K=A(K/L)^{a}(L/K)=A(K/L)^{a}(K/L)^{-1}=A(K/L)^{a-1}$

And the marginal products become:

(2.16) $dQ/dL=(1-a)\ AK^{a}L^{1-a-1}=(1-a)A(K/L)^{a}$

(2.17) $dQ/dK=AK^{a-1}L^{1-a}=aA(K/L)^{a-1}$ (aA can be written as A, because both a and A represent some unknown constant)

Yet another property of the Cobb–Douglas function is that the elasticity of substitution is always equal to unity. Using equation (2.6), the elasticity of substitution can be written as:

(2.18) $\sigma=d(K/L)/(K/L)\div d(MP_{L}/MP_{K})/(MP_{L}/MP_{K})$

Let s be the MRTS between capital and labor, which, as we learned, is equal to the ratio of marginal products of K and L. Letting k = K / L and using (2.11) and (2.12) gives us:

(2.19) $s=(1-a)A(K/L)^{a}/aA(K/L)^{a-1}=[(1-a)/a]k$

(2.20) $ds/dk=[(1-a)/a]$

Rewrite equation (2.13) as:

(2.21) $\sigma=dk/ds\div s/k$, or

(2.22) $\sigma=[a/1-a]\ [k/(1-a/a)k]=1$

This mathematical property implies that it is easy to substitute one input for another in production. A 10 percent increase in the marginal product of capital relative to the marginal product of labor will result in a 10 percent increase in the use of capital relative to the use of labor.

Another property of the Cobb–Douglas function is that the exponents of K and L represent each factor's share of total output if the factors are paid according to their marginal products. To show this we need to invoke **Euler's theorem,** which relates to a property of a **homogeneous function.** A function Y = f(x1, x2, . . . xn) is homogeneous of degree r, if it can be written as:

(2.23) $f(cx1, cx2, \ldots cxn)=c^{r} f(x1, x2, \ldots xn)$

Homogeneity means that if every term in the function is multiplied by some constant *c*, then the total value of the function will increase by the amount c^{r}, where r is the degree of homogeneity. For example, a production function exhibiting constant returns to scale is homogeneous of degree 1 (r = 1). If all inputs are increased by, say, 10 percent, total output will increase by 10 percent. For such a constant returns to scale production function (r = 1), Q = f(K,L), Euler's theorem implies:

(2.24) $K(dQ/dK)+L(dQ/dL)\equiv Q$

The amount of capital used times the marginal product of capital plus the amount of labor used times the marginal product of labor will exactly equal total output. Expressed in physical units this means that if this were a production function for corn, and the factors of production were paid in corn, paying each factor according to its marginal product would exactly exhaust the output of corn during the time period of production. In Chapter 4, when we bring in money and prices, we will see that in a competitive economy in the long run, factors of production are paid according to their marginal products. According to the results of Euler's theorem, this means that total output will be exactly used up if it is distributed to the factors producing that output according to the condition: factor price = value of the marginal product. We will return to this idea in Chapter 4 when we discuss the characteristics of a competitive economy.

Finally, we can use the results above to show another property of the Cobb–Douglas function. The exponents a and 1−a are the output shares of capital and labor.

For capital we have:

$$(2.25)\quad K(dQ/dK)/Q = K\,aA(K/L)^{a-1}/Q = aKAk^{a-1}/LAk^{a} = akAk^{a-1}/Ak^{a} = a$$

For labor the proof is:

$$(2.26)\quad L(dQ/dL)/Q = L(1-a)A(K/L)^{a}/Q = L(1-a)Ak^{a}/LAk^{a} = 1-a$$

The convenient mathematical properties of the Cobb–Douglas function have made it a real workhorse for use in statistical economic analysis. Variations of the Cobb–Douglas production function are still widely used, particularly in **total factor productivity** analysis (see the appendix).

The history of production function analysis can be seen as a steady relaxation of the restrictions of the Cobb–Douglas function. One step toward generality was relaxing the assumption that the coefficients must sum to one. A general form of the Cobb–Douglas function is:

$$(2.27)\quad Q = AK^{a}L^{b}$$

In the general form, a+b is allowed to take on any value and indicates the degree of homogeneity of the function and the returns to scale of the production process it represents. If $a+b>1$ this indicates increasing returns to scale—double all inputs and output more than doubles. If $a+b<1$ this indicates decreasing returns to scale. Double all the inputs and output will increase by some factor less than that.

A production function that became popular in the 1960s is the CES, or constant elasticity of substitution function:

$$(2.28)\quad Q = \gamma[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-1/\rho} \text{ where } \rho > -1 \text{ and } \rho \neq 0$$

In this equation, δ is a distribution parameter indicating the relative amounts of capital and labor, and ρ is a substitution parameter determining the value of the elasticity of substitution according to $\sigma = 1/(1+\rho)$. The CES function is more flexible than the Cobb–Douglas because the elasticity of substitution $(1/1-\rho)$ is not constrained to take on any particular value. However, whatever value it takes must be the same for any pair of inputs. For example, if we include three inputs, capital, labor, and energy, the elasticity of substitution between capital and labor, capital and energy, and energy and labor are all identical.

In the 1970s, a more flexible functional form of the production function came into fashion—the transcendental logarithmic function, or translog function. It is what is known as a Taylor's expansion of the general production function Q = f(X1, X2, . . . Xn). For the two input case we have been considering, it can be written as:

$$(2.29)\quad \log Q = \log\gamma_0 + \alpha_1\log K + \beta_1\log L + \alpha_2(\log K)^2 + \beta_2(\log L)^2 + \gamma_1\log K\log L$$

The difficulty with the translog function is that it is very sensitive to the data used to estimate it. In time series estimates, small changes in the amounts of capital and labor can produce wide variations in the elasticity of substitution between inputs.

▸ ASSUMPTION ALERT! *Things to think about before moving on.*

The neoclassical theory of production is a model of the static exchange of a fixed amount of inputs among firms with given techniques of production and each firm having (implicitly) perfect information about the characteristics of the inputs and the production techniques available.

The particular Pareto-efficient outcome depends on the initial distribution of the inputs among the two firms. ◂

So far we have said nothing about prices, wages, or economic rent. No money has been involved, just physical relationships among inputs. Surprisingly, we will see that although microeconomic theory is sometimes called "price theory," in the pure Walrasian model money plays no independent role. As we will see later, this has surprising implications for neoclassical macroeconomics.

APPENDIX

Separability

It is frequently useful when analyzing a particular production process to separate that process into stages or components. For example, making a bicycle might involve making the chassis, wheels, seat, and pedals and putting them all together at the end (Figure 2.6).

In the production function framework, a technology is said to be separable if the marginal rate of technical substitution between two inputs in one technology (making the bicycle wheels, for example) is unaffected by changes in the levels of other inputs.

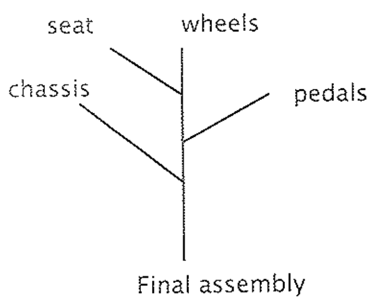


Figure 2.6. Separating assembly of a bicycle into discrete technologies

The Solow Growth Model and Total Factor Productivity

In 1957 Robert Solow developed a "dynamic" version of the Cobb–Douglas function. Solow's model essentially allocates the *growth rates* of inputs and outputs instead of the absolute amounts of inputs. Solow used the neoclassical production function to explore the relationship between the growth of output per worker (Q / L) and the growth of the capital labor ratio (K / L). Solow's work helped earn him a Nobel Prize in economics and laid the groundwork for the ***microfoundations project*** in economics, that is, establishing the rules of behavior for the macroeconomy based on the microeconomic theory of the firm.

Stated in terms of rates of growth, the Cobb–Douglas function becomes:

(2.30) $\dot{Q} = A\dot{K}^{a}\dot{L}^{1-a}$.

A dot (•) over K, L, or Q indicates that variable's rate of growth (Figure 2.7). Equation (2.31) can be used to illustrate the concept of *total factor productivity* (TFP).

Total factor productivity (TFP) is written as:

(2.31) $A = \dot{Q} - a\dot{K} - (1-a)\dot{L}$

The weights a and (1 − a) are the product shares of capital and labor. As we will see later, in competitive equilibrium these are equal to the cost shares of K and L. Notice three things about the TFP measure:

1. It is calculated as a *residual*, that is, the portion of the growth rate in output not accounted for by the weighted growth rates of the inputs of capital and labor.
2. The relative importance of the inputs of capital and labor is indicated by their output shares. These are calculated based on the assumption of linear homogeneity (constant returns to scale).

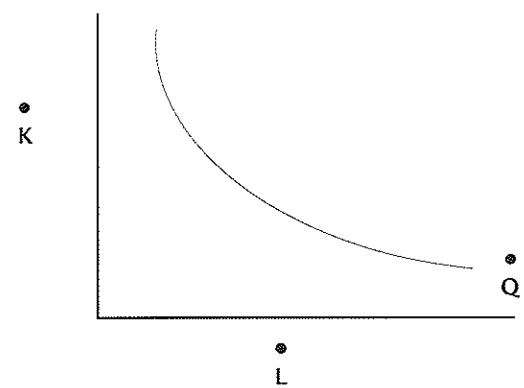


Figure 2.7. A production function in terms of growth rates

3. TFP is a reformulation of the static model of optimal allocation, using input growth rates rather than the absolute amounts of the inputs.

GLOSSARY

Constant returns to scale—If all inputs are increased by the same percentage, output will increase by that percentage. This is the mathematical property of being linearly homogeneous or homogenous of degree 1.

Contract curve for production—The locus of all Pareto-efficient points in an Edgeworth box diagram, each point representing a different initial distribution of inputs. At each point on the contract curve, the marginal rates of technical substitution between the two inputs are the same for the two producers.

Diminishing marginal productivity—As more and more of one input is used, the amounts of all other inputs held constant, the point will be reached where the increase in output from an additional unit of that input begins to decline.

Diminishing marginal rate of technical substitution—As the amount of one input increases relative to another input, the more of the first input a producer is willing to give up in exchange for the second input.

Elasticity of substitution—A mathematical expression indicating the ability of a firm (with a given technology) to substitute one input for another in production.

Euler's theorem—If a function such as $Q=f(K,L)$ is homogeneous of degree r, then we can write: $K(dQ/dK)+L(dQ/dL)\equiv rQ$

Homogeneous function—A function $Y=f(x1, x2, \ldots xn)$ is homogeneous of degree r if it can be written as: $f(cx1, cx2, \ldots cxn)=c^r f(x1, x2, \ldots xn)$. Homogeneity means that if every term in the function is multiplied by some constant c, then the total value of the function will increase by the amount c^r.

Isoquant—A curve showing all the combinations of two inputs yielding the same level of output.

Marginal product—The additional output obtained from one additional unit of a productive input, the amounts of all other inputs held constant.

Marginal rate of technical substitution—The rate at which one input can be substituted for another keeping total output constant.

Microfoundations project—The use of the Walrasian theory of the firm to describe the macroeconomy.

Pareto efficiency in exchange—In production, a situation in which no further trading of inputs can increase the output of one good without decreasing the output of another good.

Production function—Expresses output as a function of the quantities of inputs used. In this chapter, we have seen the general form $Q=f(K, L)$. Three popular forms are the Cobb–Douglas, CES, and translog functions.

Total factor productivity—The growth rate in output not accounted for by the growth rates of inputs. It is taken to be a measure of technological change.

GENERAL EQUILIBRIUM IN A BARTER ECONOMY

Exchange is political economy, it is society itself, for it is impossible to conceive of society without exchange or exchange without society.

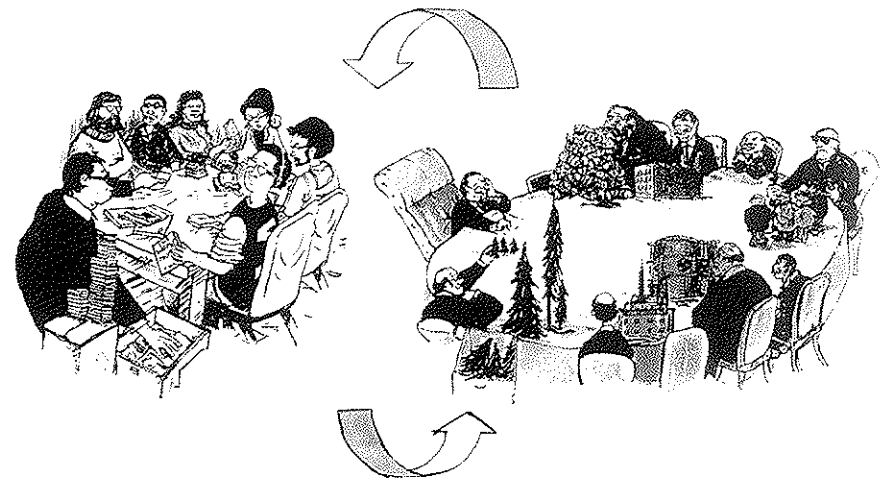
—*Frédéric Bastiat,* Economic Harmonies *[1850], translated from the French by W. Hayden Boyers, edited by George B. de Huszar (Princeton, NJ: Van Nostrand, 1964), xxv*

The major concern of Walrasian economic theory is efficiency. In Chapter 1 we established the condition for Pareto efficiency with respect to consumers exchanging market goods, namely, that the marginal rates of substitution (MRS) for the goods should be the same for both consumers. In Chapter 2 we established the condition for Pareto efficiency in production, namely, that the marginal rates of technical substitution (MRTS) for the inputs used in production should be the same for the two firms. The third and final step in our discussion of a barter economy is to establish the conditions for efficiency in general. How can we know that firms are efficiently producing the array of goods that consumers value most highly? In economic jargon, how do we know the economy is in **general equilibrium**? When an economy is in general equilibrium, the array of goods that consumers *want* (given their preferences) is the same as the array that producers *can* produce (with given technologies).

To establish general equilibrium between producers and consumers (general or *global* Pareto efficiency) we need three analytical tools: a **utility possibilities frontier**, a **production possibilities frontier**, and a **social welfare function**.

THE UTILITY POSSIBILITIES FRONTIER

As we saw in Chapter 1, employing the criterion of Pareto efficiency in exchange requires starting with a particular distribution of the two goods between the



two consumers. Referring to the *consumption space* diagram in Figure 3.1, a different initial distribution of the goods will result in a different Pareto-efficient point along the contract curve CC'. All the points on the contract curve are Pareto efficient, so how do we determine which distribution of the two goods maximizes social welfare in our simple two-person society? To answer this question, we begin by converting the commodity consumption of our two consumers into a measure of their relative utilities. Notice in the consumption space diagram in Figure 3.1 that at point 3 consumer A has more of both goods X and Y and at point 1 consumer B has more of both goods. At point 2 the goods are evenly distributed. Assuming that commodity consumption is equivalent to utility, we can use this information to construct a second diagram showing the relative utilities of A and B at each point on the contract curve. This is called a **utility possibilities frontier**. It shows the relative utilities of A and B for every possible Pareto-efficient distribution of the two goods.

THE PRODUCTION POSSIBILITIES FRONTIER

The next thing we need is a **production possibilities frontier**. This can be derived from the Edgeworth box for production showing the exchange of inputs between firms. Along the contract curve showing all the different

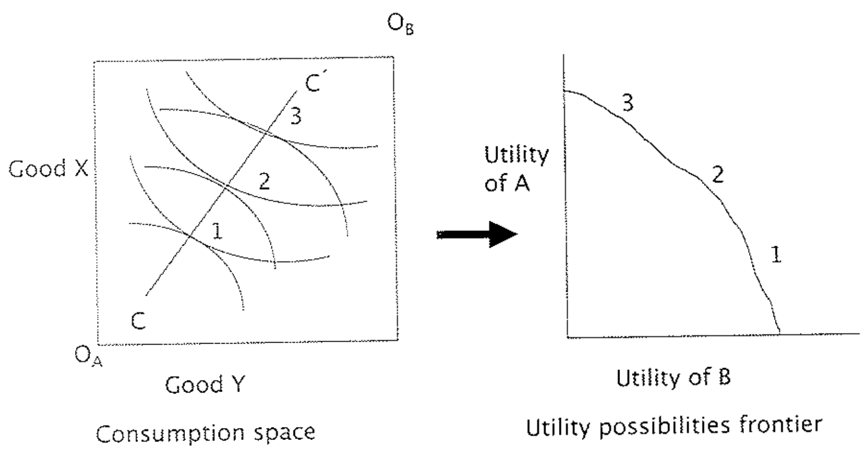


Figure 3.1. From commodity consumption to utility possibilities

Pareto-efficient combinations of K and L, notice that at point 3 in the *input space* box in Figure 3.2, most of the capital and labor available in this simple economy is used to produce good X. At point 1 most of the two inputs are used to produce good Y. This information can be transferred to the diagram on the right in Figure 3.2 showing the production possibilities frontier (PPF). The PPF shows the maximum amount of one good that can be produced given the amount produced of the other good.

Any point on the production possibilities frontier can be used to generate an Edgeworth box for consumption as shown for two points in the diagram on the left in Figure 3.3. The two Edgeworth boxes on the left in Figure 3.3 are the same as the ones in Chapter 1 (figures 1.2 and 1.3). Remember that one of the starting points for this analysis is that the total amounts of goods X and Y are fixed. All the possible combinations of X and Y are given by the points on the production possibilities frontier, and each one of these will generate a different Edgeworth box in consumption space and a different contract curve. These contract curves can be transformed into utility possibility curves as shown in Figure 3.3. Using all these utility possibility curves we can take one more step—applying the Pareto principle one more time—and construct a **grand utility possibilities frontier** (GUPF). The GUPF is an envelope

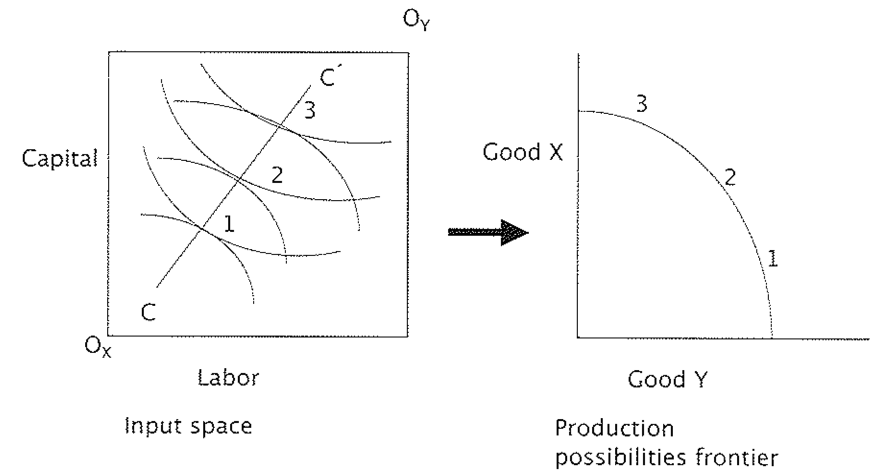


Figure 3.2. From input allocation to production

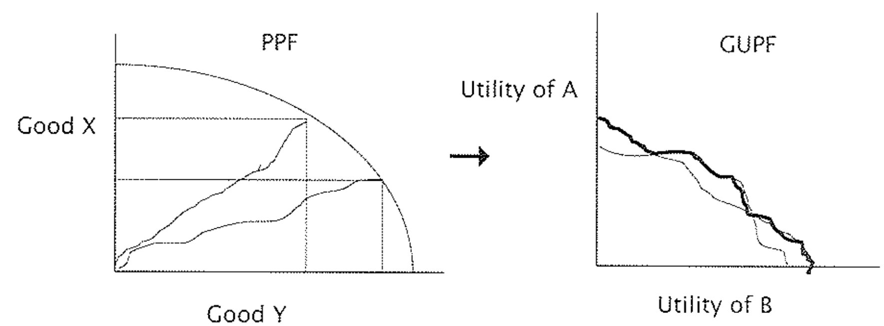


Figure 3.3. From the production possibilities frontier to the grand utility possibilities frontier

curve derived from the various utility possibility curves such as the two shown in the right-hand diagram in Figure 3.3. If we are on a curve below the GUPF (the heavy shaded line in the right-hand diagram in Figure 3.3), we can increase the utility of either A or B (or both) without decreasing the utility of either one.

THE SOCIAL WELFARE FUNCTION

Once we have constructed the grand utility possibility frontier, we have reached the limit to applying the Pareto principle. All points on the GUPF are Pareto optimal and we have no method or rule to pick one point over another. Any movement along the GUPF will make one of the consumers, either A or B, worse off. To pick the "best" point on the GUPF, that is, to choose the "socially optimal" combination of the utilities of consumers A and B, we need a **social welfare function** such as the one depicted in Figure 3.4.

Social welfare functions such as those labeled SWF1, SWF2, and SWF3 show all the combinations of utilities of A and B that are equally acceptable from society's point of view. As is the case with individual utility functions, social welfare functions further away from the origin represent higher levels of utility. In Figure 3.4, society's utility is maximized a point 1 where the social welfare function SWF2 is tangent to the grand utility possibilities frontier. This is called a *constrained bliss point*. Moving from point 2 on social welfare function SWF1 to point 1 on the higher social welfare function SWF2 increases society's total well-being. Point 3 on social welfare function SWF3 is unattainable given society's limited technology and resources.

The unspecified neoclassical (or "Bergsonian," after economist Abram Bergson) social welfare function is the weighted sum of individual welfares:

$$(3.1) \qquad W = \Sigma\, k(i)U(i)$$

The weights k(i) are unspecified, but neoclassical economists point out that any specification of the function will make neoclassical welfare theory a complete theory of social choice.

There are many possibilities and difficulties in constructing a social welfare function. The basic question is, how should society "choose" among the many possible Pareto-efficient distributions of income? We look at two possibilities here just for purposes of illustration. One possibility is just to accept the existing distribution of income, whatever it is, as "fair." This is essentially the position of Robert Nozick's *contractarian* approach (see his book *Anarchy, State, and Utopia*, 1974), which argues society has a set of rules for acquiring wealth. People who are wealthy have gained their wealth by following

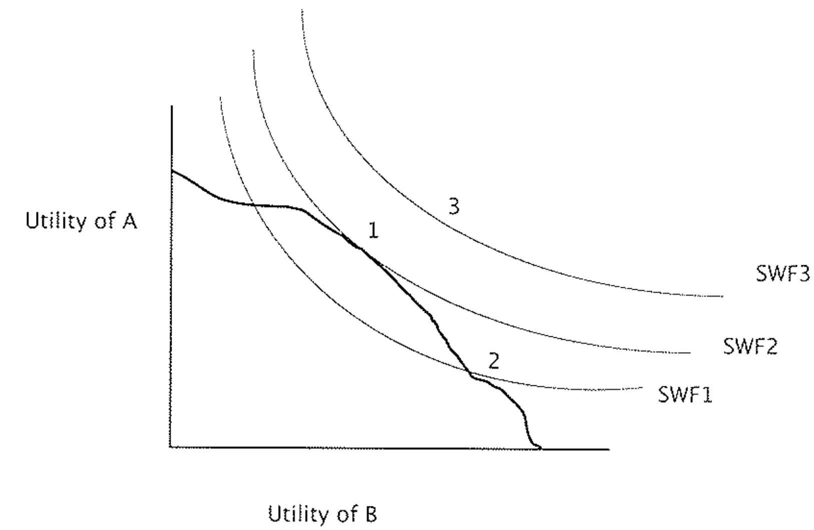


Figure 3.4. Social welfare functions

these rules, and taking money away from them and giving it to those who have not followed the rules (the poor, for example) represents an unfair taking of property—a breach of the social contract.

John Rawls, in his book *A Theory of Justice* (1971), takes a much different approach. He conducts a thought experiment and asks the question, if you had a choice of being placed in one of many different societies, each with a different income distribution, from very equal to very unequal, and you did not know ahead of time what your income would be, what sort of income distribution would you choose? Consider, for example, the points on the contract curve in Figure 3.1. If you did not know ahead of time whether your position in this society would be consumer A or consumer B, which point would you choose? Point 1 where consumer B has most of the two goods, point 3 where consumer A has most of the two goods, or point 2 where the goods are about evenly distributed? Both from a sense of fairness and a tendency toward loss aversion, most people would pick point 2.

There exists a vast literature on social welfare functions (and on the work Nozick and Rawls), and these simple examples are only an introduction to the complexity of the issue of social justice. But before we leave the issue, we need to mention the **Arrow impossibility theorem.** Basically, Arrow's theorem shows that there is no way to convert the rankings of individual preferences into a social (community-wide) ranking, given a few basic and reasonable assumptions. These include non-dictatorship, universality, and independence of irrelevant alternatives. Once again, a vast literature examines and extends Arrow's theorem. The problem raised by Arrow's theorem, like so many other difficulties in the Walrasian system, arises from the assumption of atomistic agents, that is, the "voters" (or coalitions of voters) must act independently of other voters. Arrow's paradox is discussed in more detail in Chapter 6.

THE THIRD CONDITION FOR PARETO EFFICIENCY IN EXCHANGE

To establish the final condition for Pareto efficiency in a pure exchange economy, we need to start with a point on the production possibilities frontier (Figure 3.2). This point indicates the total amounts of goods X and Y that our simple economy can produce given its technology and available resources. Each point corresponds to an Edgeworth box diagram showing all the Pareto-efficient distributions of those goods X and Y between the two consumers (the contract curve).

The slope of the production possibilities frontier gives the ***rate of product transformation*** (RPT), that is, the rate at which the output of one good can be reduced thereby freeing up resources that can be used to increase the output of the other good. For example if the slope of the PPF is 1, this indicates that we can give up one unit of good X and produce one more unit of good Y. It shows the rate at which our economy is *able* to switch production for one good to the other, given the available resources and technology. Now consider the points on the contract curve for exchange (in the Edgeworth box within the PPF) in Figure 3.5. All these points correspond to points of tangency of the indifference cures for the two consumers, that is, points where the marginal rates of substitution (MRS) for the two goods are the same for both consumers. In other words, these points show the rate at which our simple society is *willing* to substitute one good for another, given the preferences

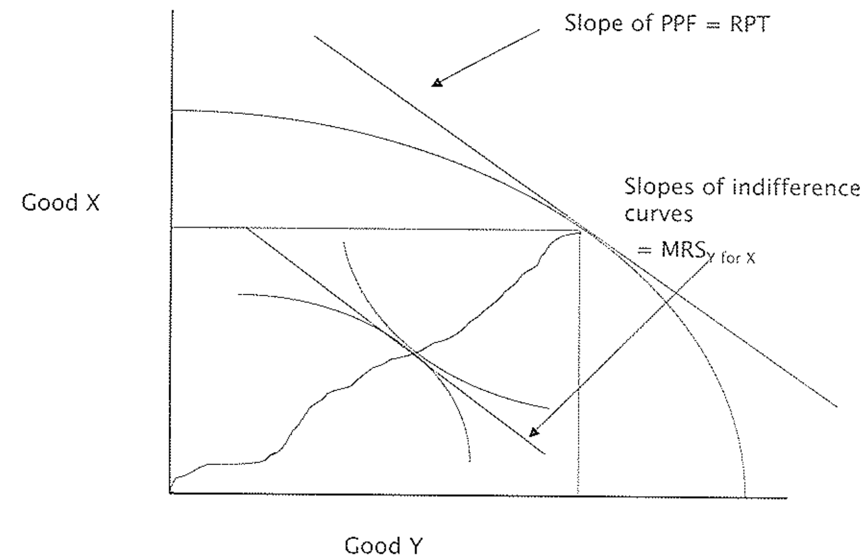


Figure 3.5. General equilibrium in a pure exchange economy

of our two consumers. This leads to the third Pareto condition for efficiency. The rate at which society is able to substitute one good for another in production must be equal to the rate at which consumers are willing to substitute one good for another in consumption.

(3.2) $MRS_{Y\ for\ X} = RPT_{Y\ for\ X}$

The best way to convince yourself that this is true is to consider a situation in which this is not the case. Assume that the $RPT_{Y\ for\ X}$ is equal to two; for example, our society can produce two more apples by giving up the production of one orange. Assume further that the marginal rate of substitution is equal to one. That is, consumers are willing to give up one orange for one apple. If this is the case, then our economy can give up the production of one orange, use the freed up resources to produce two apples, and thereby make consumers happier. They are "better off" by one apple. As long as the $RPT \neq MRS$ there are efficiency gains to be obtained by changing the mix of the production of goods X and Y.

Pareto Condition III. *The third and final condition for Pareto efficiency in a barter economy is that the rate at which the economy can stop producing one good in order to produce more of the other good, the rate of product transformation (RPT), is equal to the rate at which consumers are willing to give up consumption of one good in order to consume more of the other good, the marginal rate of substitution (MRS). When this condition is fulfilled the economy is in a state of general equilibrium.*

Given all the assumptions about the nature of preferences, the substitutability of inputs, the complete assignment of property rights, the unrestricted availability of information, and so on, we end up with the following situation.

1. Consumers are maximizing their well-being by getting the most desirable array of goods possible, given their stable preferences and their initial endowments of these goods.
2. Producers are maximizing the output of the goods they produce, given the state of technology and their initial endowment of productive inputs.
3. For any particular output of X and Y, this system will ensure not only that these goods are produced in the most efficient manner possible but also that the distribution of the goods between the consumers is the most efficient possible.

To summarize the conditions for Pareto efficiency we have established:

Pareto Condition I: In consumption, the marginal rates of substitution between the two goods are the same for the two consumers.

$$\overset{A}{MRS_{Y\ for\ X}} = \overset{B}{MRS_{Y\ for\ X}}$$

Pareto Condition II: In production, the marginal rates of technical substitution between the two inputs goods are the same for both producers.

$$\overset{X}{MRTS_{L\ for\ K}} = \overset{Y}{MRTS_{L\ for\ K}}$$

Pareto Condition III: General Pareto efficiency occurs when the rate at which consumers are willing to substitute one good for another in consumption is the same as the rate at which producers can switch from making one good to making another in production.

$$\overset{A=B}{MRS_{X\ for\ Y}} = RPT_{Y\ for\ X}$$

▶ ASSUMPTION ALERT! *The conditions for establishing general equilibrium:*

1. *The economy is operating on the contract curve for the exchange of goods. No person's utility can be increased without reducing the utility of at least one other person. All the assumptions of Homo economicus hold.*
2. *The economy is operating on the contract curve for the exchange of inputs. The production of one good cannot be increased without decreasing the output of at least one other good. All the assumptions of perfect competition hold.*
3. *The economy is operating on the production possibilities frontier. Resources and technology are being employed in the most efficient way possible.* ◀

A POTENTIAL PARETO IMPROVEMENT

Before we move from our simple face-to-face barter economy to one that is based on price signals, we need one more concept. This is the notion of a **potential Pareto improvement,** or PPI, first proposed independently by John Hicks and Nicholas Kaldor in 1939. It is sometimes called the Kaldor–Hicks criterion or the compensation principle. A severe limitation of the strict Pareto criterion is its restricted policy applicability. Almost any action affecting the economy will benefit some people and harm others.

Suppose we are in a situation such as point 1 in Figure 3.6. This is a Pareto inferior situation, because we can move to any point in the hatched area and make both consumers better off. The thick line on the contract curve within the hatched area is called the **core** of an exchange economy. We can also say that a movement to any point in the dotted areas will make both consumers worse off. But what about a movement to a point such as point 2? This is a movement from a Pareto inferior to a Pareto-efficient point, but a movement from point 1 to point 2 will make consumer A worse off and is not permitted under the strict Pareto criterion.

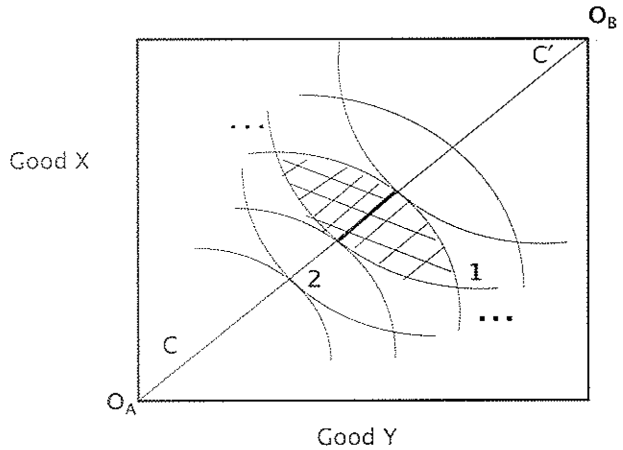


Figure 3.6. A potential Pareto improvement

The Kaldor–Hicks criterion permits such a move if the total change in utility is positive. Kaldor's version is that such a change should be made if the winner can compensate the loser and still be better off. Hicks's version is that a change should be made if the loser cannot bribe the winner not to make the change (and still be better off). According to Kaldor, whether or not the loser is actually compensated is irrelevant. Economists should be concerned with efficiency, not distribution. It is up to politicians to decide whether or not an outcome is fair.

The notion of a potential Pareto improvement revolutionized economics in two important ways. First of all, it legitimized the focus of economic analysis and economic policy on efficiency. Second, it paved the way for economists to focus on increasing per capita income as the major goal of economic policy. Identification of potential Pareto improvements, that is, efficiency gains, is the major task of cost-benefit analysis, one of the principal tools of economic analysis.

This completes our discussion of general equilibrium (Pareto efficiency or Pareto optimality) in a pure barter economy. Again, this model is the heart and soul of contemporary Walrasian economics. As the next chapter shows, the model of perfect competition adds prices and markets to the barter system but ends up with exactly the same conditions for economic efficiency. This model of a pure exchange economy is the ideal to which most economists

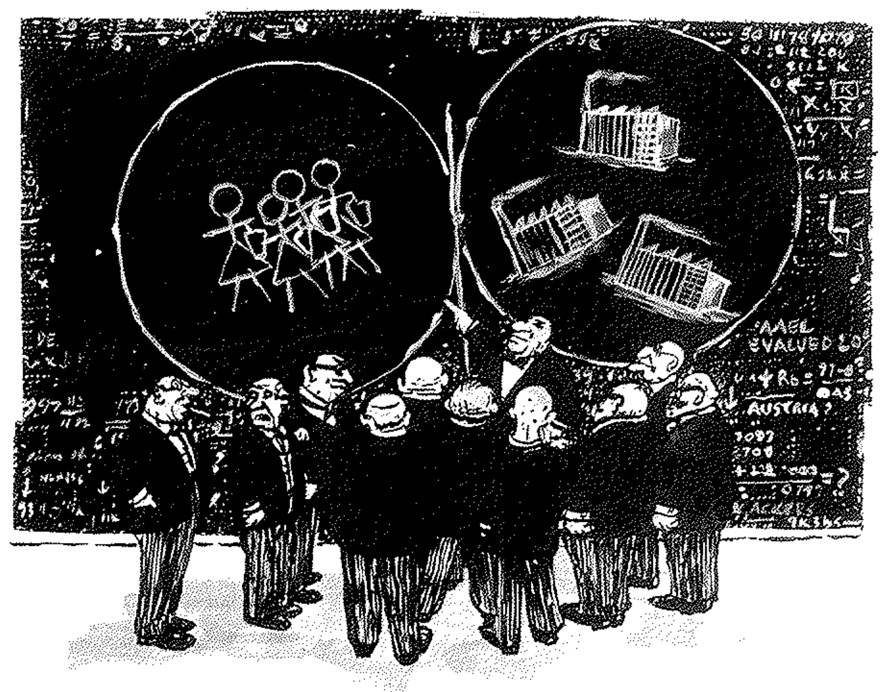
consciously or unconsciously refer when making policy recommendations about actual economies.

APPENDIX

The Existence of Equilibrium

The intellectual power of the Walrasian system lies in the fact that, given all the assumptions about consumer and producer behavior and technology, we can find a unique efficient level and mix of output. That is, we can find a point where a given set of preferences and a given set of production possibilities meet.

As economics became more and more mathematical in the middle of the twentieth century, more attention was given to the mathematical properties of the equations describing market economies. A lot of attention was given to proving that, given the mathematical assumptions of the Walrasian system, it is mathematically possible to establish an equilibrium point where the set of commodities produced exactly duplicated the set of commodities



that consumers wanted. In particular, a branch of mathematics called topology can establish the conditions under which the set of consumer preferences and the set of producer possibilities meet at a unique point as shown in the diagram above. These are called **fixed-point theorems.** One basic proof is Brouwer's fixed-point theorem, which, stated formally, demonstrates that a continuous mapping of a closed, bounded convex set onto itself has a least one fixed point. Formally, there exists a point such that $f(x)=x$. Brouwer's proof was used to show that under the assumptions of the model of perfect competition, where utility and production functions are smooth and continuous, at least one equilibrium point exists where the set of production possibilities are the same as the set of consumption possibilities.

In Figure 3.7 the diagonal represents all the points where $x=x$, and the dotted line represents some mapping (transformation) of x, denoted by $f(x)$. In the diagram there is no way to draw a continuous line from the left vertical axis to the right vertical axis of the diagram without crossing the diagonal. When the dotted line crosses the diagonal, $f(x)=x$, and this proves that in the mathematical representation of the economy, there is at least one point where a general equilibrium solution exists. One problem with Brouwer's theorem is that other systems exist that do not satisfy the conditions of perfect competition but that also have equilibrium points. More general

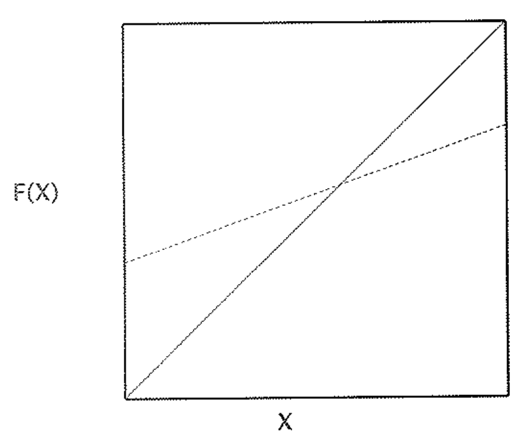


Figure 3.7. Brouwer's fixed-point theorem

versions of Brouwer's theorem are the Kakutani, Schauder, and Lefschetz fixed-point theorems.

GLOSSARY

Arrow impossibility theorem—Given the requirements of non-imposition, non-dictatorship, Pareto efficiency, and independence of irrelevant alternatives, no voting system can convert individual preferences into a society-wide ranking.

Core—In the Edgeworth box diagram, the core is the segment of the contract curve that contains all the reachable Pareto-efficient outcomes given the initial endowment.

Fixed-point theorems—Mathematical theorems used by economists to ensure the theoretical existence of a point of general equilibrium.

General equilibrium—A situation in which consumers are maximizing their utility given their initial endowment of goods, producers are maximizing output given their initial endowment of productive inputs, and producers are producing the most desirable mix of goods based on consumer preferences.

Grand utility possibilities frontier—A graph or curve showing the maximum possible utility of one person given the utility of the other person. It is an envelop curve derived from all the utility possibilities frontier curves associated with every possible contract curve for consumption.

Potential Pareto improvement—A gain in total utility that makes at least one person worse off. Sometimes it is called the compensation principle or the Kaldor–Hicks criterion.

Production possibilities frontier—A graph or curve showing the maximum possible production of one good given the production of the other good. It shows all the Pareto-efficient combinations of goods that can be produced, given society's endowment of resources and technology.

Rate of product transformation—The rate at which an economy can switch from producing one to good to another. It is equal to the slope of the production possibilities frontier.

Social welfare function—A graph or curve showing the all the possible combinations of individual utilities where social welfare is the same. The social welfare function is based on given preferences, technology and resource endowment, plus some specific ethical assumption about the fair distribution of goods among consumers.

Utility possibilities frontier—A graph or curve showing the maximum possible utility of one consumer given the utility of the other consumer. It shows all the possible Pareto-efficient combinations of utilities, given the preferences of each consumer.

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4

INTRODUCING PRICES

Perfect Competition and Pareto Efficiency

> As every individual, therefore, endeavours as much as he can both to employ his capital in the support of domestick industry, and so to direct that industry that its produce may be of the greatest value; every individual necessarily labours to render the annual revenue of the society as great as he can. He generally, indeed, neither intends to promote the publick interest, nor knows how much he is promoting it. By preferring the support of domestick to that of foreign industry, he intends only his own security; and by directing that industry in such a manner as its produce may be of the greatest value, he intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his intention.
>
> —*Adam Smith,* An Inquiry into the Nature and Causes of the Wealth of Nations *[1776], edited by R. H. Campbell and A. S. Skinner (New York: Liberty Press, 1981, IV.ii), 456*

The Walrasian representation of a barter economy presented in chapters 1–3 is the core of contemporary microeconomic theory. Interestingly, although microeconomics is sometimes called "price theory," prices play no independent role in the basic Walrasian system. In a "frictionless" economy populated by independent consumers and producers with perfect information about prices, the results of free exchange will exactly duplicate the outcome obtained in a face-to-face barter system. The price of each good contains all the information necessary to compare its desirability to the desirability of every other good. In such an economy the "invisible hand" of the market will ensure the most efficient allocation of society's scarce resources.