MICROECONOMICS AND BEHAVIOR

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CHAPTER 3

RATIONAL CONSUMER CHOICE

You have just cashed your monthly allowance check and are on your way to the local music store to buy an Eric Clapton CD you've been wanting. The price of the disc is $10. In scenario 1 you lose $10 on your way to the store. In scenario 2 you buy the disc and then trip and fall on your way out of the store; the disc shatters as it hits the sidewalk. Try to imagine your frame of mind in each scenario.

a. Would you proceed to buy the disc in scenario 1?
b. Would you return to buy the disc in scenario 2?

These questions[1] were recently put to a large class of undergraduates who had never taken an economics course. In response to the first question, 54 percent answered yes, saying they would buy the disc after losing the $10 bill. But only 32 percent answered yes to the second question—68 percent said they would *not* buy the disc after having broken the first one. There is, of course, no "correct" answer to either of these questions. The events described will have more of an impact, for example, on a poor consumer than on a rich one. Yet a moment's reflection reveals that your behavior in one scenario logically should be exactly the same as in the other. After all, in both scenarios, the only economically relevant change is that you now have $10 less to spend than before. This might well mean that you will want to give up having the disc; or it could mean saving less or giving up some other good or service that you would have bought instead. But your choice should not be affected by the particular way you hap-

[1] These questions are patterned after similar questions posed by decision theorists Daniel Kahneman and Amos Tversky (see Chapter 8).

pened to become $10 poorer. In both scenarios, the cost of the disc is $10, and the benefits you will receive from listening to it are also the same. You should either buy the disc in both scenarios or not buy it in both scenarios. And yet, as noted, many people said they would behave differently in the two scenarios.

Chapter Preview

Our task in this chapter will be to set forth the economist's basic model for answering questions like the ones posed above. This model is known as the theory of *rational consumer choice*. It underlies all individual purchase decisions, which in turn add up to the demand curves we worked with in the preceding chapter.

Rational choice theory begins with the assumption that consumers enter the marketplace with well-defined preferences. Taking prices as given, their task is to allocate their incomes to best serve these preferences. Two steps are required to carry out this task. The first is to describe the various combinations of goods the consumer is *able* to buy. These combinations depend, we will see, on both her income level and the prices of the goods. The second step is then to select from among the feasible combinations the particular one that she *prefers* to all others. Analysis of this step will require some means of describing her preferences, in particular a summary of the rank ordering she assigns to all feasible combinations. Formal development of these two elements of the theory will occupy our attention throughout this chapter. Because the first element—describing the set of possibilities—is much less abstract than the second, let us begin with it.

The Opportunity Set or Budget Constraint

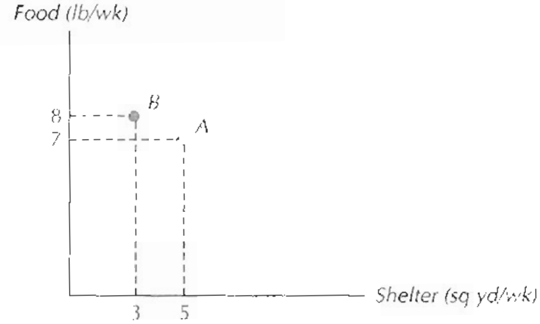
For simplicity, let us begin by considering a world with only two goods,[2] food and shelter. A *bundle* of goods is the term used to describe a particular combination of food, measured in pounds per week, and shelter, measured in square yards per week. Thus, in Figure 3-1, one bundle (bundle A) might consist of 5 sq yd/wk of shelter and 7 lb/wk of food, while another (bundle B) consists of 3 sq yd/wk of shelter and 8 lb/wk of food. For brevity's sake, we may use the notation (5, 7) to denote bundle A and the notation (3, 8) to denote bundle B. More generally, (S_0, F_0) will denote the bundle with S_0 sq yd/wk of shelter and F_0 lb/wk of food. By convention, the *first* number of the pair in any bundle represents the good measured along the horizontal axis.

Note that the units on both axes are *flows*, which means physical quantities per unit of time—pounds per week, square yards per week. Consumption is always measured as a flow. It is important to keep track of the time dimension because, without it, there would be no way to evaluate whether a given quantity of consumption was large or small. (Suppose all you know is that your food

[2] As economists use the term, a "good" may refer to either a product or a service.

FIGURE 3-1
A bundle is a specific combination of goods. Bundle *A* has 5 units of shelter and 7 units of food. Bundle *B* has 3 units of shelter and 8 units of food.

TWO BUNDLES OF GOODS



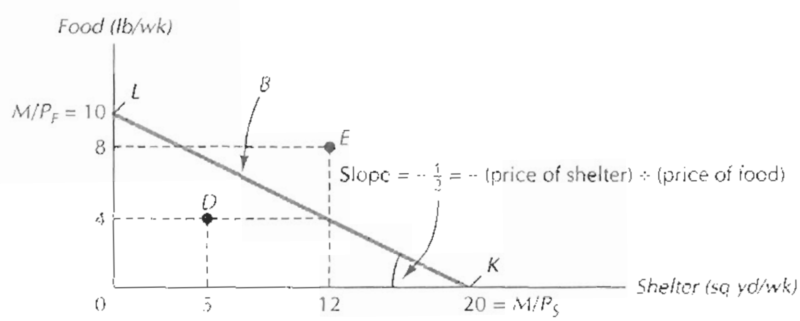
consumption is 4 lb. If that's how much you eat each day, it's a lot. But if that's all you eat in a month, you should submit your application for a tuition refund right away, because you won't be at school much longer.)

Suppose the consumer's income is M = \$100/wk, all of which she spends on some combination of food and shelter. (Note that income is also a flow.) Suppose further that the prices of shelter and food are P_S = \$5/sq yd and P_F = \$10/lb, respectively. If the consumer spent all of her income on shelter, she could buy M/P_S = (\$100/wk) ÷ (\$5/sq yd) = 20 sq yd/wk. That is to say, she could buy the bundle consisting of 20 sq yd/wk of shelter and 0 lb/wk of food, denoted (20, 0). Alternatively, suppose the consumer spent all of her income on food. She would then get the bundle consisting of M/P_F = (\$100/wk) ÷ (\$10/lb), which is 10 lb/wk of food and 0 sq yd/wk of shelter, denoted (0, 10).

In Figure 3-2, these polar cases are labeled *K* and *L*, respectively. The consumer is also able to purchase any other bundle that lies along the straight line that joins points *K* and *L*. [Verify, for example, that the bundle (12, 4) lies on this

FIGURE 3-2
Line *B* describes the set of all bundles the consumer can purchase for given values of income and prices. Its slope is the negative of the price of shelter divided by the price of food. In absolute value, this slope is the opportunity cost of an additional unit of shelter—the number of units of food that must be sacrificed in order to purchase one additional unit of shelter at market prices.

THE BUDGET CONSTRAINT, OR OPPORTUNITY SET



Budget constraint The set of all bundles that are affordable with given income and prices. Also called the *opportunity set.*

same line.] This line is called the ***budget constraint***, or ***opportunity set***, and is labeled B in the diagram.

Recall the maxim from high school algebra that the slope of a straight line is its "rise" over its "run" (the change in its vertical position divided by the corresponding change in its horizontal position). Here, note that the slope of the budget constraint is its vertical intercept (the rise) divided by its horizontal intercept (the corresponding run): $-(10\text{ lb/wk})/(20\text{ sq yd/wk}) = -\frac{1}{2}\text{ lb/sq yd}$. The minus sign signifies that the budget line falls as it moves to the right—that it has a negative slope. More generally, if M denotes the consumer's weekly income, and P_S and P_F denote the prices of shelter and food, respectively, the horizontal and vertical intercepts will be given by (M/P_S) and (M/P_F), respectively. Thus the general formula for the slope of the budget constraint is given by $-(M/P_F)/(M/P_S) = -P_S/P_F$, which is simply the negative of the price ratio of the two goods. Given their respective prices, it is the rate at which food can be exchanged for shelter. Thus, in Figure 3-2, 1 lb of food can be exchanged for 2 sq yd of shelter. In the language of opportunity cost from Chapter 1, we would say that the opportunity cost of an additional square yard of shelter is $P_S/P_F = \frac{1}{2}$ lb of food.

In addition to being able to buy any of the bundles along her budget constraint, the consumer is also able to purchase any bundle that lies within the *budget triangle* bounded by it and the two axes. D is one such bundle in Figure 3-2. Bundle D costs \$65/wk, which is well below the consumer's income of \$100/wk. The bundles on or within the budget triangle are also referred to as the *feasible*, or *affordable*, *set*. Bundles like E that lie outside the budget triangle are said to be *infeasible*, or *unaffordable*. At a cost of \$140/wk, E is simply beyond the consumer's reach.

If S and F denote the quantities of shelter and food, respectively, the budget constraint must satisfy the following equation:

$$P_S S + P_F F = M, \tag{3.1}$$

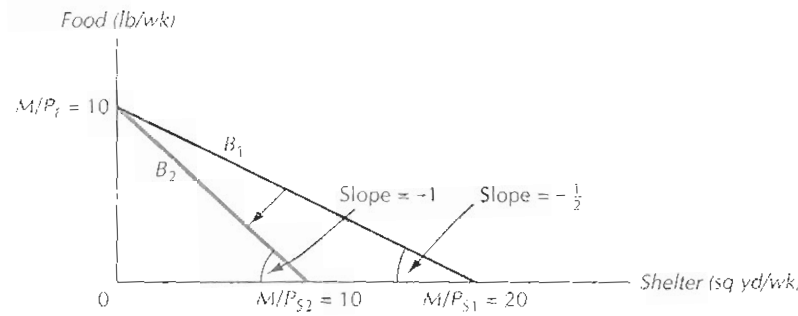
which says simply that the consumer's weekly expenditure on shelter ($P_S S$) plus her weekly expenditure on food ($P_F F$) must add up to her weekly income (M). To express the budget constraint in the manner conventionally used to represent the formula for a straight line, we solve Equation 3.1 for F in terms of S, which yields

$$F = \frac{M}{P_F} - \frac{P_S}{P_F}S. \tag{3.2}$$

Equation 3.2 provides another way of seeing that the vertical intercept of the budget constraint is given by M/P_F, and its slope by $-(P_S/P_F)$. The equation for the budget constraint in Figure 3-2 is $F = 10 - \frac{1}{2}S$.

FIGURE 3-3
When shelter goes up in price, the vertical intercept of the budget constraint remains the same. The original budget constraint rotates inward about this intercept.

THE EFFECT OF A RISE IN THE PRICE OF SHELTER



BUDGET SHIFTS DUE TO PRICE OR INCOME CHANGES

Price Changes. The slope and position of the budget constraint are fully determined by the consumer's income and the prices of the respective goods. Change any one of these and we have a new budget constraint. Figure 3-3 shows the effect of an increase in the price of shelter from P_{S1} = \$5/sq yd to P_{S2} = \$10. Since both weekly income and the price of food are unchanged, the vertical intercept of the consumer's budget constraint stays the same. The rise in the price of shelter rotates the budget constraint inward about this intercept, as shown in the diagram.

Note in Figure 3-3 that even though the price of food has not changed, the new budget constraint, B_2, curtails not only the amount of shelter the consumer can buy, but also the amount of food.[3]

EXERCISE 3-1

Show the effect on the budget constraint B_1 in Figure 3-3 of a fall in the price of shelter from \$5/sq yd to \$4/sq yd.

In Exercise 3-1, you saw that a fall in the price of shelter again leaves the vertical intercept of the budget constraint unchanged. This time the budget constraint rotates outward. Note also in Exercise 3-1 that although the price of food remains unchanged, the new budget constraint enables the consumer to buy bundles that contain not only more shelter, but also more food than she could afford on the original budget constraint.

[3] The single exception to this statement involves the vertical intercept, (0, 10), which lies on both the original and the new budget constraints.

EXERCISE 3-2

Show the effect on the budget constraint B_1 in Figure 3-3 of a rise in the price of food from \$10/lb to \$20/lb.

Exercise 3-2 demonstrates that when the price of food changes, the budget constraint rotates about its horizontal intercept. Note also that even though income and the price of shelter remain the same, the new budget constraint curtails not only the amount of food the consumer can buy, but also the amount of shelter.

When we change the price of only one good, we necessarily change the slope of the budget constraint, $-P_S/P_F$. The same is true if we change both prices by different proportions. But as Exercise 3-3 will illustrate, changing both prices by exactly the same proportion gives rise to a new budget constraint with the same slope as before.

EXERCISE 3-3

Show the effect on the budget constraint B_1 in Figure 3-3 of a rise in the price of food from \$10/lb to \$20/lb and a rise in the price of shelter from \$5/sq yd to \$10/sq yd.

Note from Exercise 3-3 that the effect of doubling the prices of both food and shelter is to shift the budget constraint inward and parallel to the original budget constraint. The important lesson of this exercise is that the slope of a budget constraint tells us only about *relative prices*, nothing about how high prices are in absolute terms. When the prices of food and shelter change in the same proportion, the opportunity cost of shelter in terms of food remains the same as before.

Income Changes. The effect of a change in income is much like the effect of an equal proportional change in all prices. Suppose, for example, that our hypothetical consumer's income is cut by half, from \$100/wk to \$50/wk. The horizontal intercept of the consumer's budget constraint then falls from 20 sq yd/wk to 10 sq yd/wk, and the vertical intercept falls from 10 lb/wk to 5 lb/wk, as shown in Figure 3-4. Thus the new budget, B_2, is parallel to the old, B_1, each with a slope of $-\frac{1}{2}$. In terms of its effect on what the consumer can buy, cutting income by one-half is thus no different from doubling each price. Precisely the same budget constraint results from both changes.

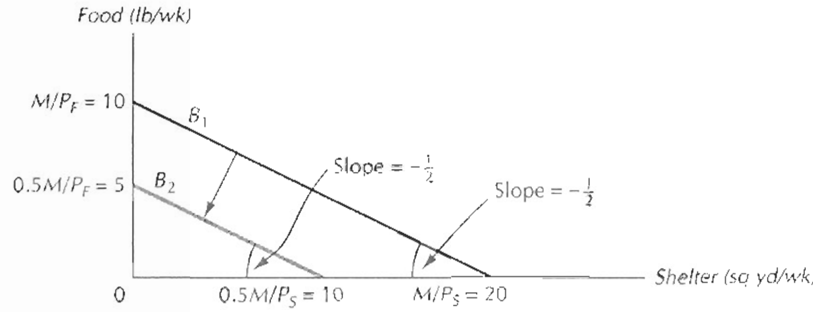
EXERCISE 3-4

Show the effect on the budget constraint B_1 in Figure 3-4 of an increase in income from \$100/wk to \$120/wk.

Exercise 3-4 illustrates that an increase in income shifts the budget constraint parallel outward. As in the case of an income reduction, the slope of the budget constraint remains the same.

FIGURE 3-4
Both horizontal and vertical intercepts fall by half. The new budget constraint has the same slope as the old, but is closer to the origin.

THE EFFECT OF CUTTING INCOME BY HALF



BUDGETS INVOLVING MORE THAN TWO GOODS

The examples discussed so far have all been ones in which the consumer is faced with the opportunity to buy only two different goods. Needless to say, not many consumers have such narrow options. In its most general form, the consumer budgeting problem can be posed as a choice between not two but N different goods, where N can be an indefinitely large number. With only two goods ($N = 2$), the budget constraint is a straight line, as we have just seen. With three goods ($N = 3$), it is a plane. When we have more than three goods, the budget constraint becomes what mathematicians call a ***hyperplane***, or ***multidimensional plane***. The only real difficulty is in representing this multidimensional case geometrically. We are just not very good at visualizing surfaces that have more than three dimensions.

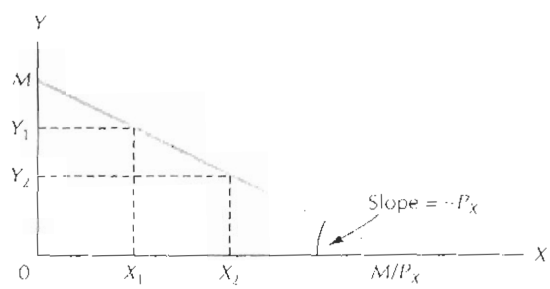
The nineteenth-century economist Alfred Marshall proposed a disarmingly simple solution to this problem. It is to view the consumer's choice as being one between a particular good—call it X—and an amalgam of other goods, denoted Y. This amalgam is generally called the ***composite good***. We may think of the composite good as the amount of income the consumer has left over after buying the good X. Equivalently, it is the amount of money the consumer spends on goods other than X.

Composite good
In an analysis of the consumption of a specific good X, the *composite good* refers to the amount of money the consumer spends on all goods other than X.

To illustrate how this concept is used, suppose the consumer has an income level of $\$M$/wk, and the price of X is given by P_X. The consumer's budget constraint may then be represented as a straight line in the X, Y plane, as shown in Figure 3-5. For simplicity, the price of a unit of the composite good is taken to be 1, so that if the consumer devotes none of his income to X, he will be able to buy M units of the composite good. All this means is that he will have $\$M$ available to spend on other goods if he buys no X. Alternatively, if he spends all his income on X, he will be able to purchase the bundle $(M/P_X, 0)$. Since the price of Y is assumed to be \$1/unit, the slope of the budget constraint is simply $-P_X$.

As before, the budget constraint summarizes the various combinations of bundles that are affordable. Thus, for example, the consumer can have X_1 units

FIGURE 3-5
The vertical axis measures the amount of money spent each month on all goods other than X.



of X and Y_1 units of the composite good in Figure 3-5, or X_2 and Y_2, or any other combination that lies on the budget constraint.

KINKED BUDGET CONSTRAINTS

The budget constraints we have seen so far have all been straight lines. When relative prices are constant, the opportunity cost of one good in terms of any other is the same, no matter what bundle of goods we already have. But sometimes the budget constraints we encounter in practice are kinked lines. By way of illustration, consider the following example of quantity discounts.

EXAMPLE 3-1

The Gigawatt Power Company charges $0.10 per kilowatt-hour (kwh) for the first 1000 kwh of power purchased by a residential customer each month, but only $0.05/kwh for all additional kwh. For a residential customer with a monthly income of $400, graph the budget constraint for electric power and the composite good.

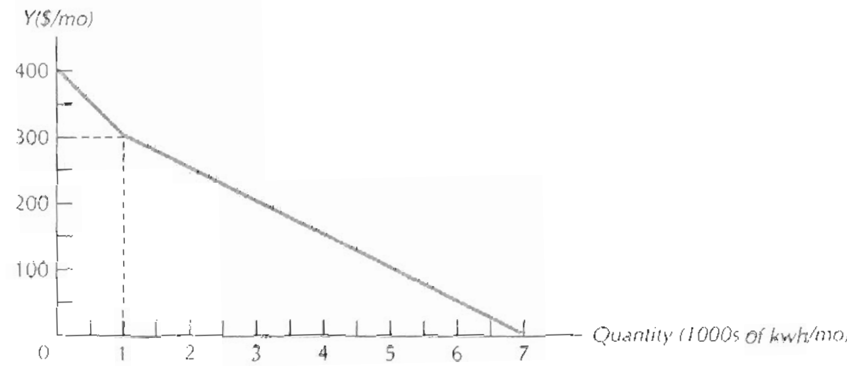
If the consumer buys no electric power at all, he will have $400/mo available for the purchase of other goods. Thus the vertical intercept of his budget constraint is the point (0, 400). As shown in Figure 3-6, for each of the first 1000 kwh he buys, he must give up $0.10, which means that the slope of his budget constraint starts out at −1/10. Then at 1000 kwh/mo, the price falls to $0.05/kwh, which means that the slope of his budget constraint from that point rightward is only −1/20.

Note that along the budget constraint shown in Figure 3-6, the opportunity cost of electricity depends on how much electricity the consumer has already purchased. Consider a consumer who now uses 1020 kwh each month and is trying to decide whether to leave his front porch light on all night, which would result in an additional consumption of 20 kwh/mo. If he leaves the light on, it will cost him an extra $1/mo. Had his usual consumption level been only 980

FIGURE 3-6

A QUANTITY DISCOUNT GIVES RISE TO A NONLINEAR BUDGET CONSTRAINT

Once electric power consumption reaches 1000 kwh/mo, the opportunity cost of additional power falls from \$0.10/kwh to \$0.05/kwh.



kwh/mo, however, the cost of leaving the front porch light on would have been \$2/mo. On the basis of this difference in the opportunity cost of additional electricity, we can predict that people who already use a lot of electricity (that is, more than 1000 kwh/mo) should be more likely than others to leave their porch lights burning at night.

IF THE BUDGET CONSTRAINT IS THE SAME, THE DECISION SHOULD BE THE SAME

Even without knowing anything about the consumer's preferences, we can use budgetary information to make certain inferences about how a rational consumer will behave. Suppose, for example, that the consumer's tastes do not change over time and that he is confronted with exactly the same budget constraint in each of two different situations. If he is rational, he should make exactly the same choice in both cases. As the following example will make clear, however, it may not always be immediately apparent that the budget constraints are in fact the same.

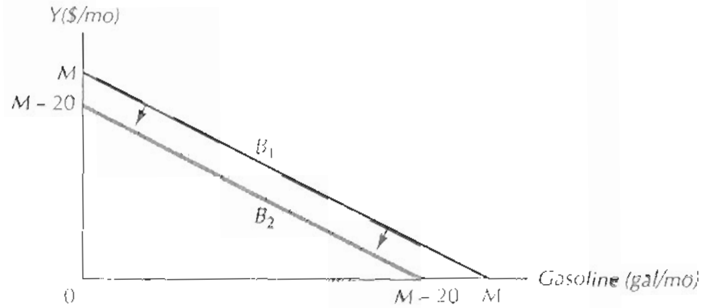
EXAMPLE 3-2

On one occasion, Gowdy fills his car's tank with gasoline on the evening before his departure on a fishing trip. He awakens to discover that a thief has siphoned out all but 1 gal from his 21-gal tank. On another occasion, he plans to stop for gas on his way out the next morning before he goes fishing. He awakens to discover that he has lost \$20 from his wallet. If gasoline sells for \$1/gal and the round-trip will consume 5 gal, how, if at all, should Gowdy's decision about whether to take the fishing trip differ in the two cases? (Assume that, monetary costs aside, the inconvenience of having to refill his tank is negligible.)

Suppose Gowdy's income is \$$M$/mo. Before his loss, his budget constraint is line B_1 in Figure 3-7. In both of the instances described, his budget constraint at the moment he discovers his loss will shift inward to B_2. If he does not take the trip,

FIGURE 3-7
A theft of $20 worth of gasoline has exactly the same effect on the budget constraint as the loss of $20 in cash. The bundle chosen should therefore be the same, irrespective of the source of the loss.

BUDGET CONSTRAINTS FOLLOWING THEFT OF GASOLINE, LOSS OF CASH



he will have $M - \$20$ available to spend on other goods in both cases. And if he does take the trip, he will have to purchase the required gasoline at $1/gal in both cases. No matter what the source of the loss, the remaining opportunities are exactly the same. If Gowdy's budget is tight, he may decide to cancel his trip. Otherwise, he may go despite the loss. But because his budget constraint and tastes are the same in the lost-cash case as in the stolen-gas case, it would not be rational for him to take the trip in one instance but not in the other.

Note that the situation described in Example 3-2 has the same structure as the one described in the broken-disc example with which we began this chapter. It, too, is one in which the decision should be the same in both instances because the budget constraint and preferences are the same in each.

Although the rational choice model makes clear that the decisions should be the same if the budget constraints and preferences are the same, people sometimes choose differently. The difficulty is often that the way the different situations are described sometimes causes people to overlook the essential similarities between them. For instance, in Example 3-2, many people erroneously conclude that the cost of taking the trip is higher in the stolen-gas case than in the lost-money case, and so they are less likely to take the trip in the former instance. Similarly, many people were less inclined to buy the disc after having broken the first one than after having lost $10 because they thought, incorrectly, that the disc would cost more under the broken-disc scenario. As we have seen, however, the amount that will be saved by not buying the disc, or by not taking the trip, is exactly the same under each scenario.

To recapitulate briefly, the budget constraint or opportunity set summarizes the combinations of bundles that the consumer is able to buy. Its position is determined jointly by income and prices. From the set of feasible bundles, the consumer's task is to pick the particular one she likes best. To identify this bundle, we need some means of summarizing the consumer's preferences over all possible bundles she might consume. To this task we now turn.

Consumer Preferences

Preference ordering A scheme whereby the consumer ranks all possible consumption bundles in order of preference.

For simplicity, let us again begin by considering a world with only two goods, shelter and food. A *preference ordering* is a scheme that enables the consumer to rank different bundles of goods in terms of their desirability or order of preference. Consider two bundles, *A* and *B*. For concreteness, suppose that *A* contains 4 sq yd/wk of shelter and 2 lb/wk of food, while *B* has 3 sq yd/wk of shelter and 3 lb/wk of food. Knowing nothing about a consumer's preferences, we can say nothing about which of these bundles he will prefer. *A* has more shelter, but less food, than *B*. Someone who spends a lot of time at home would probably choose *A*, while someone with a rapid metabolism would be more likely to choose *B*.

In general, we can say that for any two such bundles, the consumer is able to make one of three possible statements: (1) *A* is preferred to *B*, (2) *B* is preferred to *A*, or (3) *A* and *B* are equally preferred. The preference ordering enables the consumer to rank different bundles, but not to make more precise quantitative statements about their relative desirability. Thus, for example, the consumer might be able to say that he prefers *A* to *B*, but not that *A* provides twice as much satisfaction as *B*.

Preference orderings often differ widely among consumers. One person will like Rachmaninoff, another the Rolling Stones. Despite these differences, however, most preference orderings share several important features. More specifically, economists generally assume four simple properties of preference orderings. We will begin by considering the first three of these, which take us a long way toward being able to construct the concise analytical representation of preferences we need for the budget allocation problem.

1. Completeness. A preference ordering is *complete* if it enables the consumer to rank all possible combinations of goods and services. Taken literally, the completeness assumption is virtually always false, for there are many goods we know too little about to be able to evaluate decisively. It is nonetheless a useful simplifying assumption for the analysis of choices among bundles of goods with which consumers are familiar. Its real intent is to rule out instances like the one portrayed in the fable of Buridan's ass. The hungry animal was unable to choose between two bales of hay in front of him and starved to death as a result.

2. Transitivity. If you like steak better than hamburger, and you like hamburger better than hot dogs, then you are probably someone who likes steak better than hot dogs. To say that a consumer's preference ordering is *transitive* means that, for any three bundles, *A*, *B*, and *C*, if he prefers *A* to *B* and prefers *B* to *C*, then he always prefers *A* to *C*. The preference relationship is thus assumed to be like the relationship used to compare heights of people. If Ewing is taller than Jordan, and Jordan is taller than Webb, we know that Ewing must be taller than Webb.

Not all comparative relationships are transitive. The relationship "half-sibling," for example, is not. I have a half-sister who, in turn, has three half-sisters of her own. But her half-sisters are not my half-sisters. A similar nontransitivity is shown by the relationship "defeats in football." Some seasons, Michigan defeats Ohio State, and Ohio State beats Iowa, but that doesn't tell us that Michigan will necessarily beat Iowa.

Transitivity is a simple consistency property and applies as well to the relation "equally preferred to," and to any combination of it and the "preferred to" relation. Thus, for example, if *A* is equally preferred to *B* and *B* is equally preferred to *C*, it follows that *A* is equally preferred to *C*. Similarly, if *A* is preferred to *B* and *B* is equally preferred to *C*, it follows that *A* is preferred to *C*. Reasonable as the transitivity property sounds, we will see examples in later chapters of behavior that seems inconsistent with it. But it is an accurate description of preferences in most instances, and unless otherwise stated, we will adopt it throughout as a working assumption.

3. More-is-better. The more-is-better property of preference orderings means simply that, other things equal, more of a good is preferred to less. We can, of course, think of examples where more of something makes us worse off rather than better (as with someone who has overeaten). But these examples usually contemplate some sort of practical difficulty, such as having a self-control problem or being unable to store a good for future use. As long as people can freely dispose of goods they don't want, having more of something can't make them worse off.

As an example of the application of the more-is-better assumption, consider the two bundles *A*, which has 12 sq yd/wk of shelter and 10 lb/wk of food, and *B*, which has 12 sq yd/wk of shelter and 11 lb/wk of food. The assumption tells us that *B* is preferred to *A*, because it has more food and no less shelter.

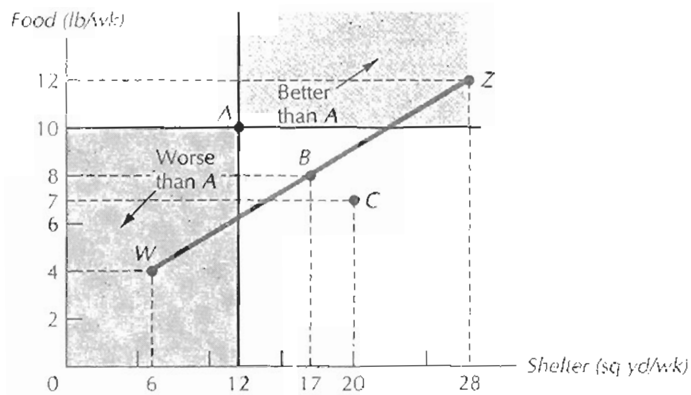
INDIFFERENCE CURVES

Before proceeding to the fourth assumption about preference orderings, let us consider briefly some of the implications of the first three. Most important, they enable us to generate a graphical description of the consumer's preferences. To see how, consider first the bundle *A* in Figure 3-8, which has 12 sq yd/wk of shelter and 10 lb/wk of food. The more-is-better assumption tells us that all bundles to the northeast of *A* are preferred to *A*, and that *A*, in turn, is preferred to all those to the southwest of *A*. Thus, for example, the more-is-better assumption tells us that *Z*, which has 28 sq yd/wk of shelter and 12 lb/wk of food, is preferred to *A* and that *A*, in turn, is preferred to *W*, which has only 6 sq yd/wk of shelter and 4 lb/wk of food.

Now consider the set of bundles that lie along the line joining *W* and *Z*. Because *Z* is preferred to *A* and *A* is preferred to *W*, it follows that as we move from *Z* to *W*, we must encounter a bundle that is equally preferred to *A*. (The intuition behind this claim is the same as the intuition that tells us that if we climb on any continuous path on a mountainside from one point at 1000 feet above sea level to another at 2000 feet, we must pass through every intermediate

FIGURE 3-8
Z is preferred to A because it has more of each good than A has. For the same reason, A is preferred to W. It follows that on the line joining W and Z there must be a bundle B that is equally preferred to A. In similar fashion, we can find a bundle C that is equally preferred to B.

GENERATING EQUALLY PREFERRED BUNDLES



altitude along the way.) Let B denote the bundle that is equally preferred to A, and suppose it contains 17 sq yd/wk of shelter and 8 lb/wk of food. (The exact amounts of each good in B will, of course, depend on the specific consumer whose preferences we are talking about.) The more-is-better assumption also tells us that there will be only one such bundle on the straight line between W and Z. Points on that line to the northeast of B are all better than B, those to the southwest of B are all worse.

In precisely the same fashion, we can find another point—call it C—that is equally preferred to B. C is shown as the bundle (20, 7), where the specific quantities in C again depend on the preferences of the consumer under consideration. By the transitivity assumption, we know that C is also equally preferred to A (since C is equally preferred to B, which is equally preferred to A).

Indifference curve A set of bundles among which the consumer is indifferent.

We can repeat this process as often as we like, and the end result will be an ***indifference curve***, a set of bundles all of which are equally preferred to the original bundle A, and hence also equally preferred to one another. This set is shown as the curve labeled I in Figure 3-9. It is called an indifference curve because the consumer is indifferent between all the bundles that lie along it.

An indifference curve also permits us to compare the satisfaction implicit in bundles that lie along it with those that lie either above or below it. It permits us, for example, to compare bundle C (20, 7) to bundle K (23, 4), which has less food and more shelter than C has. We know that C is equally preferred to D (25, 6) because both bundles lie along the same indifference curve. D, in turn, is preferred to K because of the more-is-better assumption: it has 2 sq yd/wk more shelter and 2 lb/wk more food than K has. Transitivity, finally, tells us that since C is equally preferred to D and D is preferred to K, C must be preferred to K.

By analogous reasoning, we can say that bundle L is preferred to A. *In general, bundles that lie above an indifference curve are all preferred to the bundles that lie on it. Similarly, bundles that lie on an indifference curve are all preferred to those that lie below it.*

AN INDIFFERENCE CURVE

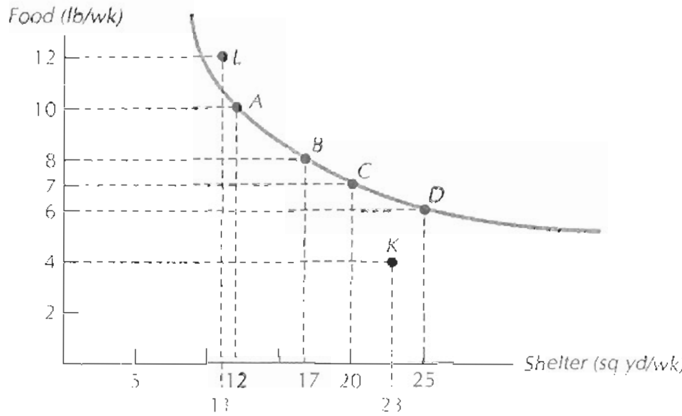


FIGURE 3-9
An indifference curve is a set of bundles that the consumer prefers equally. Any bundle, such as *L*, that lies above an indifference curve is preferred to any bundle on the indifference curve. Any bundle on the indifference curve, in turn, is preferred to any bundle, such as *K*, that lies below the indifference curve.

PART OF AN INDIFFERENCE MAP

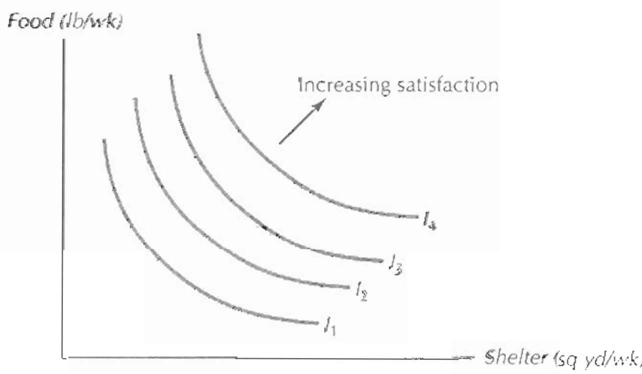


FIGURE 3-10
The entire set of a consumer's indifference curves is called the consumer's indifference map. Bundles on any indifference curve are less preferred than bundles on a higher indifference curve, and more preferred than bundles on a lower indifference curve.

THE MARGINAL RATE OF SUBSTITUTION

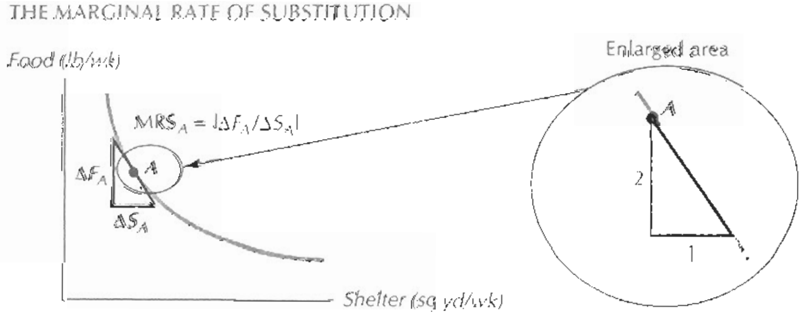


FIGURE 3-11
MRS at any point along an indifference curve is defined as the absolute value of the slope of the indifference curve at that point. It is the amount of food the consumer must be given to compensate for the loss of 1 unit of shelter.

Indifference map A representative sample of the set of a consumer's indifference curves, used as a graphical summary of her preference ordering.

The completeness property of preferences implies that there is an indifference curve that passes through every possible bundle. That being so, we can represent a consumer's preferences with an ***indifference map***, an example of which is shown in Figure 3-10. This indifference map shows just four of the infinitely many indifference curves that, taken together, yield a complete description of the consumer's preferences.

The numbers $I_1, \ldots, I_4$ in Figure 3-10 are index values used to denote the order of preference that corresponds to the respective indifference curves. Any index numbers would do equally well provided they satisfied the property $I_1 < I_2 < I_3 < I_4$. In representing the consumer's preferences, what really counts is the *ranking* of the indifference curves, not the particular numerical values we assign to them.[4]

As the next exercise will establish, two of our first three assumptions about preference orderings are enough to establish an additional important property of indifference maps.

EXERCISE 3-5

Show that the more-is-better and transitivity assumptions together rule out the possibility that any two indifference curves might cross one another. *Hint:* Assume that two indifference curves can cross and show that this implies a contradiction of one of our basic assumptions.

TRADEOFFS BETWEEN GOODS

Marginal rate of substitution (MRS) At any point on an indifference curve, the rate at which the consumer is willing to exchange the good measured along the vertical axis for the good measured along the horizontal axis; equal to the absolute value of the slope of the indifference curve.

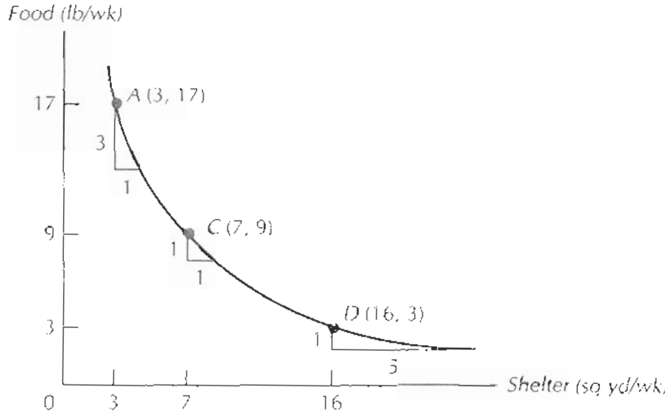
An important property of a consumer's preferences is the rate at which he is willing to exchange, or "trade off," one good for another. This property is represented at any point on an indifference curve by the ***marginal rate of substitution (MRS)***, which is defined as the absolute value of the slope of the indifference curve at that point. In the left panel of Figure 3-11, for example, the marginal rate of substitution at point A is given by the absolute value of the slope of the tangent to the indifference curve at A, which is the ratio $\Delta F_A/\Delta S_A$. (The notation ΔF_A means "small change in food from the amount at point A.") If we take ΔF_A units of food away from the consumer at point A, we have to give him ΔS_A additional units of shelter to make him just as well off as before. The right panel of the figure shows an enlargement of the region surrounding bundle A. If the marginal rate of substitution at A is 2, this means that the consumer must be given 2 lb/wk of food in order to make up for the loss of 1 sq yd/wk of shelter.

Whereas the slope of the budget constraint tells us the rate at which we can substitute food for shelter without changing total expenditure, the MRS tells us the rate at which we can substitute food for shelter without changing total

[4] For a more complete discussion of this issue, see the Appendix to this chapter.

FIGURE 3-12
The more food the consumer has, the more she is willing to give up to obtain an additional unit of shelter. The marginal rates of substitution at bundles *A*, *C*, and *D* are 3, 1, and $\frac{1}{5}$, respectively.

DIMINISHING MARGINAL RATE OF SUBSTITUTION



satisfaction.[5] Put another way, the slope of the budget constraint is the marginal cost of shelter in terms of food, while the MRS is the marginal benefit of shelter in terms of food.

Our fourth and final assumption about preference orderings concerns the behavior of the MRS as we move downward along an indifference curve.

4. Diminishing Marginal Rate of Substitution. This assumption states that, along any indifference curve, the more a consumer has of one good, the more she must be given of that good before she will be willing to give up a unit of the other good. Stated differently, MRS declines as we move downward to the right along an indifference curve. A preference ordering with a diminishing marginal rate of substitution will thus generate indifference curves that are convex—or bowed outward—when viewed from the origin. The indifference curves shown in Figures 3-9, 3-10, and 3-11 all have this property, as does the curve shown in Figure 3-12.

In Figure 3-12, note at bundle *A* that food is relatively plentiful and the consumer would be willing to sacrifice 3 lb/wk of it in order to obtain an additional square yard of shelter. Her MRS at *A* is 3. At *C*, the quantities of food and shelter are more balanced, and there she would be willing to give up only 1 lb/wk to obtain an additional square yard of shelter. Her MRS at *C* is 1. Finally, note that food is relatively scarce at *D*, where the consumer would need 5 additional sq yd/wk of shelter in return for giving up 1 lb/wk of food. Her MRS at *D* is 1/5.

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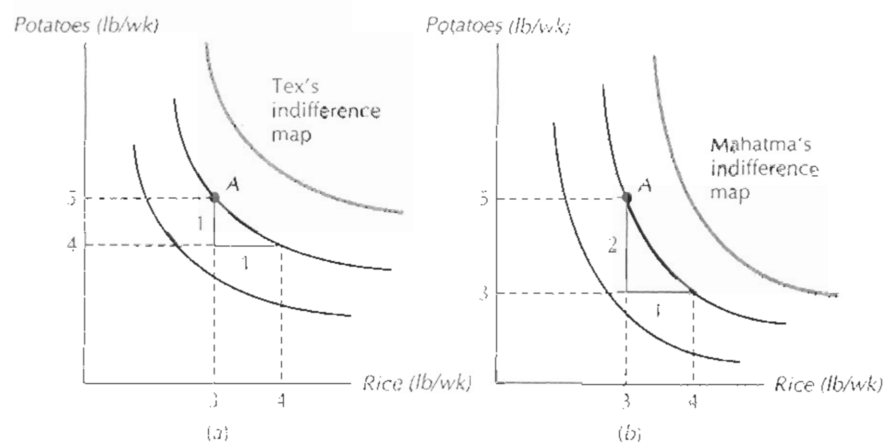
[5] More formally, the indifference curve may be expressed as a function $Y = Y(X)$, and the MRS at point *A* is defined as the absolute value of the derivative of the indifference curve at that point: $\text{MRS} = |dY(X)/dX|$.

FIGURE 3-13
Relatively speaking, Tex is a potato lover; Mahatma a rice lover. This difference shows up in the fact that at any given bundle Tex's marginal rate of substitution of potatoes for rice is smaller than Mahatma's.

PEOPLE WITH DIFFERENT TASTES



Intuitively, diminishing MRS means that consumers like variety. We are usually willing to give up goods we already have a lot of in order to obtain more of those goods we now have only a little of.

USING INDIFFERENCE CURVES TO DESCRIBE PREFERENCES

To get a feel for how indifference maps describe a consumer's preferences, it is helpful to work through a brief series of examples. As a preliminary exercise, let us see how indifference maps can be used to portray differences in preferences between two consumers. Suppose, for example, that both Tex and Mahatma like potatoes but that Mahatma likes rice much more than Tex does. This difference in their tastes is captured by the differing slopes of their indifference curves in Figure 3-13. Note in Figure 3-13*a*, which shows Tex's indifference map, that Tex would be willing to exchange 1 lb of potatoes for 1 lb of rice at bundle *A*. But at the corresponding bundle in Figure 3-13*b*, which shows Mahatma's indifference map, we see that Mahatma would trade 2 lb of potatoes for only 1 lb of rice. Their difference in preferences shows up clearly in this difference in their marginal rates of substitution of potatoes for rice.

In the examples that follow, we will explore the role of some of our assumptions about preference orderings by examining preferences that do not obey all these assumptions.

EXAMPLE 3-3

Indifference curves for perfect substitutes. Mattingly is behind in his intermediate microeconomics course and wants to stay up late studying for an exam. To remain alert, he goes out to buy some caffeinated soda. The store carries both Coca-Cola and Jolt Cola. Jolt has twice the caffeine of Coke, and for now Mattingly cares only about total caffeine content. Draw his indifference map for Coke and Jolt.

FIGURE 3-14
One good is a perfect substitute for another if the MRS is everywhere constant. Here, the MRS is 2 at each point along each indifference curve.

INDIFFERENCE CURVES FOR PERFECT SUBSTITUTES

For Mattingly, Jolt and Coke are *perfect substitutes*. This means that his indifference curves will not have the usual convex shape but will instead be linear. The top line shown in Figure 3-14 is the set of all possible Coke-Jolt combinations that provide the same satisfaction as the bundle consisting of 6 pints of Coke per day and 0 pints of Jolt per day. Since each pint of Jolt has twice the caffeine of a pint of Coke, all bundles along this line contain precisely the same amount of caffeine. The next line down is the indifference curve for bundles equivalent to bundle (0, 4); and the third line down is the indifference curve corresponding to (0, 2). Along each of these indifference curves, the marginal rate of substitution of Coke for Jolt is always 2/1 (2 pints of Coke for every pint of Jolt).

Preferences in which goods are perfect substitutes fail to satisfy the assumption of diminishing MRS. Normally, the more of one good a person has, the more of it he is willing to exchange for another. In the case of perfect substitutes, however, the MRS is everywhere constant. Note that when we say that one good is a perfect substitute for another, that does not necessarily mean that they are *one-for-one substitutes*. In the Jolt-Coke illustration, for instance, Mattingly required 2 pints of Coke to make up for the loss of 1 pint of Jolt. An example of perfect one-for-one substitutes would be red shirts and green shirts for a consumer who is color-blind.

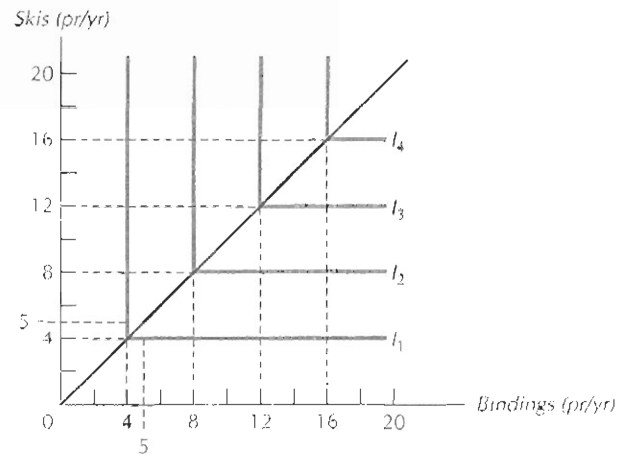
EXAMPLE 3-4

Indifference curves for perfect complements. Vreni, an aggressive skier, spends all her income on skis and bindings. She wears out 1 pair (pr) of skis for every pair of bindings she wears out. Graph Vreni's indifference curves.

Here we are confronted with the polar opposite case from the one considered in Example 3-3. Not only are skis and bindings not perfect substitutes for one another,

FIGURE 3-15
Perfect complements are always used in fixed proportions, here 1 pair of skis for every pair of bindings.

INDIFFERENCE CURVES FOR PERFECT COMPLEMENTS



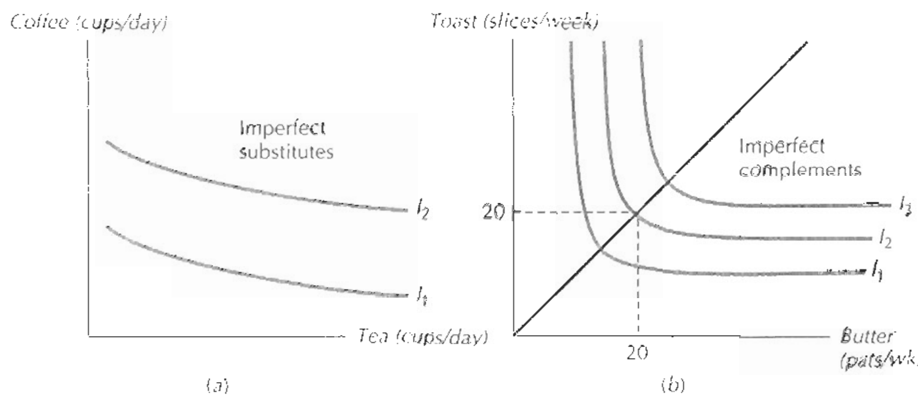
but they are not substitutes at all. To get any enjoyment from them, Vreni must consume them in exactly the right proportion. In Figure 3-15, consider the bundle that consists of 4 pr of skis per year and 4 pr of bindings. How will the satisfaction Vreni gets from that bundle compare with the satisfaction she gets from the bundle consisting of 4 pr of skis per year and 5 pr of bindings? Since she already has skis and bindings in precisely the proportion she wants, the extra pair of bindings makes her no better off than before. Provided she is not forced to use the extra pair, however, it does not make her any worse off either. She can leave it aside and continue to use 4 pr of skis per year and 4 pr of bindings. Thus the bundle consisting of 4 pr of skis per year and 5 pr of bindings lies on exactly the same indifference curve as the original bundle. By similar reasoning, the bundle consisting of 5 pr of skis per year and 4 pr of bindings lies on this indifference curve as well. Proceeding in like fashion, we can trace out the entire indifference curve passing through bundle (4, 4), which is denoted as I_1 in *Figure* 3-15.

All of Vreni's indifference curves will have the same L-shape as I_1, and their cusps will all lie along the ray with slope = 1, as shown in the diagram. Thus, for example, I_4 is the indifference curve that contains bundle (16, 16).

Preference orderings involving perfect complements do not satisfy the more-is-better assumption. Once the desired ratio is achieved, adding more of any one good does not make the consumer better off. Nor does the marginal rate of substitution diminish smoothly as we move along the indifference curve. On the contrary, it is infinite on the vertical arm of the indifference curve, zero on the horizontal, and undefined at the cusp.

FIGURE 3-16
Two goods are less-than-perfect substitutes (*a*) if the MRS diminishes only slightly with downward movements along an indifference curve. Substitution possibilities exist between less-than-perfect complements (*b*), but are very limited.

INDIFFERENCE CURVES FOR LESS-THAN-PERFECT SUBSTITUTES AND COMPLEMENTS



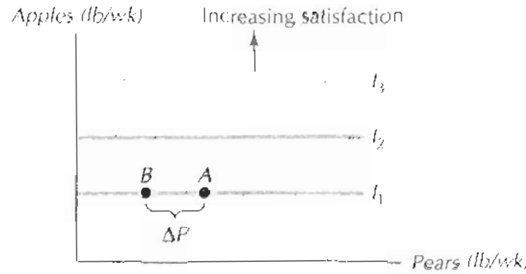
EXERCISE 3-6

Draw Vreni's indifference curves on the assumption that she is such an aggressive skier that she wears out 2 pr of skis for every pair of hindings she wears out.

Exercise 3-6 illustrates that goods can be perfect complements without being one-for-one complements.

Pairs of goods can be either substitutes or complements without being perfect substitutes or complements. For many people, coffee and tea are an example of less-than-perfect substitutes. The more nearly a perfect substitute one good is for another, the closer to straight lines will be the indifference curves for those goods. Thus a typical consumer's indifference curves for coffee and tea, shown in Figure 3-16*a*, would not be perfectly straight lines, but neither would they exhibit a great deal of curvature. Pairs of goods can also be complementary without being perfectly so. Someone may prefer a certain amount of butter on each slice of toast, for example, and yet not insist that butter and toast be combined in absolutely fixed proportions. For such a person butter and toast can be substituted for one another, but only within very narrow limits. The indifference curves in this case would look something like the ones in Figure 3-16*b*. They are not perfectly L-shaped, but they display much more pronounced curvature than we see in the usual case. This consumer normally likes a pat of butter with each slice of toast. As we swap butter for toast, his indifference curves do not become vertical (as they would in the case of perfect complements), but they quickly become very steep. If he had 100 slices of toast per week but only a single pat of butter, his satisfaction would not increase much if we gave him yet another slice of toast. By the same token, if he had 100 pats of butter per week and only 1 slice of toast, an extra pat of butter wouldn't help much.

FIGURE 3-17 INDIFFERENCE CURVES WHEN ONE GOOD IS A NEUTRAL GOOD



When the consumer doesn't care about pears, the marginal rate of substitution of apples for pears is zero. The consumer needs no additional apples to compensate for the loss of a unit of pears.

EXAMPLE 3-5

A neutral good. Eve likes apples but doesn't care about pears. If apples and pears are the only two goods available, draw her indifference curves.

As before, the technique for seeing what an indifference curve looks like is to start with some bundle, take a small quantity of one good away, and then ask how large an increase in the other good would be required to restore the original satisfaction level. In Figure 3-17, for example, suppose we start at bundle *A* and then take away ΔP units of pears. How many more units of apples would we have to give Eve to make her just as happy as at *A*? The answer is none, because she didn't care about pears in the first place, and therefore suffered no loss in satisfaction when we took ΔP units of pears away. Bundle *B* is thus on the same indifference curve as bundle *A*, as are all other bundles on the horizontal line through *A*. All of Eve's indifference curves are in fact horizontal lines, as shown in Figure 3-17. Apples alone determine her level of satisfaction, and so the amount of pears in any bundle cannot be used to identify the indifference curve on which the bundle lies. Such irrelevant goods are called *neutral goods*.

As in the perfect complements example, preferences involving neutral goods violate both the more-is-better and diminishing MRS assumptions. Having more of the neutral good neither helps nor hurts. Eve's MRS of apples for pears is everywhere zero.

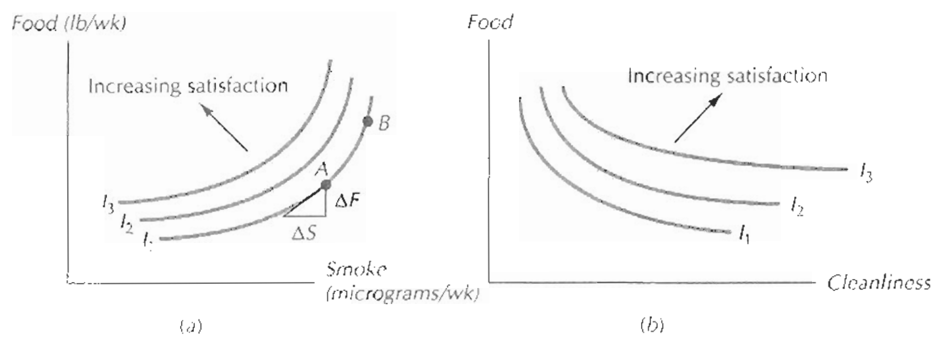
EXAMPLE 3-6

When more of a good is worse. Koop likes food but dislikes cigarette smoke. The more food he has, the more he would be willing to give up to achieve a given reduction in cigarette smoke. If food and cigarette smoke are the only two goods, draw Koop's indifference curves.

Again start at a given bundle, such as *A* in Figure 3-18*a*. Then take away a small amount of food, ΔF, and ask what change in smoke, ΔS, would be required to restore Koop's original satisfaction level. In the standard case, when we take one good away, we need to add more of the other. This time, however, we compensate

FIGURE 3-18
When the good measured along the horizontal axis is undesirable (*a*), indifference curves are positively sloped. Satisfaction increases with movements to the northwest in the diagram. The indifference map in (*a*) can be transformed to the more conventional orientation (*b*) by redefining the undesirable good.

INDIFFERENCE CURVES WHEN ONE GOOD IS UNDESIRABLE

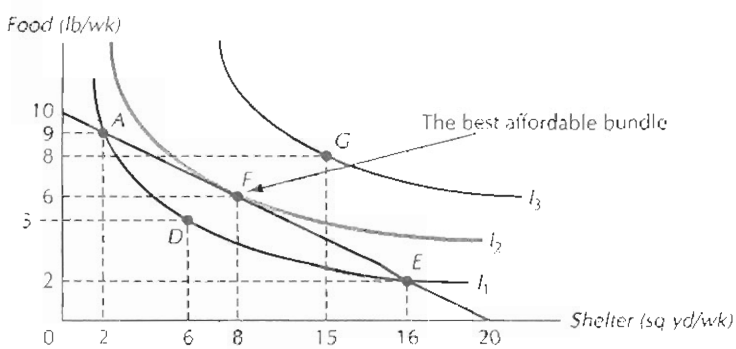


by *taking away* some of the other good. Thus, for example, when we take ΔF units of food away from Koop, we must reduce the smoke level by ΔS in order to restore his original satisfaction level. This tells us that the indifference curve through *A* slopes upward, not downward. Koop would be just as happy with a smaller meal served in a restaurant with a no-smoking section as he would with a larger meal served in a restaurant without one.

What about the curvature of Koop's indifference curves? We know that the more food Koop has, the more he is willing to trade for a given amount of smoke reduction. This tells us that the slopes of his indifference curves increase as we move to the right. Put another way, the MRS at *B* must be higher than the MRS at *A*. Koop's indifference curves for food and smoke must thus look something like the ones shown in Figure 3-18*a*. In the standard case, satisfaction increases as we move to the northeast on the indifference map. Here, it increases as we move to the northwest.

FIGURE 3-19
The best the consumer can do is to choose the bundle on the budget constraint that lies on the highest attainable indifference curve. Here, that is bundle *F*, which lies at a tangency between the indifference curve and the budget constraint.

THE BEST AFFORDABLE BUNDLE



Needless to say, preference orderings defined over undesirable goods will not satisfy the more-is-better assumption. In such cases, however, it is usually possible to portray the consumer's preferences in the usual fashion by simply redefining the undesirable good. Thus, in the previous example, we might focus not on smoke, an undesirable good, but on cleanliness (the absence of smoke), which is clearly desirable. So doing would recast the indifference map in the left panel (*a*) of Figure 3-18 as the much more conventional looking one in the right panel (*b*). Here, the cleanliness axis essentially measures the absence of smoke. Thus, for example, points near the origin would correspond to the highest smoke levels, with smoke declining as we move to the right. Note that satisfaction now increases as we move to the northeast, just as in the usual case.

The Best Feasible Bundle

We now have all the tools we need to determine how the consumer should allocate his income between two goods. The indifference map tells us how the various bundles are ranked in order of preference. The budget constraint, in turn, tells us which bundles are affordable. The consumer's task is to put the two together and to choose the most preferred affordable bundle. (Recall from Chapter 1 that we need not suppose that consumers think explicitly about budget constraints and indifference maps when deciding what to buy. It is sufficient to assume that people make decisions *as if* they were thinking in these terms, just as expert pool players choose between shots as if they knew all the relevant laws of Newtonian physics.)

For the sake of concreteness, let us again consider the choice between food and shelter that confronts a consumer with an income of M = \$100/wk facing prices of P_F = \$10/lb and P_S = \$5/sq yd. Figure 3-19 shows this consumer's budget constraint and part of his indifference map. Of the five labeled bundles—*A*, *D*, *E*, *F*, and *G*—in the diagram, *G* is the most preferred because it lies on the highest indifference curve. *G*, however, is not affordable, nor is any other bundle that lies beyond the budget constraint. The more-is-better assumption implies that the best affordable bundle must lie *on* the budget constraint, not inside it. (Any bundle inside the budget constraint would be less preferred than one just slightly to the northeast, which would also be affordable.)

Where, exactly, is the best affordable bundle located along the budget constraint? We know that it cannot be on an indifference curve that lies partly inside the budget constraint. On the indifference curve I_1, for example, the only points that are even candidates for the best affordable bundle are the two that lie on the budget constraint, namely, *A* and *E*. But *A* cannot be the best affordable bundle because it is equally preferred to *D*, which, in turn, is less desirable than *F* by the more-is-better assumption. So by transitivity, *A* is less desirable than *F*. For the same reason, *E* cannot be the best affordable bundle.

Since the best affordable bundle cannot lie on an indifference curve that lies partly inside the budget constraint, and since it must lie on the budget constraint itself, we know it has to lie on an indifference curve that intersects the budget constraint only once. In Figure 3-19, that indifference curve is the one labeled I_2, and the best affordable bundle is F, which lies at the point of tangency between I_2 and the budget constraint. With an income of \$100/wk and facing prices of \$5/sq yd for shelter and \$10/lb for food, the best this consumer can do is to buy 6 lb/wk of food and 8 sq yd/wk of shelter.

The choice of bundle F makes perfect sense on intuitive grounds. The consumer's goal, after all, is to reach the highest indifference curve he can, given his budget constraint. His strategy is to keep moving to higher and higher indifference curves until he reaches the highest one that is still affordable. For indifference maps for which a tangency point exists, as in Figure 3-19, the best bundle will always lie at the point of tangency.

In Figure 3-19, note that the marginal rate of substitution at F is exactly the same as the absolute value of the slope of the budget constraint. This will always be so when the best affordable bundle occurs at a point of tangency. The condition that must be satisfied in such cases is therefore

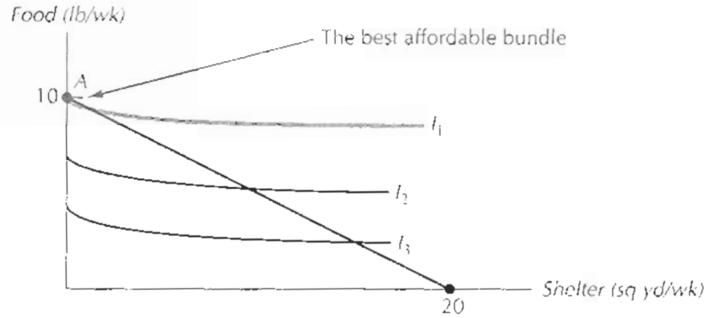
$$\text{MRS} = \frac{P_S}{P_F}. \tag{3.3}$$

The right-hand side of Equation 3.3 represents the opportunity cost of shelter in terms of food. Thus, with P_S = \$5/sq yd and P_F = \$10/lb, the opportunity cost of an additional square yard of shelter is $\frac{1}{2}$ lb of food. The left-hand side of Equation 3.3 is $|\Delta F/\Delta S|$, the absolute value of the slope of the indifference curve at the point of tangency. It is the amount of additional food the consumer must be given in order to compensate him fully for the loss of 1 sq yd of shelter. In the language of cost-benefit analysis discussed in Chapter 1, the slope of the budget constraint represents the opportunity cost of shelter in terms of food, while the slope of the indifference curve represents the benefits of consuming shelter as compared with consuming food. Since the slope of the budget constraint is $-1/2$ in this example, the tangency condition tells us that $\frac{1}{2}$ lb of food would be required to compensate for the benefits given up with the loss of 1 sq yd of shelter.

If the consumer were at some bundle on the budget line for which the two slopes are not the same, then it would always be possible for him to purchase a better bundle. To see why, suppose he were at a point where the slope of the indifference curve (in absolute value) is less than the slope of the budget constraint, as at point E in Figure 3-19. Suppose, for instance, that the MRS at E is only 1/4. This tells us that the consumer can be compensated for the loss of 1 sq yd of shelter by being given an additional $\frac{1}{4}$ lb of food. But the slope of the budget constraint tells us that by giving up 1 sq yd of shelter, he can purchase an additional $\frac{1}{2}$ lb of food. Since this is $\frac{1}{4}$ lb more than he needs to remain equally satisfied, he will clearly be better off if he purchases more food and less shelter than at point E. The opportunity cost of an additional pound of food is less than the benefit it confers.

FIGURE 3-20
When the MRS of food for shelter is always less than the slope of the budget constraint, the best the consumer can do is to spend all his income on food.

A CORNER SOLUTION



EXERCISE 3-7

Suppose that the marginal rate of substitution at point *A* in Figure 3-19 is 1.0. Show that this means that the consumer will be better off if he purchases less food and more shelter than at *A*.

CORNER SOLUTIONS

Corner solution In a choice between two goods, a case in which the consumer does not consume one of the goods.

The best affordable bundle need not always occur at a point of tangency. In some cases, there may simply *be* no point of tangency—the MRS may be everywhere greater, or less, than the slope of the budget constraint. In this case we get a ***corner solution***, like the one shown in Figure 3-20, where M, P_F, and P_S are again given by \$100/wk, \$10/lb, and \$5/sq yd, respectively. The best affordable bundle is the one labeled A, and it lies at the upper end of the budget constraint. At A the MRS is less than the absolute value of the slope of the budget constraint. For the sake of illustration, suppose the MRS at $A = 0.25$, which means that this consumer would be willing to give up 0.25 lb of food to get an additional square yard of shelter. But at market prices the opportunity cost of an additional square yard of shelter is 0.5 lb of food. He increases his satisfaction by continuing to give up shelter for more food until it is no longer possible to do so. Even though this consumer regards shelter as a desirable commodity, the best he can do is to spend all his income on food. Market prices are such that he would have to give up too much food to make the purchase of even a single unit of shelter worthwhile.

The indifference map shown in Figure 3-20 satisfies the property of diminishing marginal rate of substitution—moving to the right along any indifference curve, the slope becomes smaller in absolute terms. But because the slopes of the indifference curves start out smaller than the slope of the budget constraint here, the two never reach equality.

Recall that indifference curves that are not strongly convex are characteristic of goods that are easily substituted for one another. Corner solutions are more

EQUILIBRIUM WITH PERFECT SUBSTITUTES

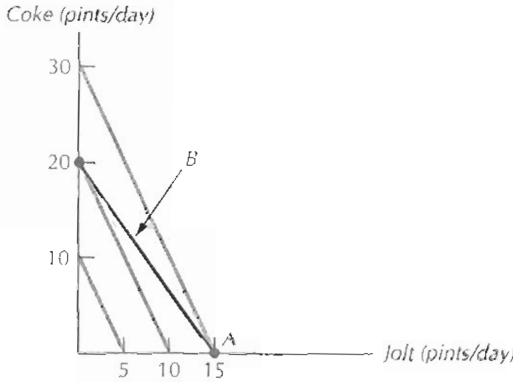


FIGURE 3-21
Here, the MRS of Coke for Jolt is 2 at every point. Whenever the price ratio P_J/P_C is less than 2, a corner solution results in which the consumer buys only Jolt. On the budget constraint *B*, the consumer does best to buy bundle *A*.

likely to occur for such goods, and indeed are almost certain to occur when goods are perfect substitutes. For such goods, recall, the MRS does not diminish at all; rather, it is everywhere the same. With perfect substitutes, indifference curves are straight lines. If they happen to be steeper than the budget constraint, we get a corner solution on the horizontal axis; if less steep, we get a corner solution on the vertical axis.

EXAMPLE 3-7

Consider again the case of Mattingly, the caffeinated cola drinker from Example 3-3. Recall that he spends all his soft drink budget on Coca-Cola and Jolt Cola, and cares only about the total caffeine content of what he drinks. If Jolt has twice the caffeine of Coke, and if Jolt costs \$1/pint and Coke costs \$0.75/pint, how will Mattingly spend his soft drink budget of \$15/wk?

Mattingly's budget constraint, denoted *B*, and indifference map are shown in Figure 3-21. The slope of his indifference curves is -2, of his budget constraint, $-4/3$. The best affordable bundle is the one labeled *A*, a corner solution in which he spends all his budget on Jolt. This makes intuitive sense in the light of Mattingly's peculiar preferences: he cares only about total caffeine content, and at the given prices, Jolt provides more caffeine per dollar than Coke does. If the Jolt-Coke price ratio, P_J/P_C, had been 3/1 (or any other amount greater than 2/1), Mattingly would have spent all his income on Coke. That is, we would again have had a corner solution, only this time on the vertical axis. Only if the price ratio had been exactly 2/1 might we have seen Mattingly spend part of his income on each good. In that case, any combination of Coke and Jolt on his budget constraint would have served him equally well.

Most of the time we will deal with problems that have not corner but *interior solutions*—that is, with problems where the best affordable bundle will lie at a point of tangency. An interior solution, again, is one where the MRS is exactly the same as the slope of the budget constraint.

INDIFFERENCE CURVES WHEN THERE ARE MORE THAN TWO GOODS

The examples discussed so far have all been ones in which the consumer cares about only two goods. Where there are more than two, we can construct indifference curves by using the same device we used earlier to represent multigood budget constraints. We simply view the consumer's choice as being one between a particular good X and an amalgam of other goods Y, which is again called the composite good. As before, the composite good is the amount of income the consumer has left over after buying the good X.

In the multigood case, we may thus continue to represent the consumer's preferences with an indifference map in the XY plane. Here, the indifference curve tells not the rate at which the consumer will exchange some particular good Y for a good X, but the rate at which he will exchange the composite good for X. Just as in the two-good case, equilibrium occurs when the consumer reaches the highest indifference curve attainable on his budget constraint.

An Application of the Rational Choice Model

As the following example will make clear, the composite good construct enables us to deal with more general questions than we could in the simple two-good case.

EXAMPLE 3-8 *Is it better to give poor people cash or food stamps?*

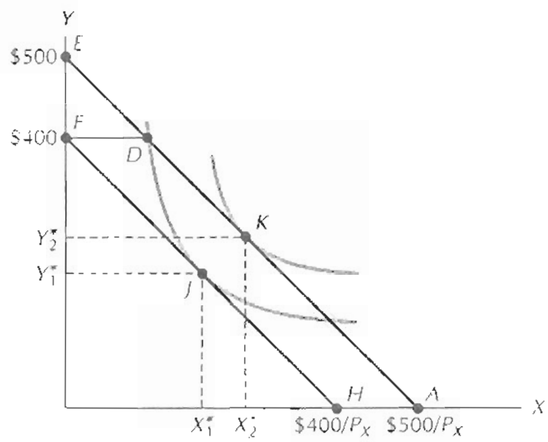
One of the objectives of the food stamp program is to alleviate hunger among poor people. Under the terms of the program, people whose incomes fall below a certain level are eligible to receive a specified quantity of food stamps. Thus, for example, a person with an income of \$400/mo might be eligible for \$100/mo worth of stamps. These stamps can then be used to buy \$100/mo worth of food. Any food he buys in excess of \$100/mo he must pay for in cash. Stamps cannot be used to purchase cigarettes, alcohol, and various other items. The government gives food retailers cash for the stamps they accept in exchange for food.

The cost to the government for the consumer in the example given was \$100—the amount it had to reimburse the store for the stamps. Would the consumer have been better off had he instead been given \$100 directly in cash? We can try to answer this question by investigating which alternative would get him to a higher indifference curve.

Suppose Y denotes the composite good and X denotes food. If the consumer's income is \$400/mo and P_X is the price of food, his initial equilibrium is the bundle

FIGURE 3-22
By comparison with the budget constraint under a cash grant (AE), the budget constraint under food stamps (ADF) limits the amount that can be spent on nonfood goods. But for the consumer whose indifference map is shown, the equilibrium bundles are the same under both programs.

FOOD STAMP PROGRAM VS. CASH GRANT PROGRAM



J shown in Figure 3-22. The effect of the food stamp program is to increase the total amount of food he can buy each month from \$400/P_X to \$500/P_X. In terms of the *maximum* amount of food he can buy, the food stamp program is thus exactly the same as a cash grant of \$100.

Where the two alternatives differ is in terms of the maximum amounts of other goods he can buy. With a cash grant of \$100, he has a total monthly income of \$500, and this is, of course, the maximum amount of nonfood goods (the composite good) he can buy. His budget constraint in this case is thus the line labeled AE in Figure 3-22.

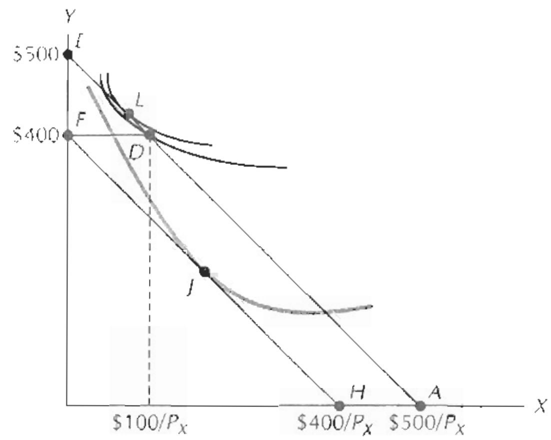
With the food stamp program, by contrast, the consumer is not able to buy \$500/mo of nonfood goods because his \$100 in food stamps can be used only for food. The maximum amount of nonfood goods he can purchase is \$400. In Figure 3-22, his budget constraint under the food stamp program is labeled ADF. For values of Y less than \$400, it is thus exactly the same as his budget constraint under the cash grant program. For values of Y larger than \$400, however, his budget constraint under the food stamp program is completely flat.

Note that the consumer whose indifference curves are shown in Figure 3-22 buys exactly the same bundle, namely, bundle K, under both programs. The effect of the food stamp program here is precisely the same as the effect of the cash grant. In general, this will be true whenever the consumer with a cash grant would have spent more on food anyway than the amount of food stamps he would have received under the food stamp program.

Figure 3-23 depicts a consumer for whom this is *not* the case. With a cash grant, he would choose the bundle L, which would put him on a higher indifference curve than he could attain under the food stamp program, which would lead him to buy bundle D. Note that bundle D contains exactly \$100 worth of food, the

FIGURE 3-23
For the consumer whose indifference map is shown, a cash grant would be preferred to food stamps, which force him to devote more to food than he would choose to spend on his own.

WHERE FOOD STAMPS AND CASH GRANTS YIELD DIFFERENT OUTCOMES



amount of food stamps he received. Bundle L, by contrast, contains less than \$100 worth of food. Here, the effect of the food stamp program is to cause the recipient to spend more on food than he would have if he had instead been given cash.

The face value of the food stamps received by most participants is smaller than what they would have spent on food. For these people, the food stamp program leads, as noted, to exactly the same behavior as a pure cash grant program.

The analysis in Example 3-8 raises the question of why Congress did not just give poor people cash grants in the first place. The ostensible reason is that Congress wanted to help poor people buy food, not luxury items or even cigarettes and alcohol. And yet if most participants would have spent at least as much on food as they received in stamps, not being able to use stamps to buy other things is a meaningless restriction. For instance, if someone would have spent \$150 on food anyway, getting \$100 in food stamps simply lets him take some of the money he would have spent on food and spend it instead on whatever else he chooses.

On purely economic grounds, there is thus a strong case for replacing the food stamp program with a much simpler program of cash grants to the poor. At the very least, this would eliminate the cumbersome step of requiring grocers to redeem their stamps for cash.

As a political matter, however, it is easy to see why Congress might have set things up the way it did. Many of the taxpayers who sponsor antipoverty programs would be distressed to see their tax dollars used to buy illicit substances. If the food stamp program prevents even a tiny minority of participants from spending more on such goods, it spares many political difficulties.

THE PUZZLE OF GIFT GIVING

Example 3-8 calls our attention to a problem that applies not just to the food stamp program but to all other forms of in-kind transfers as well. It is that while the two forms of transfer are sometimes equivalent, gifts in cash seem clearly superior on those occasions when they differ. Consider, for example, the phenomenon of gift giving. Occasionally someone receives a gift that is exactly what he would have purchased for himself had he been given an equivalent amount of money. But we are all far too familiar with gifts that miss the mark. Who, for example, has never been given an article of clothing that he was embarrassed to wear? The logic of the economic choice model seems to state unequivocally that we could avoid the problem of useless gifts by the simple expedient of giving cash. And yet virtually every society continues to engage in ritualized gift giving.

The fact that this custom has persisted should not be taken as evidence that people are stupid. Rather, it suggests that there may be something about gift giving that the rational choice model fails to capture. One purpose of giving a gift is to express affection for the recipient. A thoughtfully chosen gift accomplishes this in a way that a gift of cash cannot. Or it may be that some people have difficulty indulging themselves with even small luxuries and would feel compelled to spend cash gifts on purely practical items. For these people, a gift provides a way of enjoying a small luxury without having to feel guilty about it.[6] This interpretation is supported by the observation that we rarely give purely practical gifts like plain cotton underwear or laundry detergent.

Whatever the real reasons may be for giving in kind rather than in cash, it seems safe to assume that we do not do it because it never occurred to us to give cash. On the contrary, occasionally we do give cash gifts, especially to young relatives with low incomes. But despite the advantages of gifts in cash, people seem clearly reluctant to abandon the practice of giving in kind.

Summary

Our task in this chapter has been to set forth the basic model of rational consumer choice. In all its variants, this model retains certain common features; in particular, it takes consumers' preferences as given and assumes they will try to satisfy them in the most efficient way.

The first step in solving the budgeting problem is to identify the set of bundles of goods that the consumer is able to buy. The consumer is assumed to have an income level given in advance and to face fixed prices. Prices and income together define the consumer's budget constraint, which, in the simple two-good case, is a downward-sloping line whose slope, in absolute value, is the ratio of the two

[6] For a discussion of this interpretation, see R. Thaler, "Mental Accounting and Consumer Choice," *Marketing Science*, 4, Summer 1985.

prices. It is the set of all possible bundles that the consumer might purchase if he spends his entire income.

The second step in solving the consumer budgeting problem is to summarize the consumer's preferences. Here, we begin with a preference ordering by which the consumer is able to rank all possible bundles of goods. This ranking scheme is assumed to be complete and transitive and to exhibit the more-is-better property. Preference orderings that satisfy these restrictions give rise to indifference maps, or collections of indifference curves, each of which represents combinations of bundles among which the consumer is indifferent. Preference orderings are also assumed to exhibit a diminishing marginal rate of substitution, which means that, along any indifference curve, the more of a good a consumer has, the more he must be given to induce him to part with a unit of some other good. The diminishing MRS property is what accounts for the characteristic convex shape of indifference curves.

The budget constraint tells us what combinations of goods the consumer can afford to buy, and the indifference map summarizes the consumer's preferences concerning them. In most problems, the best affordable bundle occurs at a point of tangency between an indifference curve and the budget constraint. At that point, the marginal rate of substitution is exactly equal to the rate at which the goods can be exchanged for one another at market prices.

APPENDIX THE UTILITY FUNCTION APPROACH TO THE CONSUMER BUDGETING PROBLEM

Cardinal versus Ordinal Utility

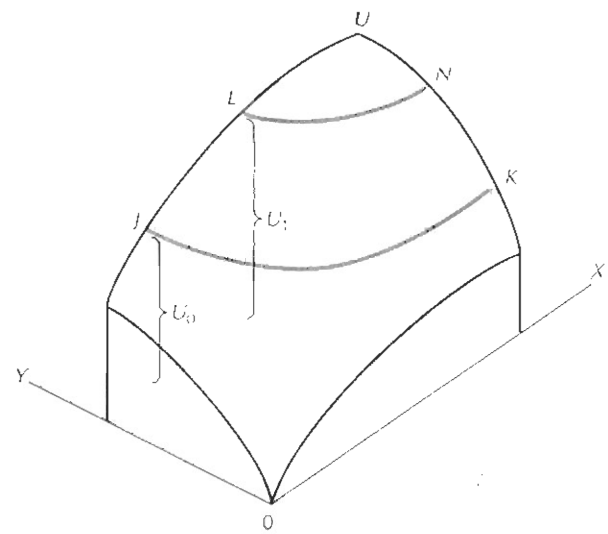
In our discussion about how to represent consumer preferences, we assumed that people are able to rank each possible bundle in order of preference. This is called the *ordinal utility* approach to the consumer budgeting problem. It does not require that people be able to make quantitative statements about how much they like various bundles. Thus it assumes that a consumer will always be able to say whether he prefers A to B, but that he may not be able to make such statements as "A is 6.43 times as good as B."

In the nineteenth century, economists commonly assumed that people could make such statements. Today we call theirs the *cardinal utility* approach to the consumer choice problem. In the two-good case, it assumes that the satisfaction provided by any bundle can be assigned a numerical, or cardinal, value by a utility function of the form

$$U = U(X, Y). \tag{A.3.1}$$

In three dimensions, the graph of such a utility function will look something

FIGURE A.3-1 A THREE-DIMENSIONAL UTILITY SURFACE



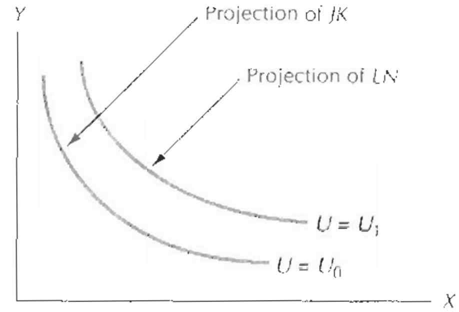
like the one shown in Figure A.3-1. It resembles a mountain, but because of the more-is-better assumption, it is a mountain without a summit. The value on the U axis measures the height of the mountain, which continues to increase the more we have of X or Y.

Suppose in Figure A.3-1 we were to fix utility at some constant amount, say, U_0. That is, suppose we cut the utility mountain with a plane parallel to the XY plane, U_0 units above it. The line labeled JK in Figure A.3-1 represents the intersection of that plane and the surface of the utility mountain. All the bundles of goods that lie on JK provide a utility level of U_0. If we then project the line JK downward onto the XY plane, we have what amounts to the U_0 indifference curve, shown in Figure A.3-2.

Suppose we then intersect the utility mountain with another plane, this time U_1 units above the XY plane. In Figure A.3-1, this second plane intersects the utility mountain along the line labeled LN. It represents the set of all bundles that confer the utility level U_1. Projecting LN down onto the XY plane, we thus get the indifference curve labeled U_1 in Figure A.3-2. In like fashion, we can generate an entire indifference map corresponding to the cardinal utility function $U(X, Y)$.

Thus we see that it is possible to start with any cardinal utility function and end up with a unique indifference map. *But it is not possible to go the other direction!* That is, it is not possible to start with an indifference map and work backward to a unique cardinal utility function. The reason is that there will always be infinitely many such utility functions that give rise to precisely the same indifference map.

FIGURE A.3-2 INDIFFERENCE CURVES AS PROJECTIONS



To see why, just imagine that we took the utility function in Equation A.3.1 and doubled it, so that utility is now given by $V = 2U(X, Y)$. When we graph V as a function of X and Y, the shape of the resulting utility mountain will be much the same as before. The difference will be that the altitude at any X, Y point will be twice what it was before. If we pass a plane $2U_0$ units above the XY plane, it would intersect the new utility mountain in precisely the same manner as the plane U_0 units high did originally. If we then project the resulting intersection down onto the XY plane, it will coincide perfectly with the original U_0 indifference curve.

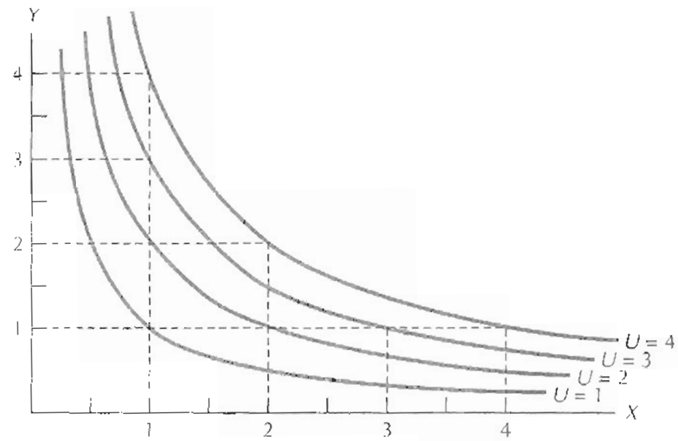
All we do when we multiply (divide, add to, or subtract from) a cardinal utility function is to relabel the indifference curves to which it gives rise. Indeed, we can make an even more general statement: If $U(X, Y)$ is any cardinal utility function and if V is any increasing function, then $U = U(X, Y)$ and $V = V[U(X, Y)]$ will give rise to precisely the same indifference maps. The special property of an increasing function is that it preserves the rank ordering of the values of the original function. That is, if $U(X_1, Y_1) > U(X_2, Y_2)$, the fact that V is an increasing function assures that $V[U(X_1, Y_1)]$ will be greater than $V[U(X_2, Y_2)]$. And as long as that requirement is met, the two functions will give rise to exactly the same indifference curves.

The concept of the indifference map was first discussed by Francis Edgeworth, who derived it from a cardinal utility function in the manner described above. It took the combined insights of Vilfredo Pareto, Irving Fisher, and John Hicks to establish that Edgeworth's apparatus was not uniquely dependent on a supporting cardinal utility function. As we have seen, the only aspect of a consumer's preferences that matters in the standard budget allocation problem is the shape and location of his indifference curves. Consumer choice turns out to be completely independent of the labels we assign to these indifference curves, provided only that higher curves correspond to higher levels of utility.

Modern economists prefer the ordinal approach because it rests on much weaker assumptions than the cardinal approach. That is, it is much easier

FIGURE A.3-3
To get the indifference curve that corresponds to all bundles that yield a utility level of U_0, set $XY = U_0$ and solve for Y to get $Y = U_0/X$.

INDIFFERENCE CURVES FOR THE UTILITY FUNCTION $U = XY$



to imagine that people can rank different bundles than to suppose that they can make precise quantitative statements about how much satisfaction each provides.

Generating Indifference Curves Algebraically

Even if we assume that consumers have only ordinal preference orderings, it will often be convenient to represent those preferences with a cardinal utility index. The advantage is that this procedure provides a compact algebraic way of summarizing all the information that is implicit in the graphical representation of preferences.

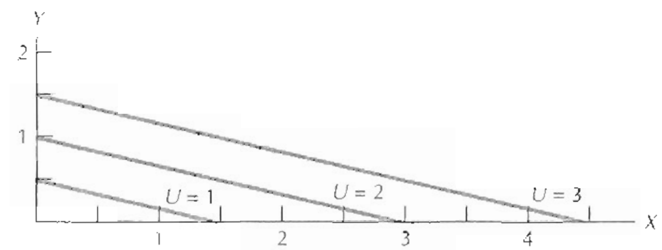
Suppose, for example, that Tom's utility function is given by $U(X, Y) = XY$, and we want to use this information to graph Tom's indifference map. In the language of utility functions, an indifference curve is all combinations of X and Y that yield the same level of utility. Suppose we look at the indifference curve that corresponds to 1 unit of utility—that is, the combinations of bundles for which $XY = 1$. Solving this equation for Y, we have

$$Y = \frac{1}{X}, \tag{A.3.2}$$

which is the indifference curve labeled $U = 1$ in Figure A.3-3. The indifference curve that corresponds to 2 units of utility is generated by solving $XY = 2$ to get $Y = 2/X$, and it is shown by the curve labeled $U = 2$ in Figure A.3-3. In similar fashion, we generate the indifference curves for $U = 3$ and $U = 4$, which are

FIGURE A.3-4
The indifference curve that corresponds to all bundles yielding a utility level of U_0 is given by $Y = (U_0/2) - (\frac{1}{3})X$.

INDIFFERENCE CURVES FOR THE UTILITY FUNCTION $U(X,Y) = (\frac{2}{3})X + 2Y$



correspondingly labeled in the diagram. More generally, we get the indifference curve corresponding to a utility level of U_0 by solving $XY = U_0$ to get $Y = U_0/X$.

Consider another illustration, this time with $U(X, Y) = (\frac{2}{3})X + 2Y$. The bundles of X and Y that yield a utility level of U_0 are again found by solving $U(X, Y) = U_0$ for Y. This time we get $Y = (U_0/2) - (\frac{1}{3})X$. The indifference curves corresponding to $U = 1$, $U = 2$, and $U = 3$ are shown in Figure A.3-4. Note that they are all linear, which tells us that this particular utility function describes a preference ordering in which X and Y are perfect substitutes.

Using Calculus to Maximize Utility

Students who have had calculus are able to solve the consumer's budget allocation problem without direct recourse to the geometry of indifference maps. Let $U(X, Y)$ be the consumer's utility function; and suppose M, P_X, and P_Y denote income, the price of X, and the price of Y, respectively. Formally, the consumer's allocation problem can be stated as follows:

$$\underset{X,\,Y}{\text{Maximize}}\ U(X, Y) \text{ subject to } P_X X + P_Y Y = M. \qquad \text{(A.3.3)}$$

The appearance of the terms X and Y below the "maximize" expression indicates that these are the variables the consumer must choose. The price and income values in the budget constraint are given in advance.

THE METHOD OF LAGRANGIAN MULTIPLIERS

As noted earlier, the function $U(X, Y)$ itself has no maximum; it simply keeps on increasing with increases in X or Y. The maximization problem defined in Equation A.3.3 is called a *constrained maximization problem*, which means we want to find the values of X and Y that produce the highest value of U *subject to the constraint that the consumer spend only as much as his income.* We will examine two different approaches to this problem.

One way of making sure that the budget constraint is satisfied is to use the so-called method of *Lagrangian multipliers*. In this method, we begin by transforming the constrained maximization problem in Equation A.3.3 into the following unconstrained maximization problem:

$$\underset{X,\,Y,\,\lambda}{\text{Maximize}}\ \pounds = U(X, Y) - \lambda(P_X X + P_Y Y - M). \tag{A.3.4}$$

The term λ is called a Lagrangian multiplier, and its role is to assure that the budget constraint is satisfied. (How it does this will become clear in a moment.) The first-order conditions for a maximum of £ are obtained by taking the first partial derivatives of £ with respect to X, Y, and λ and setting them equal to zero:

$$\frac{\partial \pounds}{\partial X} = \frac{\partial U}{\partial X} - \lambda P_X = 0, \tag{A.3.5}$$

$$\frac{\partial \pounds}{\partial Y} = \frac{\partial U}{\partial Y} - \lambda P_Y = 0, \tag{A.3.6}$$

and

$$\frac{\partial \pounds}{\partial \lambda} = M - P_X X - P_Y Y = 0. \tag{A.3.7}$$

The next step is to solve Equations A.3.5–A.3.7 for X, Y, and λ. The solutions for X and Y are the only ones we really care about here. The role of the equilibrium value of λ is to guarantee that the budget constraint is satisfied. Note in Equation A.3.7 that setting the first partial derivative of £ with respect to λ equal to zero guarantees this result.

Specific solutions for the utility-maximizing values of X and Y require a specific functional form for the utility function. We will work through an illustrative example in a moment. But first note that an interesting characteristic of the optimal X and Y values can be obtained by dividing Equation A.3.5 by Equation A.3.6 to get

$$\frac{\partial U/\partial X}{\partial U/\partial Y} = \frac{\lambda P_X}{\lambda P_Y} = \frac{P_X}{P_Y}. \tag{A.3.8}$$

Equation A.3.8 is the utility function analog to Equation 3.3 from the text, which says that the optimal values of X and Y must satisfy MRS $= P_X/P_Y$. The terms $\partial U/\partial X$ and $\partial U/\partial Y$ from Equation A.3.8 are called the *marginal utility of X* and the *marginal utility of Y*, respectively. In words, the marginal utility of a good is the extra utility obtained per additional unit of the good consumed. Equation

A.3.8 tells us that the ratio of these marginal utilities is simply the marginal rate of substitution of Y for X.

If we rearrange Equation A.3.8 in the following form,

$$\frac{\partial U/\partial X}{P_X} = \frac{\partial U/\partial Y}{P_Y}, \quad \text{(A.3.9)}$$

another interesting property of the optimal values of X and Y emerges. In words, the left-hand side of Equation A.3.9 may be interpreted as the extra utility gained from the last dollar spent on X. Similarly, the right-hand side of the equation is the extra utility gained from the last dollar spent on Y. It is easy to see intuitively why, for the optimal values of X and Y, the extra utility gained from the last dollar spent on each must be the same. Suppose, to the contrary, that the extra utility gained from the last dollar spent on Y exceeded the extra utility from the last dollar spent on X. The consumer could then spend a dollar less on X and a dollar more on Y and end up with more utility than he had under the original allocation. The conclusion is that the original allocation could not have been optimal. Only when the extra utility gained from the last dollar spent on each good is the same will it not be possible to carry out a similar utility-augmenting reallocation.

An Example. To illustrate the Lagrangian method, suppose that $U(X, Y) = XY$ and that $M = 40$, $P_X = 4$, and $P_Y = 2$. Our constrained maximization problem would then be written as

$$\underset{X,\,Y,\,\lambda}{\text{Maximize}}\ £ = XY - \lambda(4X + 2Y - 40). \quad \text{(A.3.10)}$$

The first-order conditions for a maximum of £ are given by

$$\frac{\partial £}{\partial X} = \frac{\partial(XY)}{\partial X} - 4\lambda = Y - 4\lambda = 0, \quad \text{(A.3.11)}$$

$$\frac{\partial £}{\partial Y} = \frac{\partial(XY)}{\partial Y} - 2\lambda = X - 2\lambda = 0, \quad \text{(A.3.12)}$$

and

$$\frac{\partial £}{\partial \lambda} = 40 - 4X - 2Y = 0. \quad \text{(A.3.13)}$$

Dividing Equation A.3.11 by Equation A.3.12 and solving for Y, we get $Y = 2X$;

substituting this result into Equation A.3.13 and solving for X, we get $X = 5$, which in turn yields $Y = 2X = 10$. Thus (5, 10) is the utility-maximizing bundle.[7]

AN ALTERNATIVE METHOD

There is an alternative way of making sure that the budget constraint is satisfied, one that involves less cumbersome notation than the Lagrangian approach. In this alternative method, we simply solve the budget constraint for Y in terms of X and substitute the result wherever Y appears in the utility function. Utility then becomes a function of X alone, and we can maximize it by taking its first derivative with respect to X and equating that to zero.[8] The value of X that solves that equation is the optimal value of X, which can then be substituted back into the budget constraint to find the optimal value of Y.

To illustrate, again suppose that $U(X, Y) = XY$, with $M = 40$, $P_X = 4$, and $P_Y = 2$. The budget constraint is then $4X + 2Y = 40$, which solves for $Y = 20 - 2X$. Substituting this expression back into the utility function, we have $U(X, Y) = X(20 - 2X) = 20X - 2X^2$. Taking the first derivative of U with respect to X and equating the result to zero, we have

$$\frac{dU}{dX} = 20 - 4X = 0, \qquad \text{(A.3.14)}$$

which solves for $X = 5$. Plugging this value of X back into the budget constraint, we discover that the optimal value of Y is 10. So the optimal bundle is again (5, 10), just as we found using the Lagrangian approach. For these optimal values of X and Y, the consumer will obtain $(5)(10) = 50$ units of utility.

Both algebraic approaches to the budget allocation problem yield precisely the same result as the graphical approach described in the text. Note in Figure A.3-5 that the $U = 50$ indifference curve is tangent to the budget constraint at the bundle (5, 10).

A SIMPLIFYING TECHNIQUE

Suppose our constrained maximization problem is of the general form

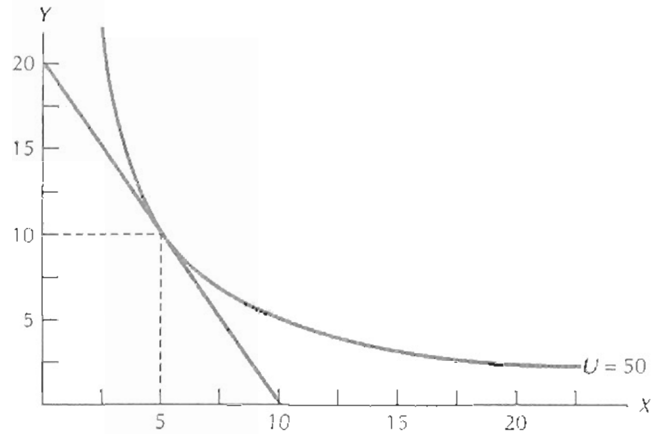
$$\underset{X,\,Y}{\text{Maximize}}\ U(X, Y) \text{ subject to } P_X X + P_Y Y = M. \qquad \text{(A.3.15)}$$

If (X^*, Y^*) is the optimum bundle for this maximization problem, then we know it will also be the optimum bundle for the utility function $V[U(X, Y)]$, where

[7] Assuming that the second-order conditions for a local maximum are also met.

[8] Here, the second-order condition for a local maximum is that $d^2U/dX^2 < 0$.

FIGURE A.3-5 THE OPTIMAL BUNDLE WHEN $U = XY$, $P_X = 4$, $P_Y = 2$, AND $M = 40$



V is any increasing function.[9] This property often enables us to transform a computationally difficult maximization problem into a simple one. By way of illustration, consider the following example:

$$\underset{X,\,Y}{\text{Maximize}}\ X^{1/3}Y^{2/3} \text{ subject to } 4X + 2Y = 24. \qquad \text{(A.3.16)}$$

First note what happens when we proceed with the untransformed utility function given in Equation A.3.16. Solving the budget constraint for $Y = 12 - 2X$ and substituting back into the utility function, we have $U = X^{1/3}(12 - 2X)^{2/3}$. Calculating dU/dX is a bit tedious in this case, but if we carry out each step carefully we get the following first-order condition:

$$\frac{dU}{dX} = \left(\tfrac{1}{3}\right)X^{-2/3}(12 - 2X)^{2/3} + X^{1/3}\left(\tfrac{2}{3}\right)(12 - 2X)^{-1/3}(-2) = 0, \qquad \text{(A.3.17)}$$

which, after a little more tedious rearrangement, solves for $X = 2$. And from the budget constraint we then get $Y = 8$.

Now suppose we transform the utility function by taking its logarithm:

$$V = \ln[U(X, Y)] = \ln(X^{1/3}Y^{2/3}) = \left(\tfrac{1}{3}\right)\ln X + \left(\tfrac{2}{3}\right)\ln Y. \qquad \text{(A.3.18)}$$

Since the logarithm is an increasing function, when we maximize V subject to

[9] Again, an increasing function is one for which $V(X_1) > V(X_2)$ whenever $X_1 > X_2$.

the budget constraint, we will get the same answer we got using U. The advantage of the logarithmic transformation here is that the derivative of V is much easier to calculate than the derivative of U. Again, solving the budget constraint for $Y = 12 - 2X$ and substituting the result into V, we have $V = (\frac{1}{3})\ln X + (\frac{2}{3})\ln(12 - 2X)$. This time the first-order condition follows almost without effort:

$$\frac{dV}{dX} = \frac{\frac{1}{3}}{X} - \frac{2(\frac{2}{3})}{12 - 2X} = 0, \qquad (A.3.19)$$

which solves easily for $X = 2$. Plugging $X = 2$ back into the budget constraint, we again get $Y = 8$.

The best transformation to make will naturally depend on the particular utility function you start with. The logarithmic transformation greatly simplified matters in the example above, but will not necessarily be helpful for other forms of U.

Questions for Review

1. If the prices of all products are rising at 20 percent per year, and your employer gives you a 20 percent salary increase, are you better off, worse off, or equally well off in comparison with your situation a year ago?
2. If you were president of a conservation organization, which rate structure would you prefer the Gigawatt Power Company to use: the one described in Example 3-1, or one in which all power sold for $0.08/kwh? (Assume that each rate structure would exactly cover the company's costs.)
3. True or false: The downward slope of indifference curves is a consequence of the diminishing marginal rate of substitution.
4. Construct an example of a preference ordering over Coke, Diet Coke, and Diet Pepsi that violates the transitivity assumption.
5. Explain in your own words how the slope of an indifference curve provides information about how much a consumer likes one good relative to another.
6. Explain why a consumer will often buy one bundle of goods even though he prefers another.
7. Why are corner solutions especially likely in the case of perfect substitutes?

Problems

1. The Acme Seed Company charges $2/lb for the first 10 lb you buy of marigold seeds each week and $1/lb for every pound you buy thereafter. If your income is $100/wk, draw your budget constraint for the composite good and marigold seeds.
2. Same as Problem 1, except now the price for every pound after 10 lb/wk is $4/lb.
3. Smith likes cashews better than almonds and likes almonds better than walnuts. He