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Incorporation of Spinal Flexibility Measurements Into Finite Element Analysis

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This technical note demonstrates two methods of incorporating the experimental stiffness of spinal motion segments into a finite element analysis of the spine. The first method is to incorporate the experimental data directly as a stiffness matrix. The second method approximates the experimental data as a beam element.

General Stiffness Matrix Representation

Several investigators [1, 4–7, 9, 11, 12, 14] have experimentally determined flexibility coefficients of human intervertebral motion segments. Generally, the center of the upper vertebral body was used as the reference point for the experimental axis system, and the lower vertebra was fixed. The experimental data form a 6×6 stiffness matrix for motion of the free (moving) end of the motion segment. The matrix relates the six degrees of freedom (three translations and three rotations) of the free vertebra to the forces and moments acting on it. These matrices make it possible to quantify the "coupling" within motion segments [16].

Panjabi et al. [7] published complete thoracic motion segment 6×6 flexibility matrices and the corresponding stiffness matrices obtained by inversion. There was a small difference in the flexibility matrices for the positive and negative directions of loading (flexion versus extension, etc.). The diagonal values in these matrices were compared with those in other published reports for both thoracic and lumbar motion segments [1, 4, 5, 6, 11]. The flexibilities agreed within an order of magnitude.

To incorporate these measurements into a finite element model requires the fixed end stiffness for the two vertebral centers and also the relative stiffness between the two vertebral centers. This results in a full 12×12 stiffness matrix relating the six degrees of freedom and the forces and moments at each vertebral center. A 12×12 stiffness matrix is generated from a 6×6 stiffness matrix by invoking equilibrium [12, 15]. In matrix notation, the equilibrium condition gives

$$K_{12 \times 12} = L_{12 \times 6} K_{6 \times 6} L^{T}{}_{6 \times 12}$$

where L depends on the distance between the ends of the motion segments and the particular coordinate system used. Using Panjabi et al. [7] coordinate system (*X*-left, *Y*-vertical, and *Z*-forwards) L is

$$L = \begin{bmatrix} \multicolumn{2}{c}{I} \\ \hline -I & 0 \\ \Lambda & -I \end{bmatrix}_{12 \times 6}$$

where I is a 6×6 identity submatrix, $-I$ is a 3×3 negative identity submatrix, and Λ is the following 3×3 submatrix

$$\Lambda = \begin{bmatrix} 0 & 0 & -\lambda \\ 0 & 0 & 0 \\ \lambda & 0 & 0 \end{bmatrix}$$

where λ is the distance between the vertebral centers.

The proper length λ to use in the transformation should be the actual distance between the centers of the vertebral bodies. Since the published flexibility and stiffness data were obtained with an immobilized lower vertebra, the position of the lower "node" is unknown. Varying the value of this distance λ changes some of the matrix elements. Using the assumption that the relative stiffness of the motion segment should be independent of which vertebra is "free," a length λ can be determined by minimizing the differences between the apparent stiffness of the two nodes. Two diagonal elements and four off-diagonal elements in the 6×6 matrix of the opposite node are dependent on λ. The difference between these elements and the corresponding elements of the experimentally determined 6×6 matrix was minimized by using least squares. For the Panjabi et al. [7] negative load stiffness matrix, this resulted in an effective length of 30.4 mm. This length is similar in magnitude to the distance between centers of adjacent thoracic vertebrae.

In actual motion segments, the effective length may not be constant because of the nonlinear stiffness and large displacements effects. The length is also approximate because of measurement errors and the experimental assumptions used to determine the stiffness values.

The Panjabi et al. [7] negative load stiffness matrix was selected to illustrate the calculation of a 12×12 matrix. With the corresponding length $\lambda = 30.4$ mm, the equilibrium condition yields the following 12×12 symmetrical stiffness matrix (units of N, mm, and rad)

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$$
\mathbf{K} = 10^4 \times \left[\begin{array}{cccccc|cccccc}
0.011 & 0 & 0 & 0 & 0.062 & 0.164 & -0.011 & 0 & 0 & 0 & -0.062 & 0.171 \\
0 & 0.124 & -0.002 & -0.073 & 0 & 0 & 0 & -0.124 & 0.002 & 0.134 & 0 & 0 \\
0 & -0.002 & 0.010 & -0.156 & 0 & 0 & 0 & 0.002 & -0.010 & -0.148 & 0 & 0 \\
0 & -0.073 & -0.156 & 18.600 & 0 & 0 & 0 & 0.073 & 0.156 & -13.85 & 0 & 0 \\
0.062 & 0 & 0 & 0 & 14.500 & 0.200 & -0.062 & 0 & 0 & 0 & -14.50 & 1.687 \\
0.164 & 0 & 0 & 0 & 0.200 & 19.700 & -0.164 & 0 & 0 & 0 & -0.200 & -14.71 \\
\hline
-0.011 & 0 & 0 & 0 & -0.062 & -0.164 & 0.011 & 0 & 0 & 0 & 0.062 & -0.171 \\
0 & -0.124 & 0.002 & 0.073 & 0 & 0 & 0 & 0.124 & -0.002 & -0.134 & 0 & 0 \\
0 & 0.002 & -0.010 & 0.156 & 0 & 0 & 0 & -0.002 & 0.010 & 0.148 & 0 & 0 \\
0 & 0.134 & -0.148 & -13.85 & 0 & 0 & 0 & -0.134 & 0.148 & 18.370 & 0 & 0 \\
-0.062 & 0 & 0 & 0 & -14.50 & -0.200 & 0.062 & 0 & 0 & 0 & 14.500 & -1.687 \\
0.171 & 0 & 0 & 0 & 1.687 & -14.71 & -0.171 & 0 & 0 & 0 & -1.687 & 19.909
\end{array}\right]
$$

The absolute magnitudes in the top left 6×6 are the Panjabi et al. [7] values. However, the absolute magnitudes in the lower right 6×6 (corresponding to performing the experiment upside-down) are somewhat different, despite the minimization of these differences. The sign differences in the off-diagonal terms in the submatrices are due to the axes convention. The values in the two submatrices would be identical if the motion segments were symmetric. A tapered beam is an example of a structure with different 6×6 submatrices. Its deflections and rotations are different at each end for the same magnitude of forces or moments.

In using experimental data in a finite element model, the model is only valid under the assumptions used to obtain the data. The motion segment stiffness is nonlinear. The reported linear stiffnesses is either from 1) the stiffness at a point on the curve (tangent stiffness), or 2) a line between two points on the curve (secant stiffness), or 3) the stiffness from a relatively linear portion of the curve. Several experimental variables can affect the reported stiffness. Preconditioning the motion segments [2, 7], testing with a preload [8, 11], and inclusion of large displacements effects [4] are examples. Assumptions about the form of the matrix also affect the reported results. Conservation of energy requires a symmetric matrix. The assumption of geometric symmetry in the motion segment about the sagittal plane results in additional zero terms in the matrix [7].

The experimental matrices only describe the flexibility (or stiffness) between the two vertebral bodies. The matrices do not include the geometric and anatomical detail of motion segments (disk, facet joints, ligaments, etc.). While the forces acting at the vertebral centers of the motion segment can be calculated, the forces and stresses in individual spinal components of the motion segment are not available. Using the forces acting on the motion segments, the corresponding stresses are obtainable from an additional model having the required geometric and anatomic detail.

Stokes and Laible [13] report on spinal modeling using Panjabi et al. [7] experimental data as stiffness matrices in a finite element model. The 6×6 experimental data was first transformed into a 12×12 stiffness matrix. The length used in the equilibrium transformation was determined by the lengths in the finite element model. Then, the 12×12 stiffness matrices were rotated by a matrix transformation from the stiffness matrix coordinate system to the global finite element coordinate system.

Equivalent Beam Model

An alternative way to represent the motion segment is by selecting a beam model that fits the experimental data. Although the equivalent beam may not be able to fit exactly all the terms in the experimental matrix, a close approximation is possible. A shear beam [10] can exactly match the diagonal terms of any experimental 6×6 stiffness matrix. The off-diagonals associated with beam bending may also closely approximate the corresponding experimental off-diagonals. Advantages of this method is that the stiffness matrix is no longer dependent on the element length (element length may differ from experimental length) and beam elements are easily incorporated into finite element analysis packages. The ten nonzero elements of the 6×6 matrix for one free node contain eight variables, but yield only six independent equations for a shear beam of fixed length. The variables of a shear beam representing experimental data can be calculated after arbitrarily choosing the modulus of elasticity and Poisson's ratio. A shear beam approximation to Panjabi et al. [7] negative load stiffness matrix using a length of 30.4 mm, a modulus of elasticity of $120 N/mm^2$, and a Poisson's ratio of 0.2, yields the following area properties: area $= 314.6 mm^2$, shear area along X-axis $= 70.47 mm^2$, shear area along Z-axis $= 63.92 mm^2$, inertia about X-axis $= 41{,}309 mm^4$, inertia about Z-axis $= 43{,}512 mm^4$, and torsional constant $= 88{,}286 mm^4$. The 6×6 portion of the 12×12 shear beam stiffness corresponding to the experimental matrix is shown below (units of N, mm, and rad)

$$
\mathbf{K}_{6\times 6} = 10^4 \times \begin{bmatrix}
0.011 & 0 & 0 & 0 & 0 & 0.167 \\
0 & 0.124 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.010 & -0.152 & 0 & 0 \\
0 & 0 & -0.152 & 18.600 & 0 & 0 \\
0 & 0 & 0 & 0 & 14.500 & 0 \\
0.167 & 0 & 0 & 0 & 0 & 19.700
\end{bmatrix}
$$

While the beam element will produce displacements corresponding to the experimental motion segment data, any stress output is unrelated to stresses generated in a motion segment.

A shear beam with offsets of the shear axis from the neutral axis [15] is a more general model which can approximate some of the other off-diagonal terms. The z offset is determined by using a parallel axes transformation (15) to displace the z axis to minimize the sum of squares of the off-diagonal terms. The assumption of bilateral symmetry about the sagittal plane precludes an offset along the x axis. Dimnet et al. [3] provide a method for performing this minimization. For the Panjabi et al. [7] negative load matrix, this resulted in an offset of $z = 0.8 mm$. Since the shear area and inertia are related, an iterative solution is required to obtain these properties. The necessary area properties to approximate Panjabi et al. [7] negative load matrix ($z_{offset} = 0.8 mm$) are the same as for the shear beam without offsets; except, shear area along Z-axis $= 63.93 mm^2$, inertia about X-axis $= 41{,}105 mm^4$, and torsional constant $= 88{,}242 mm^4$. Showing only the corresponding 6×6 yields (units of N, mm, and rad)

$$K_{6\times 6} = 10^4 \times \begin{bmatrix} 0.011 & 0 & 0 & 0 & 0.009 & 0.167 \\ 0 & 0.124 & 0 & -0.100 & 0 & 0 \\ 0 & 0 & 0.010 & -0.152 & 0 & 0 \\ 0 & -0.100 & -0.152 & 18.600 & 0 & 0 \\ 0.009 & 0 & 0 & 0 & 14.500 & 0.135 \\ 0.167 & 0 & 0 & 0 & 0.135 & 19.700 \end{bmatrix}$$

The matrix elements are negligibly different from those in the simple shear beam in this case because of the small offset ($z_{\text{offset}} = 0.8$mm). The small offset is probably within the experimental error for specifying the origin of axes. Other possible beam models include a curved beam model which has additional off-diagonal terms. Since the simpler shear beam fits the data reasonably well and some of the off-diagonal terms may be within experimental error [11], more general beam models were not pursued.

The stiffness matrices and equivalent shear beams were tested in finite element models of a spine under lateral bending moments. There was little difference between the behavior of the two representations. Apparently, this was because the effects of the initial geometry predominated over constitutive effects.

These methods provide a way to simplify a model formulation by using experimental data to provide the flexibility (or stiffness) properties of a model component, in this case the spinal motion segment.

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