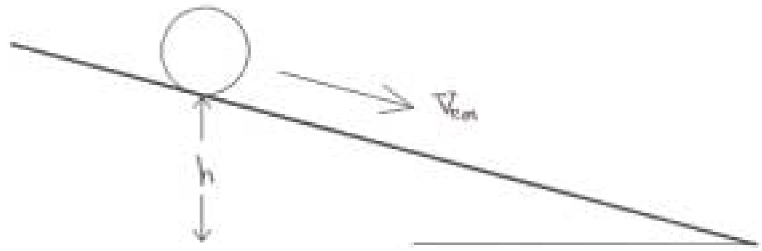
Rotational Inertia on the Inclined Plane



To determine which of the objects, the disk, the hollow cylinder, or the ball, reaches the end first, one can find the final velocity of each and conclude that since each object accelerates uniformly, the one with the highest final velocity must reach the end first.

The following considerations give the needed equations:

1) energy conservation:

$$PE_{top} = Mgh$$

$$KE_{bottom} = Mv^2_{cm}/2 + I\omega^2/2$$

2) geometry:

$$v_{cm} = \omega R$$

3) moment of inertia:

$$I_{disk} = MR^2/2 : I_{sphere} = 2MR^2/5 : I_{hol.\ cyl.} = MR^2$$

Consider first the disk. The energy expression at the bottom is:

$$Mgh = Mv^2_{cm}/2 + (1/2)(MR^2/2)(v_{cm}/R)^2$$

$$gh = v^2_{cm}/2 + v^2_{cm}/4$$

So

$$v_{cm} = gh/0.75$$

For the sphere:

$$v_{cm} = gh/0.70$$

And for the hollow cylinder:

$$v_{cm} = gh/1.0$$

The sphere reaches the bottom first followed closely by the disk and not so closely by the hollow cylinder. More energy is used in rotating the hollow cylinder than the others such that the energy left for translation is much smaller.

Note that the above expressions are void of any mass or radial dependence.

Ref. Halliday and Resnick, Fundamentals of Physics, Third Edition, John Wiley and Sons, New York, 1988, pp. 260-261.